The 4th International Conference
"Computational Mechanics
and Virtual Engineering"
COMEC 2011
20-22 OCTOBER 2011, Brasov, Romania

ON THE ACOUSTIC CLOAKING

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Abstract: *In this paper, the sound invisibility performance is discussed for spherical cloaks. The original domain consists of an alternation of concentric layers made from piezoelectric ceramics and epoxy resin, following a triadic Cantor sequence. The spatial compression, obtained by applying the geometric transformation, leads to an equivalent domain with an inhomogeneous and anisotropic distribution of the material parameters*
Keywords: *Transformation acoustics, Cantor sequence, Acoustic cloaking.*

1. INTRODUCTION

Substantial recent progress has been made in the application of the geometric transformations is designing of metamaterials, which are subwavelength scale materials having anisotropic and heterogeneous parameters. A finite size object surrounded by a coating consisting of a specially designed metamaterial would become invisible for electromagnetic waves at any frequency [1], [2]. In acoustics, the idea of the invisibility cloak is that the sound sees the space differently [3]. For the sound, the concept of distance is modified by the acoustic properties of the regions through which the sound travels. In geometrical acoustics, we are used to the idea of the acoustical path; when travelling an infinitesimal distance $\mathrm{d}s$, the corresponding acoustical path length is $c^{-1}\mathrm{d}s$, where $c^{-1}=\sqrt{\rho/\kappa}$ with ρ is the fluid density and κ is the compression modulus of the fluid [4]. The 3D acoustic equation for the pressure waves propagating in a bounded fluid region $\Omega \subset \mathrm{R}^3$ is

$$\nabla\cdot(\underline{\underline{\rho}}^{-1}\nabla p)+\frac{\omega^2}{\kappa}p=0\,, \tag{1}$$

where p is the pressure, $\underline{\underline{\rho}}$ is the rank-2 tensor of the fluid density, κ is the compression modulus of the fluid, and ω is the wave frequency.

If we consider the geometric transformation from the original coordinate system (x,y,z) to the compressed space coordinate system (x',y',z'), given by $x'(x,y,z)$, $y'(x,y,z)$ and $z'(x,y,z)$, the change of coordinates is characterized by the transformation of the differentials through the Jacobian $J_{x'x}$ of this transformation

$$\begin{pmatrix}\mathrm{d}x\\ \mathrm{d}y\\ \mathrm{d}z\end{pmatrix}=J_{xx'}^{T}\begin{pmatrix}\mathrm{d}x'\\ \mathrm{d}y'\\ \mathrm{d}z'\end{pmatrix},\quad J_{x'x}=(J_{xx'})^{-1}=\frac{\partial(x',y',z')}{\partial(x,y,z)}\,. \tag{2}$$

From the geometrical point of view, the change of coordinates implies that, in the transformed region, one can work with an associated metric tensor [5-8]

$$T=\frac{J_{xx'}^{\mathrm{T}}J_{xx'}}{\det(J_{xx'})}\,. \tag{3}$$

In other words, we can replace the material from the original domain (homogeneous and isotropic) by an equivalent compressed one that is inhomogeneous (its characteristics depend on the spherical (r',θ',ϕ') coordinates) and anisotropic (described by a tensor), and whose properties, in terms of $J_{x'x}$, are given by

$$\underline{\underline{\rho}}'=J_{x'x}^{-\mathrm{T}}\cdot\rho\cdot J_{x'x}^{-1}\cdot\det(J_{x'x})\,,\quad \kappa'=\kappa\det(J_{x'x})\,, \tag{4}$$

or, equivalently, in terms of $J_{xx'}$

$$\underline{\underline{\rho}}' = \frac{J^{\mathrm{T}}_{xx'} \cdot \rho \cdot J_{x'x}}{\det(J_{xx'})}, \ \kappa' = \frac{\kappa}{\det(J_{xx'})}, \tag{5}$$

where $\underline{\underline{\rho}}'$ is a second order tensor [9]-[12]. In this paper, we apply the 3D concave-down transformation [9, 13] which compresses a sphere of radius R_2 in the original space Ω into a shell region $R_1 < r' < R_2$ in the compressed space Ω'

$$r = \frac{R_2^{1.1}}{R_2^{0.1} - R_1^{0.1}} \left(1 - \left(\frac{R_1}{r'} \right)^{0.1} \right), \tag{6}$$

where the number 0.1 denotes the degree of the nonlinearity in the transformation. This transformation is applied in order to design a spherical cloak which surrounds a noisy machine (Fig.1). The original domain is a sphere of radius R_2, consisting of an alternation of concentric layers made from piezoelectric ceramics and epoxy resin, following a triadic Cantor sequence. After the transformation, the cloak contains a region $r < R_1$ which is filled with air and contains the noisy source, while the shell $R_1 < r < R_2$ is filled by the nonlinearly transformed material.

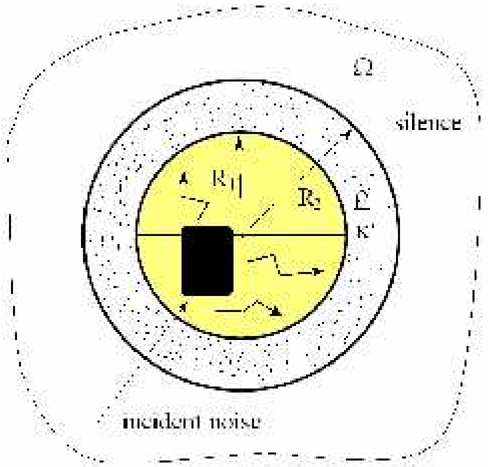

Figure 1: Spherical cloak surrounding a noisy machine

2. SPHERICAL ACOUSTIC CLOAK

The goal of this paper is to replace a material made from concentric homogeneous and isotropic layers situated in the original spherical domain by an equivalent compressed inhomogeneous anisotropic material described by the transformation matrix (3) [13]. Recent advances in metamaterials are encouraging for such an approach to constitutive parameters required for cloaking. Metamaterials are materials with subwavelength microstructures that are designed to have desired physical and acoustical properties. Despite the latest advances in metamaterials, we do not currently have the ability to manufacture a cloak with ideal constitutive parameters [12].

The versatility of geometric transforms bridges wave phenomena (the different colours of waves) ranging from electromagnetism and optics to acoustics [6]. The Helmholtz equation is able to reveal *the invisibility* scene *and the wave colours* in acoustics.

We consider that the original domain Ω is a sphere of radius R_2. The sphere consists of an alternation of concentric layers made from piezoelectric ceramics and epoxy resin, following a triadic Cantor sequence up to the fourth generation. A sketch of this material is represented in Fig.2. The dashed regions are occupied by piezoelectric ceramics, while the white regions are occupied by epoxy-resins.

The motivation of this choice goes back to [14]-[17] where it is shown the experimental evidence of extremely low thresholds for the subharmonic generation of ultrasonic waves in one-dimensional artificial piezoelectric plates with Cantor-like structure, as compared to the corresponding homogeneous and periodical plates. An anharmonic coupling between the extended-vibration (phonon) and the localized-mode (fracton) regimes explained this phenomenon. The aforementioned authors proved that the large enhancement of nonlinear interaction results from the more favorable frequency and spatial matching of coupled modes (fractons and phonons) in the Cantor-like structure. The equations which govern the subharmonic ultrasonic wave phenomenon were solved by using the cnoidal method, which employed the cnoidal wave as the fundamental basis function [18]-[20]. Cnoidal waves are much richer than sine waves, i.e. the modulus *m* of the cnoidal wave ($0 \le m < 1$) can be varied to obtain a sine wave $m = 0$, Stokes wave ($m = 0.5$) or soliton ($m = 1$).

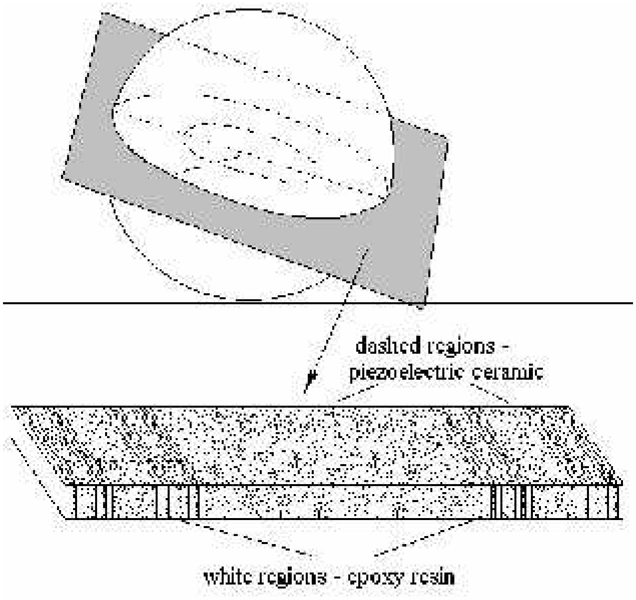

Figure 2: The Cantor-like structure [20]

The idea is to apply the transformation (6) to the governing equations of the Cantor like plate, which can be reduced to Helmholtz equations [13]. The property of Helmholtz equation to be invariant under geometric transformations is exploited in order to transform an original domain made of an initial material, into a shell domain made of a new inhomogeneous and anisotropic material.

The results show that the new materials can cloak regions of space, making them invisible to sound. We refer to acoustic cloaking which occurs when a medium contains a region in which noisy objects can be acoustically hidden.

This transformation compresses the original domain Ω occupied by a sphere of radius R_2 into a shell region $R_1 < r' < R_2$ in the compressed space Ω' , characterized by

$$\underline{\underline{\zeta}}'^{-1}_{p,e}(r') = J^{\mathrm{T}}_{rr'}\zeta^{-1}_{p,r}(r)J_{rr'}/\det(J_{rr'}),\ \underline{\underline{\Lambda}}'^{-1}(r') = J^{\mathrm{T}}_{rr'}\Lambda^{-1}(r)J_{rr'}/\det(J_{rr'}),\ J_{rr'} = \partial r/\partial r', \qquad \textbf{(7)}$$

For numerical simulations, we consider that the cloak has the inner radius $R_1 = 0.5\mathrm{m}$ and outer radius $R_2 = 1\mathrm{m}$, with a noisy machine situated inside the shell of the cloak. The parameters of the acoustic-metamaterial shell can be obtained accprding to the geometric transformation (6) and (7). The absence of the scattering of waves generated by an external source outside the cloak is observed in Fig. 3. The waves are smoothly bent around the central region inside the cloak..

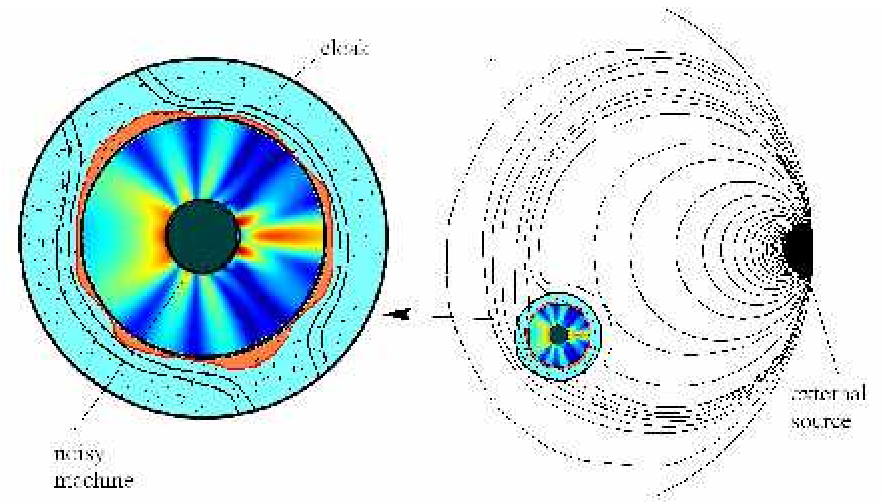

Figure 3: Wave field inside and outside the cloak [13]

The results reported in Fig.3 show that the wave field inside the cloak, i.e. the inner region of radius R_1 which surrounds the noisy machine, is completely isolated from the region situated outside the cloak. The waves generated by a noisy source are smoothly confined inside the inner region of the cloak. The inner region is acoustically isolated and the sound is not detectable by an exterior observer because the amplitudes on the boundary vanish. The domain $r < R_1$ is an acoustic invisible domain for exterior observers.

The waves generated by the exterior source outside the cloak do not interact with the interior field of waves. Actual, the invisibility cloaking means coating of an object with a special material so that sound goes around the object. Then the coated object is acoustically invisible. Fig.3. shows the behavior of the sound rays around a coated noisy machine.
A possible interaction or coupling between the internal and external wave fields is cancelled out by the presence of the shell region $R_1 < r < R_2$ filled with metamaterial, as we see in Fig. 4. In this figure, the interaction of wave is simulated on the surfaces $R = R_1$ and $R = R_2$, respectively. The color does not mean the intensity of the wave, but only the possible near-field interaction of different waves. The result of this interaction explained the noise machine shielding.

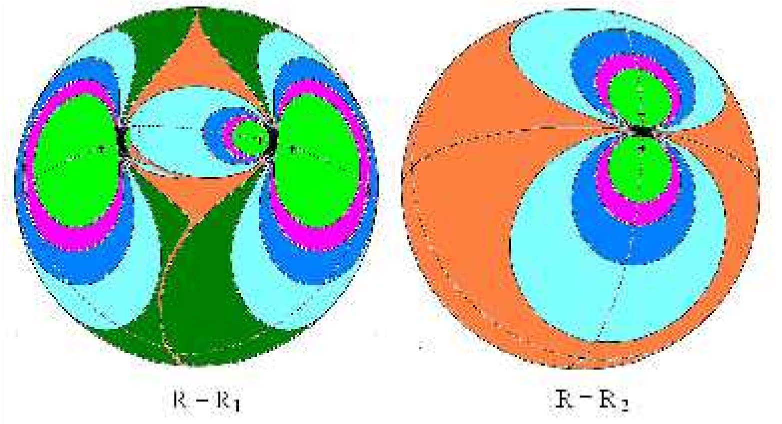

Figure 4: Near-field wave interaction on the surfaces $R = R_1$ and $R = R_2$, respectively

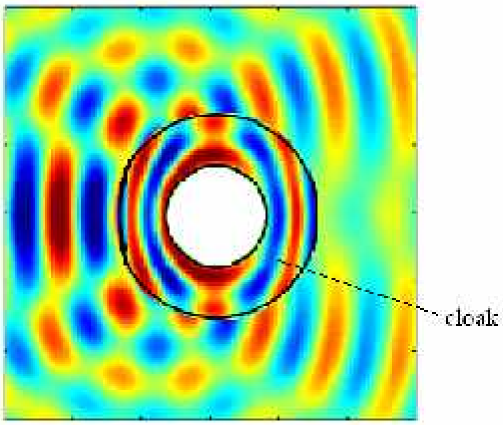

Figure 5: Wave field inside and outside the cloak in the absence of the noisy machine

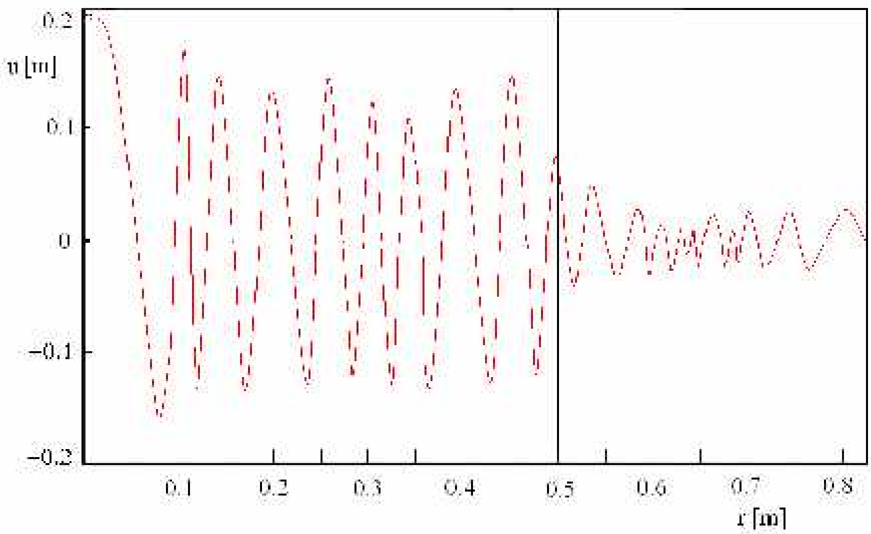

Figure 6: Wave field inside of the shell

Fig. 5 shows the simulated wave field in the absence of the noisy machine in the interior of the shell. In this case, the interior of the shell is completely isolated from the region situated outside the cloak.

We can believe that the geometric transformations can be used in designing new materials. By manipulating the spatial compression, different performances can be achieved under inhomogeneous and anisotropic materials that might be useful in the design of elastic cloaking devices [21].
Fig. 6 illustrates the wave amplitudes u inside of the shell. Seeing this figure, we notice that the wave field for $r' < 0.5\text{m}$ is decreasing in amplitude for $0.5\text{m} \le r' < 1\text{m}$. Obviously, the waves generated inside of the cloak are smoothly confined inside of the inner region of the shell. The inner region is acoustically isolated and the sound is not propagating outside of the shell because the amplitudes on the boundary almost vanish. The domain $r < R_1'$ is an acoustic invisible domain for the exterior observers.

3. CONCLUSION

In summary, the theory of the acoustic cloak is the key of designing the acoustic metamaterials. Such materials would do for sound what the acoustic invisibility cloak does for microwaves, allowing sound waves to travel seamlessly around it and emerge on the other side without distortion. The research on acoustic metamaterials began in the year 2000 with the fabrication and demonstration of sonic crystals in a liquid [22]. The results show that the metamaterials can cloak regions of space, making them invisible to sound. We refer to acoustic cloaking which occurs when a medium contains a region in which noisy objects can be acoustically hidden.
The existence of an acoustic cloaking indicates that cloaks might possibly be built for other wave systems, including seismic waves that travel through the earth and the waves at the surface of the ocean [7, 8], or to avoid sonar radars which pick up on the noise that ships emit. Metamaterials could be also used in concert halls or even as a way to deal with different noise [23-25].
Therefore, in order to design a particular property/sound trajectory, it is possible to design an inhomogeneous and anisotropic material with prescribed material properties calculated through a particular geometric transformation.

ACKNOWLEDGEMENT. This paper was written in honor of the Romanian scientist Dr. Nicolae Nicorovici (1944-2010) who was very enthusiastic about the study of cloaking effects with the emerging field of metamaterials. This research was financially supported by the National Authority for Scientific Research (ANCS, UEFISCSU), Romania, through project code PN-II-ID-PCE-2011-3-0608.

REFERENCES

[1] Pendry, J. B., Shurig, D., Smith, D. R.: Controlling electromagnetic fields, Science, 312, 1780–1782, 2006.
[2] Milton, G. W., Nicorovici, N. A.: On the cloaking effects associated with anomalous localized resonance, Proc. Roy. Soc. A **462**, 3027–3059, 2006.
[3] Dupont, G., Farhat, M., Diatta, A., Guenneau, S., Enoch, S.: Numerical analysis of three-dimensional acoustic cloaks and carpets, Wave Motion, 48(6), 483–496, 2011.
[4] Munteanu, L., Donescu, St.: Introduction to Soliton Theory: Applications to Mechanics, Book Series "Fundamental Theories of Physics", 143, Kluwer Academic Publishers, 2004.
[5] Zolla, F., Guenneau, S., Nicolet, A., Pendry, J. B.: Electromagnetic analysis of cylindrical invisibility cloaks and the mirage effect, Opt. Letters, 32, 1069–1071, 2007.
[6] Guenneau, S., McPhedran, R. C., Enoch, S., Movchan, A. B., Farhat, M., Nicorovici, N. A.: The colours of cloaks, Journal of Optics, 13(2), 024014, 2011.
[7] Cummer, S. A., Popa, B. I ., Schurig, D., Smith, D. R., Pendry, J., Rahm, M., Starr, A.: Scattering theory derivation of a 3D acoustic cloaking shell, Physical Review Letters, 100, 024301, 2008.
[8] Cummer, S. A., Schurig, D.: One path to acoustic cloaking, New Journal of Physics, 9, 45, 2007.
[9] Qiu, C. W., Hu, L., Zhang, B., Wu, B. I., Johnson, S. G., Joannopoulos, J. D.: Spherical cloaking using nonlinear transformations for improved segmentation into concentric isotropic coatings Optics Express 17(16), 13467–13478, 2009.
[10] Milton, G. W.: New metamaterials with macroscopic behavior outside that of continuum elastodynamics, New Journal of Physiscs, 9, 359– 372, 2007.
[11 Milton, G. W., Briane, M., Willis, J. R: On cloaking for elasticity and physical equations with a transformation invariant form, New Journal of Physics, 8, 248, 2006.
[12] McGuirk, J. S., Collins, P. J.: Controlling the transmitted field into a cylindrical cloak's hidden region, Optics Express, 16, 22, 17560, 2008.
[13] Munteanu, L., Chiroiu, V.: On the three-dimensional spherical acoustic cloaking, New Journal of Physics, 13(8), 083031, 2011
[14] Alippi, A., Shkerdin, G., Berttucci, A., Craciun, F., Molinari, E., Petri, A.: Low threshold subharmonic generation in composite in composite structures with Cantor-like code, Phys. Rev. Lett., 69, 3318–3321, 1992
[15] Alippi, A., Craciun, F., Molinari, E.: Stopband edges in the dispersion curves of Lamb waves propagating in piezoelectric periodical structures, Appl. Phys. Lett. 53, 19, 1806–1808, 1988.
[16] Alippi, A.: Nonlinear acoustic propagation in piezoelectric crystals, Ferroelectrics, 42, 109–116, 1982.

[17] Craciun, F., Bettucci, A., Molinari, E., Petri, A., Alippi, A.: Direct experimental observation of fracton mode patterns in one-dimensional Cantor composites, Phys. Rev. Lett., 68(10), 1992.
[18] Chiroiu, V., Chiroiu, C.: Inverse Problems in Mechanics (Bucharest: Publishing House of the Romanian Academy) (in Romanian), 2003.
[19] Chiroiu, V., Donescu, St., Munteanu, L.: Subharmonic generation of Love waves in a ferrite-dielectric plate with Cantor-like structure, Proceedings of the Romanian Academy, Series A: Mathematics, Physics, Technical Sciences, Information Science, 7(1), 47–54, 2006.
[20] Chiroiu, V., Delsanto, P. P., Scalerandi, M., Chiroiu, C., Sireteanu, T.: Subharmonic generation in piezoelectrics with Cantor-like structure, Journal of Physics D: Applied Physics, Institute of Physics Publishing, 34(3), 1579–1586, 2001.
[21] Munteanu, L., Dumitriu, D., Chiroiu, V.: Geometric transformations in designing new materials, Revue Roumaine des Sciences Techniques – Série de Mécanique Appliquée, 56(3), 2011.
[22] Zhengyou, L., Xixiang, Z., Yiwei, M., Zhu, Y. Y., Yang, Z., Chan, C. T., Sheng, P.: Locally Resonant Sonic Materials, Science, 289, 5485, 1734–1736, 2000.
[23] Zhang, S., Yin, L., Fang. N.: Focusing Ultrasound with an Acoustic Metamaterial Network, Physical Review Letter, 102, 194301, 2009.
[24] Fang, N., Xi, D., Xu, J., Ambati, M., Srituravanich, W., Sun, C., Zhang, X.: Ultrasonic metamaterials with negative modulus, Nature Materials, 5, 452–456, 2006.
[25] Lakes, R., Lee, T., Bersie, A., Wang. Y.C.: Extreme damping in composite materials with negative-stiffness inclusions, Nature, 410, 565–567, 2001.