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FUNCTIONAL GEOMETRY OF THE PUMPS WITH AXIAL PISTONS AND WITH ROTATING MOTION TRANSMITTING BY CONNECTING RODS

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Abstract: *In this paper, for the first time in a complete form, the functional geometry of the mechanism in case of pumps with axial pistons, with rotating motion transmitting by connecting rods is presented, taking into account the peculiarity of these types of hydraulic machines, and namely the rotary movement transmitting from the driving flange to the cylinder block, by means of a connecting rod called driving connecting rod. On the basis of some geometrical and working, structural-kinematical, respectively, arguments and of some calculations, the geometrical (linear and angular) sizes which characterize from functional (kinematical) point of view the analyzed mechanism are determined. These geometrical-functional considerations and determinations are useful for the study of the distribution process and realization of the distribution slots and for studying the accurate kinematics and dynamics of these hydraulic machines.*
Keywords: *axial piston pump, mechanism, functional geometry*

1. INTRODUTION

From the bibliographical analysis started in the paper [1], may be found the followings: the special literature widely used in Romania does not mention the mode of rotary motion transmitting by means of connecting rods from the driving shaft to the cylinder block in case of working analysis of the pumps with axial pistons, with rotating motion transmitting by connecting rods (PAPRMTCR), and solves the kinematics problem of these hydraulic machines, allowing the hypothesis of the infinite length of the connecting rods, which is in a whole non-agreement with reality; in the great majority of papers published abroad, studying "the accurate kinematics" of the PAPRMTCR, there is a contradiction between the approach mode of this kinematics and the considered synchronous character of the rotary motion transmitting from the driving shaft to the cylinder block; when a structural analysis of the axial piston pump mechanism with and without double cardan coupling between the driving shaft and the cylinder block is made, it is not made a principle difference between the two types of mechanisms regarding the rotary motion transmitting to the cylinder block; and so on.

In the papers [2] and [3] a structural and kinematical analysis of the axial piston pump mechanism, with inclined block, without double cardan coupling between the driving shaft and the cylinder block, taking into account its functional peculiarity, that is the cylinder block driving in the rotary motion by means of connecting rods, is presented. Each of the mechanism elements is studied from constructive and functional point of view and structural schemes equivalent to constructive schemes are drawn up, by identifying the unnecessary movement liberties. It is clearly shown that at a given moment can not exist than only a driving connecting rod, by its contact with the inner wall of the piston to which is coupled.

On the basis of these papers, the functional geometry of the PAPRMTCR mechanism may be approached.

2. FUNCTIONAL GEOMETRY OF THE MECHANISM

By using the representation shown in fig. 1, the structural scheme of the mechanism shown in fig. 2 is drawn. In fig. 3 the position of the mechanism elements in case of PAPRMTCR is presented.

To each of the kinematical elements (E_i), i=1,2,3,4, a system of Cartesian coordinates is attached, being right directed, $(\mathcal{R}_i) \equiv Q_i x_i y_i z_i$, i=1,2,3,4, with the axis unit vectors $\bar{i}_i$, $\bar{j}_i$ and $\bar{k}_i$, respectively, and a direct sense of rotation of the axis $Q_i x_i$ is chosen (see fig. 1, 2 and 3). The fixed system $(\mathcal{R}) \equiv OXYZ$ is considered as attached to the driving shaft (fig. 1), having origin in the spherical joint centre of the axle 5 (see fig. 2), axis *OZ* being oriented according to the driving shaft axis, with the positive sense inward the cylinder block of the hydraulic machine, axis *OX* oriented radially in the driving flange plan, so that the upright plan *OXZ* contains the inclined axis of the cylinder block $OZ' \equiv Q_4z_4$ (where $Q_4 \in OZ'$), which with axis *OZ* makes up angle γ. The unit vectors of the axis *OX*, *OY* and *OZ* are $\bar{i}$, $\bar{j}$ and $\bar{k}$, respectively. Element 1 (E_1) is the rigid block made up by the driving shaft and driving flange where the connecting rods are jointed. This rotates in a directly sense round axis (Δ_1) with angle φ_1 (see fig. 1 and 3). The axis system ($\mathcal{R}_1$) $\equiv Q_1x_1y_1z_1$ is chosen for (E_1) so that $Q_1 \equiv O$; $Q_1z_1 \equiv OZ$; axis Q_1x_1, radially oriented against the driving flange, passes through cardan joint centre of the connecting rod (see farther), that is $C_{(1.1).2} \equiv C_{2.(1.1)} \in Ox_1$. Element ($E_1$) turns round, therefore, in direct sense round axis (Δ_1) $\equiv$ *OZ* with the angle φ_1 (which is angle made up by axis Q_1x_1 with axis *OX* at a given moment).

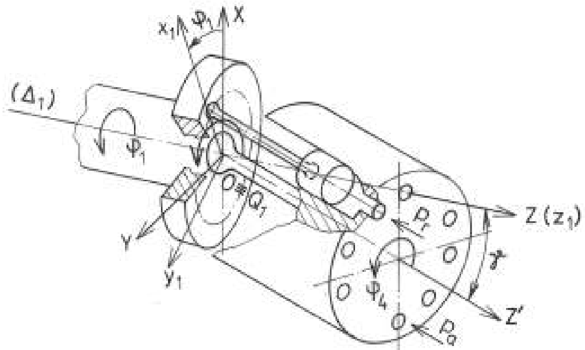

Figure 1: The kinematical element ensemble of the axial piston pump with inclined cylinder block, with motion transmitting by connecting rods: γ – tipple angle of the cylinder block; φ_1 – rotating angle of the driving shaft flange; φ_4 – rotating angle of the cylinder block; p_a – aspiration pressure; p_r – discharge pressure

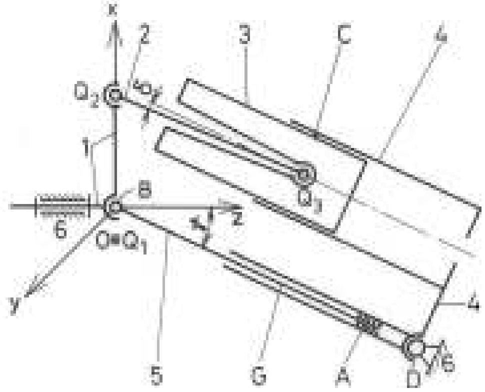

Figure 2: Structural schema of a mechanism with a piston, of an axial piston pump, with the motion transmitting by connecting rods: 1 – driving shaft and flange/crank; 2 – connecting rod; 3 – piston; 4 – cylinder/cylinder block; 5 – guiding axis; 6 – fixed element; L – bearing of the driving shaft; O ≡ Q_1 – rotating centre of the driving flange; Q_2 – the joint between the connecting rod and the driving flange; Q_3 – the joint between the connecting rod and the piston; C – the piston inside the cylinder; D – the spherical surface ensemble of the cylinder block and of the distribution plate; A – the pressing arch of the cylinder block on the distribution plate; G – the cylinder block guiding; B – the joint between the driving shaft/flange and the guiding axis

It is considered that between the driving flange (1) and the connecting rod (2) there is a passive binding having the type of a cardan joint (according to [4], [6], and [7]), where the driving fork is fixed on the driving flange in the way that the arm of this fork is radially oriented against the flange. The symmetry axis of this fork, situated at a distance R_f of the driving shaft axis (see fig. 3), is noted as ($\Delta_{1.1}$). The connecting rod is solidly to the driven fork. The connecting rod axis, which is also the symmetry axis of the driven fork, is noted as (Δ_2). The axes ($\Delta_{1.1}$) and (Δ_2) are concurrent, in the way that: $Q_{1.1} \equiv Q_2 \equiv C_{(1.1)2} \equiv C_{2(1.1)}$.

Axis $Q_{1.1}x_{1.1}$, which is the axis of the driving fork arm, coincides with axis Q_1x_1 ($Q_{1.1}x_{1.1} \equiv Q_1x_1$), having a radial direction against the driving flange, and axis $Q_{1.1}y_{1.1}$ has, as a following, the direction of the tangent to the circle designed by the cardan joint centre in the point which coincides with this centre, therefore, it is parallel to axis

Q_1y_1 (see fig. 3). The axis ($\Delta_{1.1}$), therefore $Q_{1.1}z_{1.1}$, turns (at the same time with the connecting rod joint with the flange, $Q_{1.1}$) round axis $OZ \equiv Q_1z_1$, in the way that it remains parallel with this, at a distance of R_f.

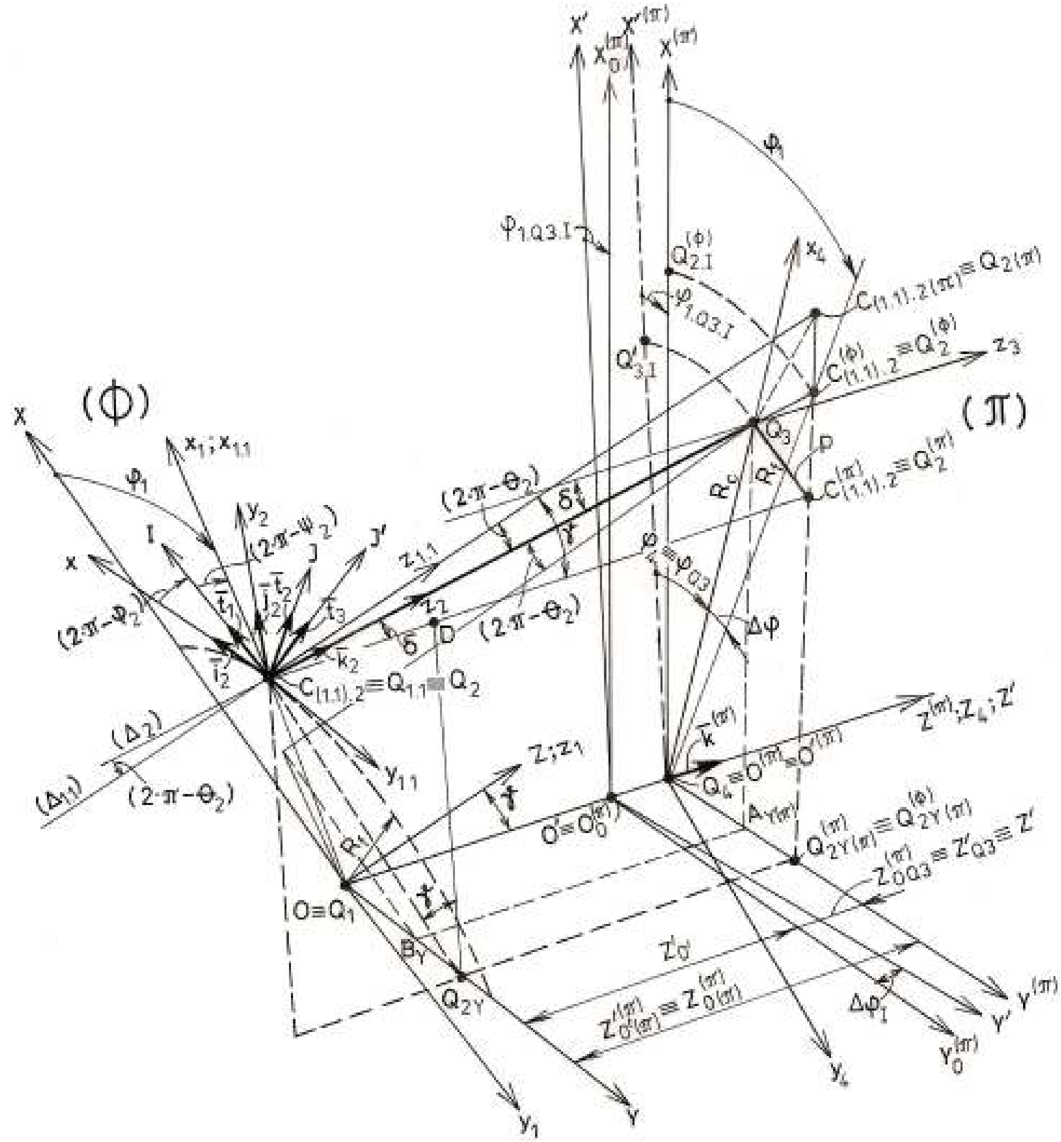

Figure 3: Positioning of the elements in case of PAPRMTCR mechanism

In this way, the position vector of the pole $Q_{1.1} \equiv Q_2$ has the following expression:

$$\bar{r}_{Q1.1} \equiv \bar{r}_{Q2} = R_f \cdot \left(\bar{i} \cdot \cos\varphi_1 + \bar{j} \cdot \sin\varphi_1\right), \tag{1}$$

and its speed will result at once

$$\bar{v}_{Q1.1} = \omega_1 \cdot R_f \cdot \bar{j}_{1.1}, \tag{2}$$

where ω_1 represents the angular speed of the driving shaft/flange

$$\omega_1 = \dot{\varphi}_1 \equiv \frac{\mathrm{d}\varphi_1}{\mathrm{d}t} \quad . \tag{3}$$

The angles φ_2, ψ_2 and θ_2 (see fig. 3) represent Euler's angles, namely: the proper rotating angle, precession angle and nutation angle, respectively. Among them there is the following relationship:

$$\mathrm{tg}\varphi_2 = \frac{\mathrm{ctg}\psi_2}{\cos\theta_2}. \tag{4}$$

For element 3 (piston) the system of rectangular axes $Q_3x_3y_3z_3$ is chosen in the following mode (see fig. 2 and 3): origin Q_3 is lying in the spherical joint centre; axis Q_3z_3 is oriented in accordance with the piston/cylinder axis; axis Q_3x_3 has a radial direction against the cylinder; axis Q_3y_3 is perpendicular on the plane ($x_3Q_3z_3$). As a result, the rectangular system $Q_3x_3y_3z_3$ will remove and turn round the axis Q_3z_3, at the same time with the piston, and also turns with the cylinder where it is lying, at the same time with the cylinder block. Further, it is not subjected to analyze the rotary movement of the piston round its own axis. It is interesting only the movement of the point Q_3, belonging both to connecting rod and piston, namely: the translation movement between the dead points (of the piston) and the rotary movement round the cylinder block axis, on a circle of radius R_c (being the radius of the circle of disposition of the cylinder centers from the cylinder block).

The dead points of the piston, therefore of the Q_3, are those points where the piston translation speed is zero. Among them the piston stroke is realized. The lower dead point (PMI) is the point representing the starting or lower position of the piston upsetting stroke. This point is also an inside one with regard to the pump. The upper dead point (PMS) is the point representing the maximal (upper) position of the piston outlet stroke. It is a point lying outside with regard to the pump. The starting of the intake stroke is lying, certainly, in PMS, and its end is lying in PMI.

For the element 4 (the cylinder block) it is taken in consideration the axis rectangular system $Q_4x_4y_4z_4$, oriented (according to fig. 3) as it follows: origin Q_4 is placed on the cylinder block axis in a point being the projection of the point Q_3 on this axis; axis Q_4z_4 coincides to the cylinder block axis (which makes with the axis OZ of the fixed system – the shaft axis – the tipple angle γ); axis Q_4x_4 has a radial direction against the cylinder block, passing diametrically through the considered cylinder (through point Q_3), therefore $Q_3 \in Q_4x_4$; axis Q_4y_4 has also a radial direction to the cylinder block and it is perpendicular on the plane ($Q_4x_4z_4$). Will result that the axes Q_4z_4 and Q_3z_3 are parallel. Distance between Q_3 and Q_4, that is the segment $|Q_3Q_4|$, lying on the axis Q_4x_4, is the radius of the circle of disposition of the cylinder centers:

$$|Q_3Q_4| = R_c . \tag{5}$$

Therefore, the axis system $Q_4x_4y_4z_4$ has both a rotary movement at the same time with the cylinder block round the axis Q_4z_4, and a translation one at the same time with the piston. The starting position of the axis Q_4x_4 is that one for which Q_3 it is found at PMI.

It is chosen the rectangular system ($\mathcal{R}^{(\Pi)}$) ≡ $O^{(\Pi)}X^{(\Pi)}Y^{(\Pi)}Z^{(\Pi)}$, with unit vectors ($\bar{i}^{(\Pi)}$, $\bar{j}^{(\Pi)}$, $\bar{k}^{(\Pi)}$) (see fig, 3), in the way that the plane ($X^{(\Pi)}O^{(\Pi)}Y^{(\Pi)}$) represents the projection of the plane (XOY) (of the fixed system $OXYZ$) on the transversal section plan of the cylinder block, noted as (Π), and $O^{(\Pi)}Z^{(\Pi)} \equiv Q_4z_4$. The axis system $O^{(\Pi)}X^{(\Pi)}Y^{(\Pi)}Z^{(\Pi)}$, having the origin $O^{(\Pi)} \equiv Q_4$, makes only a translation movement, having in view the movement of the spherical joint between the connecting rod and the piston (and does not the cylinder/piston rotation which is developed at the same time with the cylinder block rotation). Then, axis $O^{(\Pi)}Z^{(\Pi)}$ coincides to Q_4z_4, as a result it is parallel to axis Q_3z_3. Will result that the plane ($X^{(\Pi)}O^{(\Pi)}Y^{(\Pi)}$) is superposed on the plan ($x_4Q_4y_4$), the axes Q_4x_4 and Q_4y_4 carrying out a rotary movement, having the same angle against $O^{(\Pi)}X^{(\Pi)}$ and $O^{(\Pi)}Y^{(\Pi)}$, respectively. The starting position of the system $O^{(\Pi)}X^{(\Pi)}Y^{(\Pi)}Z^{(\Pi)}$, that for which the rotation angle of the driving flange is zero ($\varphi_1 = 0$) (the considered piston is not at PMI), is $O_0^{(\Pi)}X_0^{(\Pi)}Y_0^{(\Pi)}Z_0^{(\Pi)}$.

It is taken into consideration the fixed system $O'X'Y'Z'$ (see fig. 3), with $O'Z' \equiv O_0^{(\Pi)}Z^{(\Pi)}$ ($O_0^{(\Pi)} \equiv O'$), with axis $O'X'$ on which is lying Q_3, at PMI, noted with Q'_{3I}, and axis $O'Y'$ perpendicular to the plane ($X'O'Z'$). It is suggested that Q_3 at PMI, therefore Q'_{3I}, is not lying on the axis $O_0^{(\Pi)}X_0^{(\Pi)}$, contrary to the usually considered case, by using the elementary kinematics theory, but on axis $O'X'$, which is lagged with angle $\varphi_{1.Q3.I}$ against the axis $O_0^{(\Pi)}X_0^{(\Pi)}$. Therefore:

$$Q'_{3I} \in O'X' \text{ and } \varphi_{1Q3I} = \angle\left(O_0^{(\Pi)}X_0^{(\Pi)}, O'X'\right) = \angle\left(O_0^{(\Pi)}Y_0^{(\Pi)}, O'Y'\right).$$

Observation: When the movement of the pump mechanism is subjected to an analysis, by using an opening made in a carcass, may be found out that there is a lagging between the rotary movement of the driving shaft/flange and of the cylinder block. It follow to demonstrate that the allowed hypothesis is true and to determine this lagging.

Accordingly to all that has been mentioned above, it means that the starting position of the axis system $Q_4x_4y_4z_4$ (of the cylinder block) coincides to the fixed system $O'X'Y'Z'$.

Let be the plane ($X'O'Y'$) projection on the plan (Π), noted by ($X'^{(\Pi)}O'^{(\Pi)}Y'^{(\Pi)}$) (see fig.3). In this way, the plane ($X'^{(\Pi)}O'^{(\Pi)}Y'^{(\Pi)}$) is superposed on the plane ($X^{(\Pi)}O^{(\Pi)}Y^{(\Pi)}$), but the axes $O'^{(\Pi)}X'^{(\Pi)}$ and $O'^{(\Pi)}Y'^{(\Pi)}$ are lagged by angle $\varphi_{1.Q3.I}$ against the axes $O^{(\Pi)}X^{(\Pi)}$ and $O^{(\Pi)}Y^{(\Pi)}$, respectively, in the way the axes $O'X'$ and $O'Y'$ are lagged against the axes $O_0^{(\Pi)}X_0^{(\Pi)}$ and $O_0^{(\Pi)}Y_0^{(\Pi)}$, respectively. The point Q'_{3I} it is found on the axis $O'^{(\Pi)}X'^{(\Pi)}$. Therefore:

$$Q'_{3I} \in O'^{(\Pi)}X'^{(\Pi)} \text{ and } \varphi_{1Q3I} = \angle\left(O^{(\Pi)}X^{(\Pi)}, O'^{(\Pi)}X'^{(\Pi)}\right) = \angle\left(O^{(\Pi)}Y^{(\Pi)}, O'^{(\Pi)}Y'^{(\Pi)}\right).$$

The perpendicular in Q_2 on the plane (XOY) (direction of axis $Q_2z_2 \equiv (\Delta_{1.1})$), which makes an angle ($2\cdot\pi-\theta_2$) with the connecting rod axis direction $Q_2Q_3 \equiv (\Delta_2)$, stings the plane ($X^{(\Pi)}O^{(\Pi)}Y^{(\Pi)}$) in the point $Q_{2(\Pi)}$ (accordingly to fig. 3). It will result that $Q_2Q_{2(\Pi)}$ is parallel to axis OZ: $Q_2Q_{2(\Pi)} \parallel OZ$.

The point Q_2 is projected on the plane ($X^{(\Pi)}O^{(\Pi)}Y^{(\Pi)}$), getting the point $Q_2^{(\Pi)}$ (see fig. 3). It results that the segment $\left|Q_2Q_2^{(\Pi)}\right|$ is parallel to axis $O(O^{(\Pi)})Z^{(\Pi)}$: $\left|Q_2Q_2^{(\Pi)}\right| \| OZ^{(\Pi)}$.

As the axes OZ and $O(O^{(\Pi)})Z^{(\Pi)}$ are found in the plane ($XOO^{(\Pi)}X^{(\Pi)}$),, will result that the plane $\left(Q_2Q_2^{(\Pi)}Q_{2(\Pi)}\right)$ is parallel to ($XOO^{(\Pi)}X^{(\Pi)}$) and because the points $Q_{2(\Pi)}$ and $Q_2^{(\Pi)}$ are found in plane ($X^{(\Pi)}O^{(\Pi)}Y^{(\Pi)}$), it means that direction given by $\left|Q_{2(\Pi)}Q_2^{(\Pi)}\right|$ is parallel to axis $O^{(\Pi)}X^{(\Pi)}$ and farther perpendicular on axis $O^{(\Pi)}Y^{(\Pi)}$: $\left|Q_{2(\Pi)}Q_2^{(\Pi)}\right| \perp O^{(\Pi)}Y^{(\Pi)}$.

As the axes OZ and $O(O^{(\Pi)})Z^{(\Pi)}$ built up among them the angle γ, will result that the same angle it is also found among the directions $Q_2Q_{2(\Pi)}$ and $Q_2Q_2^{(\Pi)}$ (see fig. 3): $\gamma = \angle\left(Q_2Q_{2(\Pi)}, Q_2Q_2^{(\Pi)}\right)$.

The position of Q_2 (of the connecting rod joint with the driving flange) (in plane $(\Phi) \equiv XOY$) is determined, at a given moment, by the angle φ_1 against the starting position lying on the axis OX and is expressed by the relationship (1). It is remade this position, given by (1), in plane ($X^{(\Pi)}O^{(\Pi)}Y^{(\Pi)}$), noted by $Q_2^{(\Phi)}$. As the axes OY and $O^{(\Pi)}Y^{(\Pi)}$ are parallel, each of them being perpendicular on the axes OX and $O^{(\Pi)}X^{(\Pi)}$, respectively, building up a plane, will result that they are found on the same plane, and namely, the plane ($O(O^{(\Pi)})Y^{(\Pi)}Z^{(\Pi)}$). It results that the plane $\left(Q_2Q_2^{(\Pi)}Q_{2(\Pi)}\right)$, which is parallel to ($XOO^{(\Pi)}X^{(\Pi)}$) and as a result perpendicular on the plane ($YOO^{(\Pi)}Y^{(\Pi)}$), is situated at a distance (see fig. 3):

$$\left|OQ_{2Y}\right| = \left|O^{(\Pi)}Q_{2Y(\Pi)}^{(\Pi)}\right| \equiv \left|O^{(\Pi)}Q_{2Y(\Pi)}^{(\Phi)}\right| = Y_{Q2} = R_f \cdot \sin\varphi_1 \qquad (6)$$

against ($XOO^{(\Pi)}X^{(\Pi)}$).

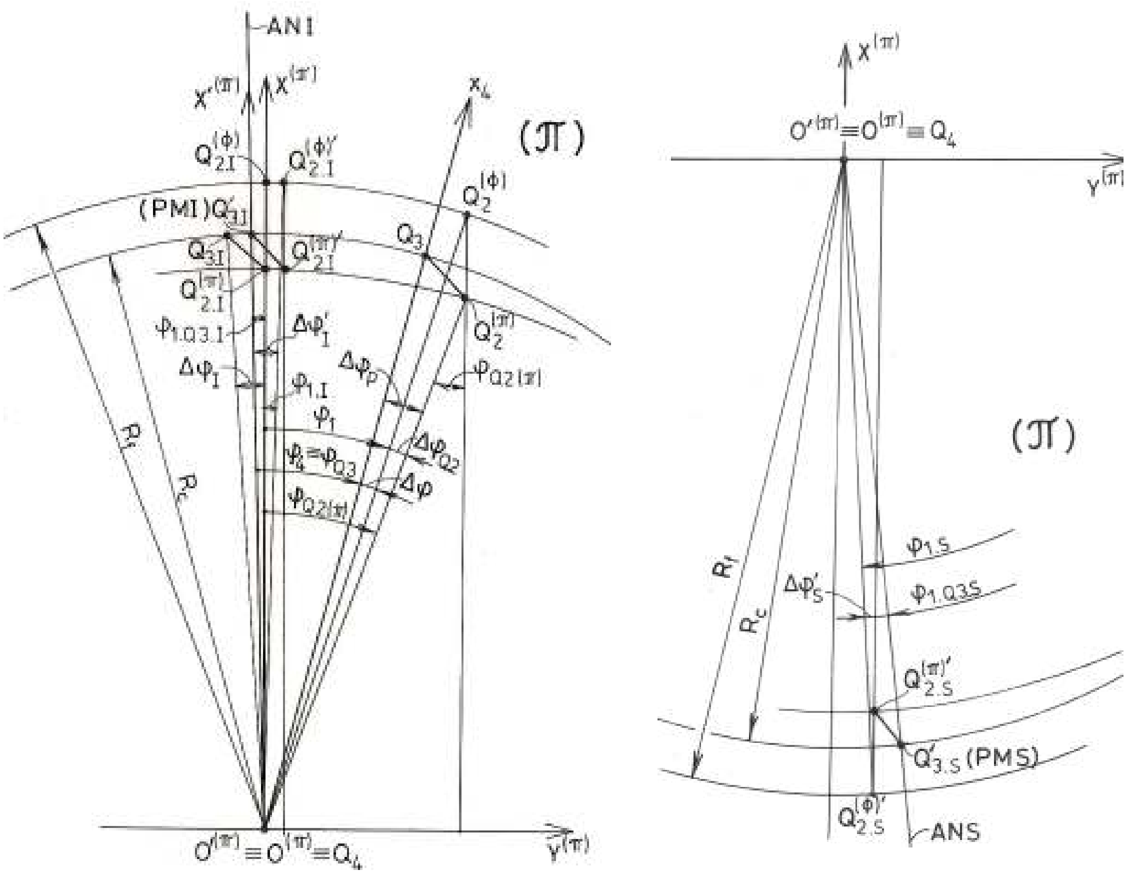

Figure 4: Representation of the connecting rod joint with the driving flange ($Q_2^{(\Phi)}$ and $Q_2^{(\Pi)}$) and with the piston (Q_3) in the plan (Π), of the connecting rod projection on the same plan ($|Q_3\,Q_2^{(\Pi)}|$), of the driving flange rotation angle (φ_1), of the cylinder block rotation angle ($\varphi_4 \equiv \varphi_{Q3}$), of the lag angle between the cylinder block and the driving flange ($\Delta\varphi$), of the angles ($\varphi_{1.Q3.I}$ and $\varphi_{1.Q3.S}$) locating the lower neutral axis (ANI) and the upper ones (AMS), a.s.o.

As the axes OY and $O^{(\Pi)}Y^{(\Pi)}$ are found on the same plane, $(O(O^{(\Pi)})Y^{(\Pi)}Z^{(\Pi)})$, being parallel, and because the projection of the point Q_2 on the axis OY, noted by Q_{2Y}, has the same quota, Y_{Q2}, like the projection of point $Q_2^{(\Phi)}$ on the axis $O^{(\Pi)}Y^{(\Pi)}$, noted with $Q_{2Y(\Pi)}^{(\Phi)}$, will result that the point $Q_2^{(\Phi)}$ is found on the direction of the segment $\left|Q_{2(\Pi)}Q_2^{(\Pi)}\right|$, that is the points $Q_{2(\Pi)}$, $Q_2^{(\Phi)}$ and $Q_2^{(\Pi)}$ are co-linearly, their direction being perpendicular on axis $O^{(\Pi)}Y^{(\Pi)}$ (accordingly to fig. 3).

May be considered a some position of the rotating mechanism, so that the driving shaft/flange has been in rotation state with angle φ_1,

$$\varphi_1 = \angle(Q_1x_1, OX); \tag{7}$$

therefore also the connecting rod and flange joint, that is the point Q_2, being on the axis Q_1x_1, has been in rotation with the same angle with regard to the shaft axis. This means that in the plane (Π) it is found again the angle φ_1 between the right line $O^{(\Pi)}\ Q_2^{(\Phi)}$ and the axis $O^{(\Pi)}X^{(\Pi)}$ (see fig. 3 and 4). In this time, the cylinder block, that is also point Q_3, the joint of the connecting rod with the piston, has been in rotation with the angle

$$\varphi_4 \equiv \varphi_{Q3} = \angle\left(O^{(\Pi)}Q_3, O^{(\Pi)}X^{(\Pi)}\right). \tag{8}$$

If the starting lagging between $Q_2^{(\Phi)}$ and Q_3 in comparison with the cylinder block rotation $Q_4z_4 \equiv O^{(\Pi)}Z^{(\Pi)}$ is $\Delta\varphi'_I$ (accordingly to fig. 4):

$$\Delta\varphi'_I = \angle\left(O^{(\Pi)}Q_{2I}^{(\Phi)'}, O'^{(\Pi)}Q'_{3I}\right), \tag{9}$$

in the new situation the lagging is $\Delta\varphi$:

$$\Delta\varphi = \angle\left(O^{(\Pi)}Q_2^{(\Phi)}, Q_4(O^{(\Pi)})x_4\right) \equiv \angle\left(O^{(\Pi)}Q_2^{(\Phi)}, Q_4\left(O^{(\Pi)}\right)Q_3\right). \tag{10}$$

May be noted (as the computations will shown) that Q_3 is found at PMI ($Q'_{3I} \in O'^{(\Pi)}X'^{(\Pi)}$) after the driving flange has been in rotation with angle $\varphi_{1.I}$.

Taking into account all the above mentioned situations, the following relationship between the rotation angles of the cylinder block and of the driving shaft/flange may be written as:

$$\varphi_1 - \Delta\varphi = \varphi_{Q3} - \left(\Delta\varphi'_I - \varphi_{1I}\right). \tag{11}$$

Point Q_3 is in rotation round the cylinder block axis, on the circle of radius R_c, having the position vector (in the plan (Π)) expressed as:

$$\bar{r}_{Q3}^{(\Pi)} = R_c \cdot \left[\bar{i}^{(\Pi)} \cdot \cos(\varphi_1 - \Delta\varphi) + \bar{j}^{(\Pi)} \cdot \sin(\varphi_1 - \Delta\varphi)\right]. \tag{12}$$

Point $Q_2^{(\Phi)}$ is moved on the circle of radius R_f, in the plane (Π), having the coordinates:

$$X_{Q2(\Phi)}^{(\Pi)} = R_f \cdot \cos\varphi_1;\ Y_{Q2(\Phi)}^{(\Pi)} = R_f \cdot \sin\varphi_1. \tag{13}$$

Point $Q_2^{(\Pi)}$ is moved on an ellipse with the little semi-axis, having a length of $R_f\cos\gamma$, situated on the axis $O^{(\Pi)}X^{(\Pi)}$, and the large semi-axis, having the length R_f, found on the axis $O^{(\Pi)}Y^{(\Pi)}$. The equation of this ellipse is the following:

$$\frac{\left(Y_{Q2(\Pi)}^{(\Pi)}\right)^2}{R_f^2} + \frac{\left(X_{Q2(\Pi)}^{(\Pi)}\right)^2}{R_f \cdot \cos^2\gamma} = 1, \tag{14}$$

where

$$Y_{Q2(\Pi)}^{(\Pi)} \equiv Q_{2Y(\Pi)}^{(\Pi)} = Y_{Q2(\Phi)}^{(\Pi)} \equiv Q_{2Y(\Pi)}^{(\Phi)} = R_f \cdot \sin\varphi_1. \tag{15}$$

From the two equations will result the other coordinate, accordingly to the axis $O^{(\Pi)}X^{(\Pi)}$, of the point $Q_2^{(\Pi)}$, as:

$$X_{Q2(\Pi)}^{(\Pi)} \equiv Q_{2X(\Pi)}^{(\Pi)} = R_f \cdot \cos\gamma \cdot \cos\varphi_1. \tag{16}$$

The following angles are defined in the shape:

• $\varphi_{Q2(\Pi)}$ is the angle built up by the direction of the right line $O^{(\Pi)}\ Q_2^{(\Pi)}$ with the direction of the axis $O^{(\Pi)}X^{(\Pi)}$,

$$\varphi_{Q2(\Pi)} = \angle\left(O^{(\Pi)}Q_2^{(\Pi)}, O^{(\Pi)}X^{(\Pi)}\right); \tag{17}$$

• $\Delta\varphi_{Q2}$ – the angle between the right line $O^{(\Pi)}\ Q_2^{(\Phi)}$ and the direction of the line $O^{(\Pi)}\ Q_2^{(\Pi)}$, opposed to the segment $|Q_2^{(\Phi)}\ Q_2^{(\Pi)}|$,

$$\Delta\varphi_{Q2} = \angle\left(O^{(\Pi)}Q_2^{(\Phi)}, O^{(\Pi)}Q_2^{(\Pi)}\right); \tag{18}$$

• $\Delta\varphi_p$ – the angle built up by the right lines $O^{(\Pi)}Q_3$ şi $O^{(\Pi)}\ Q_2^{(\Pi)}$, opposed to the segment $|Q_3\ Q_2^{(\Pi)}|$, called as the angle characteristic to the connecting rod projection on the plan (Π),

$$\Delta\varphi_p = \angle\left(O^{(\Pi)}Q_3, O^{(\Pi)}Q_2^{(\Pi)}\right). \tag{19}$$

By using the first relationship from (13) and the relationship (16), the length of the segment $|Q_2^{(\Phi)}\ Q_2^{(\Pi)}|$ is determined:

$$\left|Q_2^{(\Phi)}Q_2^{(\Pi)}\right| = R_f \cdot (1-\cos\gamma)\cdot\cos\varphi_1 . \tag{20}$$

The segment $|Q_3\,Q_2^{(\Pi)}|$ represents the connecting rod projection on the plan (Π) and has the length determined by means of the relationships (12), (15) and (16):

$$p \equiv \left|Q_3 Q_2^{(\Pi)}\right| = \sqrt{\left[R_c\cdot\cos(\varphi_1-\Delta\varphi) - R_f\cdot\cos\gamma\cdot\cos\varphi_1\right]^2 + \left[R_c\cdot\sin(\varphi_1-\Delta\varphi) - R_f\cdot\sin\varphi_1\right]^2} . \tag{21}$$

Applying the generalized Pitagora's theorem in $\Delta O^{(\Pi)}Q_3\,Q_2^{(\Pi)}$, by means of the above relationships, will result the characteristic angle of the connecting rod projection on the plan (Π) ($\Delta\varphi_p$):

$$\cos\Delta\varphi_p = \frac{\cos\gamma\cdot\cos\varphi_1\cdot\cos(\varphi_1-\Delta\varphi)+\sin\varphi_1\cdot\sin(\varphi_1-\Delta\varphi)}{\sqrt{\cos^2\gamma\cdot\cos^2\varphi_1+\sin^2\varphi_1}} . \tag{22}$$

By using the same theorem, but for $\Delta O^{(\Pi)}\ Q_2^{(\Phi)}\ Q_2^{(\Pi)}$, the expression of the angle $\Delta\varphi_{Q2}$ is obtained:

$$\cos\Delta\varphi_{Q2} = \frac{\cos\gamma\cdot\cos^2\varphi_1+\sin^2\varphi_1}{\sqrt{\cos^2\gamma\cdot\cos^2\varphi_1+\sin^2\varphi_1}} . \tag{23}$$

By expressing $\mathrm{tg}\varphi_{Q2(\Pi)}$ in $\Delta O^{(\Pi)}\ Q_{2Y(\Pi)}^{(\Pi)}\ Q_2^{(\Pi)}$ and $\mathrm{tg}\varphi_1$ in $\Delta O^{(\Pi)}\ Q_{2Y(\Pi)}^{(\Pi)}\ Q_2^{(\Phi)}$ and then by using the expressions of the respective segment lengths, on the basis of the above stated relationships, the angle $\varphi_{Q2(\Pi)}$ is determined:

$$\varphi_{Q2(\Pi)} = \mathrm{arctg}\frac{\mathrm{tg}\varphi_1}{\cos\gamma} . \tag{24}$$

On the basis of this relationship, where $\varphi_{Q2(\Pi)}$ is substituted by the evident expression (see fig. 4):

$$\varphi_{Q2(\Pi)} = \varphi_1 - \Delta\varphi_{Q2} , \tag{25}$$

an other formula for the angle $\Delta\varphi_{Q2}$ determination is obtained:

$$\Delta\varphi_{Q2} = \mathrm{arctg}\frac{(\cos\gamma\text{-}1)\cdot\sin2\varphi_1}{2\cdot\left(\cos\gamma\cdot\cos^2\varphi_1+\sin^2\varphi_1\right)} . \tag{26}$$

By means of the segment $|Q_2Q_3|$ the connecting rod in a some position is presented. This position is determined by the driving flange rotation with angle φ_1 and, at the same time, by the cylinder block rotation with angle φ_{Q3}. By l the connecting rod length is noted:

$$l = \left|Q_2Q_3\right| . \tag{27}$$

As the segment $|Q_2Q_3|$ is oriented accordingly to axis Q_2z_2, by unit vector $\bar{k}_2$, the vector $\overline{Q_2Q_3}$ is written in the way:

$$\overline{Q_2Q_3} = l\cdot\bar{k}_2 , \tag{28}$$

where $\bar{k}_2$ has an expression in accordance with direction cosines of the connecting rod axis against the fixed system ($\mathcal{R}$) ≡ $OXYZ$, that is

$$\bar{k}_2 = \bar{i}\cdot\cos\alpha_2 + \bar{j}\cdot\cos\beta_2 + \bar{k}\cdot\cos\gamma_2 . \tag{29}$$

As the right line $Q_2Q_2^{(\Pi)}$ is perpendicular on the plan ($X'^{(\Pi)}O'^{(\Pi)}Y'^{(\Pi)}$), in the way it has been shown above, it means that it is parallel with the cylinder axis, with the piston axis, respectively, whose movement is interesting, that is $Q_2Q_2^{(\Pi)} \parallel O^{[\Pi]}Z^{(\Pi)}$, the length of the segment $|Q_2Q_2^{(\Pi)}|$ representing the distance of the point Q_2 (of the connecting rod joint with the driving flange) against the mentioned plan. As a result, from the rectangular triangle $Q_2Q_3Q_2^{(\Pi)}$, the following relationships are obtained:

$$\left|Q_2Q_2^{(\Pi)}\right| = \sqrt{l^2-p^2}\ ; \left|Q_2Q_2^{(\Pi)}\right| = l\cdot\cos\delta , \tag{30}$$

from which will result the expression

$$\cos\delta = \frac{1}{l}\cdot\sqrt{l^2-p^2} , \tag{31}$$

by which the connecting rod inclination angle (δ) is determined against the piston/cylinder axis.
For $\delta = \delta_M$ (in the case when the connecting rod comes in contact with the piston cup) p_M accordingly to the relationship (31) is obtained.

3. CONCLUSIONS

In this paper, for the first time in a complete form, the functional geometry of the PAPRMTCR mechanism is presented, taking into account the peculiarity of these types of hydraulic machines, and namely the rotary movement transmitting from the driving flange to the cylinder block, by means of a connecting rod called driving connecting rod. By using Cartesian axis systems, rightly oriented for kinematical elements, on the basis of some geometrical and working, structural-kinematical, respectively, arguments and of some calculations, the geometrical (linear and angular) sizes which characterize from functional (kinematical) point of view the analyzed mechanism are determined. From the considerations and determinations included in the second part of the paper the following conclusions may be drawn.

● There is a lagging between the rotary movements of the cylinder block and the driving flange/shaft, the cylinder block remaining behind the shaft; so after when the pump driving shaft has rotated with the angle $\varphi_{1.I}$, the piston reaches at the lower dead point (PMI) (Q_{3I}').

● The piston dead points (PMI and PMS) may not be found in the positions occupied by the pistons at $\varphi_1 \in \{0°, 180°\}$.

● These positions are given by the angle $\varphi_{1.Q3.I}$, for PMI, and angle $\varphi_{1.Q3.S}$, for PMS, accordingly to expressions (see fig. 4):

$$\varphi_{1.Q3.I} = \varphi_{1.I} - \Delta\varphi_I' \;;\; \varphi_{1.Q3.S} = \varphi_{1.S} - \Delta\varphi_S' \,, \tag{32}$$

respectively by the angles $\varphi_{Q3.I}$, for PMI, and $\varphi_{Q3.S}$, for PMS, accordingly to formula (see fig. 4):

$$\varphi_{Q3} = \varphi_1 - \Delta\varphi - \left(\varphi_{1.I} - \Delta\varphi_I'\right), \tag{33}$$

from which will result

$$\varphi_{Q3.I} = 0° \;;\; \varphi_{Q3.S} = \varphi_{1.Q3.S} - \varphi_{1.Q3.I} \,. \tag{34}$$

● In this way, there are two neutral axes of the distribution plate (see fig. 4): the lower neutral axis (ANI) and the upper ones (AMS). ANI represents the axis from the distributing plate plane which passes through the points of projection on this plan of the cylinder block axis ($Q_4^{(\Pi)}$) and of the cylinder axis ($Q_{3.I}'$), in the moments when the pistons are found, successively, at PMI. ANI coincides to the axis $O'^{(\Pi)}X'^{(\Pi)}$ (ANI ≡ $O'^{(\Pi)}X'^{(\Pi)}$) and is determined by the angle $\varphi_{1.Q3.I}$ against $O^{(\Pi)}X^{(\Pi)}$. ANS is the axis from the same distributing plate plane which passes through the points of projection on this plane of the cylinder block axis ($Q_4^{(\Pi)}$) and of the cylinder axis ($Q_{3.S}'$), in the moments when the pistons are found, successively, at PMS. ANS is, at its turn, determined by the angle $\varphi_{1.Q3.S}$ against the same axis, $O^{(\Pi)}X^{(\Pi)}$. The positions of the axes ANI and ANS are dependent on the tipple angle of the cylinder block (γ).

● As $\varphi_{Q3.S}$ is the cylinder rotation angle corresponding to the piston outlet stroke,

$$\varphi_{Q3.S} = \varphi_{Q3}^{(r)} \equiv \varphi_r \,, \tag{35}$$

and $\varphi_{Q3}^{(a)}$ is the angle corresponding to the same piston intake stroke,

$$\varphi_{Q3}^{(a)} = 2 \cdot \pi - \varphi_{Q3}^{(r)} \equiv \varphi_a \,, \tag{36}$$

accordingly to the above mentioned, will result that, generally, there are the following inequality:

$$\varphi_r < \varphi_a \,, \tag{37}$$

the values of those angles being changed with variation of the angle γ.

● It results that PAPRMTCR will present a kinematical asymmetry (KAs), characterized by the coefficient of kinematical asymmetry, noted with k_{KAs} and defined by the following relationship:

$$k_{KAs} = \frac{\varphi_r}{\varphi_a} \,. \tag{38}$$

It is found that, generally, the value of this coefficient is sub-unitary.

● The connecting rod inclination angle (δ) against the piston/cylinder axis is dependent on the driving flange rotation angle and depends also on the tipple angle (γ).

● These geometrical-functional considerations and determinations of the analyzed mechanism are useful for the study of the distribution process and realization of the distribution slots and for studying the accurate kinematics and dynamics of the PAPRMTCR, which will be treated in the frame of some future papers.

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