EIGEN VALUE PROBLEMS IN AXIALSYMMERICAL CURVED PLATES

RADU N. Gheorghe, COMANESCU S. Ioana

Transilvania University of Brasov

rngh@unitbv.ro; ioanacom@unitbv.ro

Abstract: As it is well known, the finite element method (FEM) has witnessed a wide spread due to the advantages it presents. The method of finite elements is remarkably efficient mainly in cases of linear problems, especially if the computer program is used together with pre-processor and post processor programs. In this way the effort required in preparing the data and in processing the results is reduced to a minimum. However, the utilization of general finite element programs presupposes as a prerequisite access to powerful computers with rapid and large memories.

In many cases it is advisable to use programs specialized for a certain type of problems or eigen value problems, whenever the employment of a general-type programe becomes extremely clumsy.

Starting from these considerations, the authors have proposed to elaborate a program specialized in solving eigen value problems for axially symmetrical curved plates.

Key words: membrane tension, eigen value problems, curved plates.

1. INTRODUCTION AND EIGEN VALUE PROBLEMS

Two principal types of eigen value problems were considered:

- The determination of eigen frequencies and eigen modes:

$$([K]-\omega^2 [M]) \{u\}=0 \qquad (1)$$

- The determination of the critical load for loss of stability (bifurcation) and of the corresponding modes:

$([K]+\lambda[K_G])\{u\}=0$ (2)

The following notations were used:

[K]-the elastic rigidity matrix;

$[K_G]$-the geometrical rigidity matrix;

[M]-the mass matrix;

ω – the pulsation

λ – the load proportionality factor;

The problem given by equation (1) may be generalized thus:

$([K]+ \lambda[K_G]- \omega^2 [M])\{u\}=0$ (3)

i.e. the influence of the membrane stresses (constant throughout the plate thickness) upon the eigen frequencies may be studied.

As it is known, for the determination of the rigidity matrix of finite element the expression of the total potential energy is started from:

$$\pi = -\int_{Ae} qwdA + \frac{1}{2}\int_{Ae}\{\gamma\}^T[D_1]\{\gamma\}dA + \frac{1}{2}\int_{Ae}\{\varepsilon_0\}^T[D]\{\varepsilon_0\}dA + \frac{1}{2}\int_{Ae}\{w'\}^T[\sigma_0]\{w'\}, \quad (4)$$

where q is the force normally distributed over the plate surface,

w- the normal displacement in the median plane of the plate

$\{\varkappa\}$- the curvature and torsion vector

[D]- the matrix of Hooke's law

$[\sigma_0]$ - the vector of stresses in the median plane (of the membrane)

$\{\varepsilon\}$□-the vector of deformation in the median plane

{w'}- the vector of displacement derivates

h- the plate thickness

A_e-the surface of the element

The finite element adopted is a semi analytical finite element with two nodes and four unknown displacements per node, $\vec{u}, \vec{v}, \vec{w}, and\ \frac{\partial\vec{w}}{\partial s}, \vec{u}, \vec{w}$ denoting the displacements related to a local reference point attached to the finite element.

The finite element interpolation function is a polynomial function of the form:

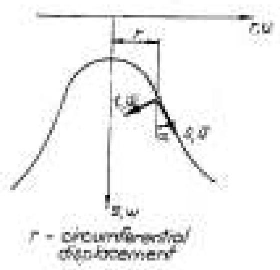

Fig 1

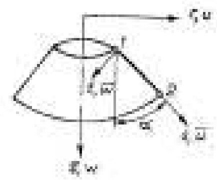

Fig 2

$$\overrightarrow{u_0} = (c_1 + c_2 s)\cos n\theta$$
$$\vec{v} = (c_3 + c_4 s)\sin n\theta$$
$$\vec{w} = (c_5 + c_6 s + c_7 s^2 + c_8 s^3)\cos n\theta$$

The rigidity matrix is determined by the usual way, by minimizing the expression of total potential energy

$$[K] = \int_{AC} [B]^T [D][B] dA$$

$$[K_G] = \int_{AC} [G]^T [\sigma_0][G] dA$$

[B]- the matrix of the nodal displacement

[G]-the matrix connecting the derivates $\frac{\partial \omega}{\partial s}, \frac{1}{r}\frac{\partial \omega}{\partial \theta}$ to the nodal displacement and $[\sigma_0] = \begin{bmatrix} \sigma_{0s} & \tau_{0s\theta} \\ \tau_{0st} & \sigma_{ot} \end{bmatrix}$

In the actual version of the program the tension field $[\sigma_0]$ is axially-symmetrical

For instance, for R_1=40, R_2=200, h=2 the following results were obtained:

- The lowest eigen frequency is 41,1 Hz;
- For a linear temperature variation along the radius the smallest difference of critical stability loss is Δ_{tcr} 25,8K.

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