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ON THE ADVANCED AUXETIC COMPOSITES

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Abstract : *The paper's target is to make more understandable the properties of polymer network composites made up of re-entrant cell structures (auxetic hexagons) with demonstrable auxetic properties. The behavior of auxetic composites is interpreted in the light of Cosserat elasticity which admits degrees of freedom not present in the classical elasticity: the rotation of points in the material, and a couple per unit area or the couple stress. The prediction of the Young's modulus is developed for a laminated periodic material made up of alternating aluminum and auxetic material, by using the Laplace and Fourier transforms.*
Keywords : *Auxetic material, Cosserat elasticity, Young's modulus, Laplace and Fourier transforms.*

1. INTRODUCTION

Materials with negative Poisson's ratio are termed by Evans [1] as auxetics or auxetic materials. The term *auxetic* is coming from the Greek word *auxetos*, meaning *that which may be increased*. Instead of getting thinner like an elongated elastic band when stretched, the auxetic material gains volume, expanding laterally. Auxetic materials and their negative Poisson's ratios have not been well understood. Materials of this type are expected to have interesting mechanical properties, such as high energy absorption, fracture toughness, indentation resistance and enhanced shear moduli, which may be useful in some applications [2]-[5]. The aforementioned authors studied the application of auxetic materials to damping devices, medical anchors and cushions. Scientists have been aware of the existence of auxetic materials for over a hundred years, though without very special attention, and treating them as an accident or a curiosity. In the case of an isotropic material, the range of Poisson's ratio is from -1.0 to 0.5, based on thermodynamic considerations of strain energy in the theory of elasticity. Love [6] presented an example of a cubic single crystal pyrite as having the Poisson's ratio of -0.14, and he suggested that the effect may be caused by twinned crystals.

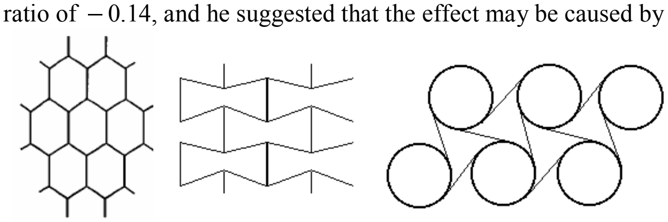

Figure 1: Conventional honeycomb network, re-entrant honeycomb and hexagonal structures, with negative Poisson's ratio.

The auxetic behavior is found in materials from molecular and microscopic levels up to the macroscopic level. Negative Poisson's ratios are observed in real materials with a high degree of anisotropy, such as conventional honeycomb network, re-entrant honeycomb and hexagonal structures (figure 1), reticulated metal foams, the skin covering a cow's teats, certain rocks and minerals, living bone tissue, etc. Fabrication of man-made auxetic materials and structures has succeeded, i.e. composite laminates, micro-porous polymers, 2D honeycombs and 3D foams.

All major classes of materials (polymers, composites, metals and ceramics) can exist in auxetic form. A specific feature exhibeted by auxetic materials in comparison with other foams is their significant damping capacity at various loading levels, with increase up to 16 times compared to conventional foams [7]-[13].

This paper focuses on the application of the Cosserat theory to derive the effective Young's modulus of the laminated composite plate based on auxetic materials.

2. THE THEORY

Consider a chiral Cosserat medium, in a Cartesian coordinates system (x,y,z). The equations of motion in the absence of body forces and body couples are [14], [15]

$$\sigma_{kl,k}-\rho\ddot{u}_l=0\,,\;\; m_{rk,r}+\varepsilon_{klr}\sigma_{lr}-\rho j\ddot{\varphi}_k=0\,. \tag{1}$$

Here σ_{kl} is the stress tensor, m_{kl} is the couple stress tensor, u is the displacement vector, φ_k is the microrotation vector which in Cosserat elasticity is kinematically different from the macrorotation vector $r_k=\frac{1}{2}\varepsilon_{klm}u_{m,l}$, and ε_{klm} is the permutation symbol. We remember that φ_k refers to the rotation of points themselves, while r_k refers to the rotation associated with movement of nearby points. In (1) ρ is the mass density and j the microinertia. The constitutive equations are given by

$$\sigma_{kl}=\lambda e_{rr}\delta_{kl}+(2\mu+\kappa)e_{kl}+\kappa\varepsilon_{klm}(r_m-\varphi_m)+C_1\varphi_{r,r}\delta_{kl}+C_2\varphi_{k,l}+C_3\varphi_{l,k}\,,$$

$$m_{kl}=\alpha\varphi_{r,r}\delta_{kl}+\beta\varphi_{k,l}+\gamma\varphi_{l,k}+C_1e_{rr}\delta_{kl}+(C_2+C_3)e_{kl}+(C_3-C_2)\varepsilon_{klm}(r_m-\varphi_m), \tag{2}$$

where $e_{kl}=\frac{1}{2}(u_{k,l}+u_{l,k})$ is the macrostrain vector. λ, and μ are Lamé elastic constants, κ is the Cosserat rotation modulus, α,β,γ, the Cosserat rotation gradient moduli, and C_i, $i=1,2,3$ are the chiral elastic constants associated with noncentrosymmetry. For $C_i=0$ the equations of isotropic micropolar elasticity are recovered. For $\alpha=\beta=\gamma=\kappa=0$, (1) reduces to the constitutive equations of classical isotropic linear elasticity theory. From the requirement that the internal energy must be nonnegative (the material is stable), we obtain restrictions on the micropolar elastic constants $0\le 3\lambda+2\mu+\kappa$, $0\le 2\mu+\kappa$, $0\le\kappa$, $0\le 3\alpha+\beta+\gamma$, $-\gamma\le\beta\le\gamma$, $0\le\gamma$, and any positive or negative C_1,C_2,C_3. The initial conditions are given by

$$u_i(x,y,z,0)=u_i^0(x,y,z)\,,\;\varphi_i(x,y,z,0)=0\,,\; i=1,2,3\,,\;\; m_{ij}(x,y,z,0)=0\,,\;\; \sigma_{ij}(x,y,z,0)=0\,,\;\; i=j\neq 3\,. \tag{3}$$

By introducing the dimensionless quantities

$$x'=\frac{\omega}{c_1}x\,,\; z'=\frac{\omega}{c_1}z\,,\; v_i'=\frac{\omega}{c_1}\hat{u}_i\,,\; i=1,2\,,\; \phi_2'=\frac{\mu K_0^2}{\rho j\omega^{*2}}\hat{\varphi}_2\,,\; t'=\omega t\,,\; \sigma_{ij}'=\frac{1}{\mu K_0^2}\hat{\sigma}_{ij}\,,$$

$$m_{ij}'=\frac{c_1}{\gamma\omega K_0^2}\hat{m}_{ij}\,,\; \omega^2=\frac{\kappa(1-K_0^2)}{\rho j}\,,\; c_1^2=\frac{\lambda+2\mu+\kappa}{\rho}\,,\; i,j=1,3\,,$$

the motion equations are obtained from (1) and (2) as

$$v_{1,xx}+(1-a^2)v_{3,xz}+a^2v_{1,zz}-s_4^*\phi_{2,z}=\frac{1}{s_1+s_2}\ddot{v}_1\,,\; v_{3,zz}+(1-a^2)v_{1,xz}+a^2v_{3,xx}+s_4^*\phi_{2,x}=\frac{1}{s_1+s_2}\ddot{v}_3\,,$$

$$\phi_{2,xx}+\phi_{2,zz}-\frac{2c_1^2\kappa(1-K_0^2)}{\omega^2\gamma K_0^2}\phi_2+\frac{c_1^2\mu}{\omega^2\gamma}(v_{1,z}-v_{3,x})=\frac{1}{s_4}\ddot{\phi}_2\,,\;\; K_0^2=1+\frac{(C_1+C_2+C_3)^2}{(\lambda+2\mu+\kappa)(\alpha+\beta+\gamma)}\,, \tag{4}$$

where

$$s_1=\frac{\lambda+\mu K_0^2}{\rho c_1^2}\,,\; s_2=\frac{\kappa(1-K_0^2)+\mu K_0^2}{\rho c_1^2}\,,\; s_3=\frac{\kappa j(1-K_0^2)\omega^{*2}}{\mu K_0^2c_1^2}\,,\; s_4=\frac{\gamma K_0^2}{\rho j c_1^2}\,,\; a^2=\frac{s_2}{s_1+s_2}\,,\; s_4^*=\frac{s_3}{s_1+s_2}\,. \tag{5}$$

The initial conditions (3) becomes

$$v_i(x,y,0)=v_i^0\,,\;\; i=1,3\,,\;\; \phi_2(x,y,0)=0\,. \tag{6}$$

Consider the case of laminated plates made up of a periodic layering of sheets normal to the direction x of wave propagation, each elastic material having constant properties. For simplicity, without any loss of generality, the particular 2D case in which all quantities depend only on x and z is considered.

3. SOLUTIONS

To solve equations (4)-(6), consider the Laplace and Fourier transforms

$$\left\{\overline{v}_i(x,z,p),\overline{\phi}_2(x,z,p)\right\}=\int_0^\infty\{v_i(x,z,t),\phi_2(x,z,t)\}\exp(-pt)\mathrm{d}t\,,\; i=1,3\,,$$

$$\left\{\tilde{v}_i(\xi,z,p),\tilde{\phi}_2(\xi,z,p)\right\}=\int_0^{\infty}\{\overline{v}_i(x,z,p),\overline{\phi}_2(x,z,p)\}\exp(\mathrm{i}\xi x)\mathrm{d}x\,,\ i=1,3\,.$$

By applying these transforms to (4)-(6), we obtain

$$\tilde{v}_1''=\frac{1}{a^2}\left[\xi^2+\frac{p^2}{s_1+s_2}\right]\tilde{v}_1+\frac{\mathrm{i}\xi(1-a^2)}{a^2}\tilde{v}_3'+\frac{s_4^*}{a^2}\tilde{\phi}_2'\,,\quad \tilde{v}_3''=[a^2\xi^2+\frac{p^2}{s_1+s_2}]\tilde{v}_3+\mathrm{i}\xi s_4^*\tilde{\phi}_2+\mathrm{i}\xi(1-a^2)\tilde{v}_1'\,,$$

$$\tilde{\phi}_2''=\frac{c_1^2\mu}{\omega^2\gamma}\tilde{v}_1'-\frac{\mathrm{i}\xi c_1^2\mu}{\omega^2\gamma}\tilde{v}_3+\left[\xi^2+\frac{2c_1^2\kappa(1-K_0^2)\mu}{\omega^2\gamma K_0^2}+\frac{\rho^2}{s_4}\right]\tilde{\phi}_2\,. \qquad (7)$$

An eigenvalue problem is obtained by taking the solutions of (3.3) of the form $W(\xi,z,p)=X(\xi,p)\exp(qz)$, where $W(\xi,z,p)=\{\tilde{v}_1,\tilde{v}_3,\tilde{\phi}_2\}$. The characteristic equation becomes

$$q^3-\lambda_1q^2+\lambda_2q+\lambda_3=0\,, \qquad (8)$$

with

$$\lambda_1=\left(1+\frac{1}{a^2}\right)\frac{p^2}{s_1+s_2}+3\xi^2+\frac{2c_1^2\kappa(1-K_0^2)}{\omega^2\gamma K_0^2}+\frac{p^2}{s_4}-\frac{2c_1^2\mu s_4^*}{\omega^2\gamma a^2}\,,$$

$$\lambda_2=\left[\xi^2+\frac{2c_1^2\kappa(1-K_0^2)}{\omega^2\gamma K_0^2}+\frac{p^2}{s_4}\right]\left[\frac{p^2}{s_1+s_2}\left(1+\frac{1}{a^2}\right)+2\xi^2\right]-\frac{c_1^2\mu s_4^*}{\omega^2\gamma a^2}\left[\frac{p^2}{s_1+s_2}+2\xi^2\right]+\frac{1}{a^2}\left[\frac{p^2}{s_1+s_2}+\xi^2\right]\left[\frac{p^2}{s_1+s_2}+a^2\xi^2\right],$$

$$\lambda_3=\frac{1}{a^2}\left[\xi^2+\frac{p^2}{s_1+s_2}\right]\left[\frac{p^2}{s_1+s_2}+a^2\xi^2\right]\left[\frac{p^2}{s_4}+\xi^2+\frac{2c_1^2\kappa(1-K_0^2)}{\omega^2\gamma K_0^2}\right]-\frac{s_4^*}{a^2}\left[\frac{p^2}{s_1+s_2}+\xi^2\right]\xi^2\frac{c_1^2\mu}{\omega^2\gamma}. \qquad (9)$$

The roots of (8) are q_i, $i=1,2,3$, with real parts positive. The eigenvector $X(\xi,p)$ is given by

$$X_{i1}(\xi,p)=\begin{vmatrix}a_iq_i\\ b_i\\ -\xi\end{vmatrix},\quad X_{i2}(\xi,p)=\begin{vmatrix}a_iq_i^2\\ b_iq_i\\ -\xi q_i\end{vmatrix},\quad i=1,2,3,\quad a_i=\left[\xi(a^2-1)\left\{\left(\xi^2+\frac{2c_1^2\kappa(1-K_0^2)}{\omega^2\gamma K_0^2}+\frac{p^2}{s_4}\right)-q_i^2\right\}-\frac{c_1^2\mu s_4^*\xi}{\omega^2\gamma}\right]/\Delta_i\,,$$

$$b_i=-\mathrm{i}\left[\left(\xi^2+\frac{p^2}{s_1+s_2}\right)\left(\xi^2+\frac{2c_1^2\kappa(1-K_0^2)}{\omega^2\gamma K_0^2}+\frac{p^2}{s_4}\right)+a^2q_i^2[q_i^2-(\xi^2+\frac{2c_1^2\kappa(1-K_0^2)}{\omega^2\gamma K_0^2}+\frac{p^2}{s_4})\}-(\xi^2+\frac{p^2}{s_1+s_2})q_i^2+\frac{c_1^2\mu s_4^*q_i^2)}{\omega^2\gamma}\right]/\Delta_i,$$

$$\Delta_i=\frac{c_1^2\mu}{\omega^2\gamma}[q_i^2-(\xi^2+\frac{p^2}{s_1+s_2})]\,,i=1,2,3\,. \qquad (10)$$

Therefore, the solutions become $W(\xi,z,p)=\sum_{i=1}^{3}B_iX_i(\xi,p)\exp(q_i(\xi,p)z)$, where B_i, $i=1,2,3$, are arbitrary constants. The transformed displacement, microrotation and stresses are

$$\tilde{v}_1(\xi,z,p)=a_1(\xi,p)q_1(\xi,p)B_1\exp(q_1(\xi,p)z)+a_2(\xi,p)q_2(\xi,p)B_2\exp(q_2(\xi,p)z)+a_3(\xi,p)q_3(\xi,p)B_3\exp(q_3(\xi,p)z)\},$$

$$\tilde{v}_3(\xi,z,p)=b_1(\xi,p)B_1\exp(q_1(\xi,p)z)+b_2(\xi,p)B_2\exp(q_2(\xi,p)z)+b_3(\xi,p)B_3\exp(q_3(\xi,p)z)\},$$

$$\tilde{\phi}_2(\xi,z,p)=-\xi\{B_1\exp(q_1(\xi,p)z)+B_2\exp(q_2(\xi,p)z)+B_3\exp(q_3(\xi,p)z)\}\,. \qquad (11)$$

To obtain the unknowns B_i, $i=1,2,3$, we apply the Laplace and Fourier transforms on the initial conditions (6)

$$\tilde{v}_i(\xi,z,0)=\tilde{v}_i^0\,,\ i=1,3\,,\ \phi_2(x,y,0)=0\,. \qquad (12)$$

The transformed quantities are functions of z, the parameters of Laplace and Fourier transforms p and ξ, and are of the form $f(\xi,z,p)$. To obtain the function $f(x,z,t)$, first we invert the Fourier transform by using

$$\overline{f}(x,z,p)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp(-\mathrm{i}\xi x)\tilde{f}(\xi,z,p)\mathrm{d}\xi=\frac{1}{\pi}\int_0^{\infty}\{\cos(\xi x)f_e-\mathrm{i}\sin(\xi x)f_0\}\mathrm{d}\xi\,, \qquad (13)$$

where f_e and f_0 are even and odd parts of the function $\tilde{f}(\xi,z,p)$ respectively. Expression (13) gives the Laplace $\overline{f}(x,z,p)$ of the function $f(x,z,t)$.

4. THE YOUNG' MODULUS OF THE COMPOSITE

Consider a laminated 2D composite plate which occupies the region $x \in [0, L]$, $z \in [-c, c]$, is made up of alternating N aluminum and auxetic material layers, normal to the direction x of wave propagation (figure 2). The layers are parallel, planar, periodical, across which the displacements are continuous. The length of each layer is l. The interfaces between the layers are located at nl, $n = 1, 2, ..., N$, and each joint has two faces identified by $+$ and $-$. Choose coordinates so that the waves lie in the (x, z) plane. The plate is assumed to be in plane strain and to support waves running along the x-direction. Suppose that all material constants are functions of x. The continuity of solutions v_1, v_2 and ϕ_2 at the interface nl is given by

$$v_i\left(\frac{\omega}{c_1}nl^-, \frac{\omega}{c_1}z, \omega t\right) = v_i\left(\frac{\omega}{c_1}nl^+, \frac{\omega}{c_1}z, \omega t\right),\ i = 1,3,\ \phi_2\left(\frac{\omega}{c_1}nl^-, \frac{\omega}{c_1}z, \omega t\right) = \phi_2\left(\frac{\omega}{c_1}nl^+, \frac{\omega}{c_1}z, \omega t\right),$$

for $n = 1, 2, ..., N$. To predict Young' modulus from the Lamé elastic constants λ, μ, we have the formula which is not valid in our case $E_0 = \frac{(3\lambda + 2\mu)\mu}{\lambda + \mu}$. We are interested in knowing the influence of the Cosserat rotation modulus κ, the Cosserat rotation gradient moduli α, β, γ, and the chiral elastic constants C_i, $i = 1, 2, 3$, on the effective Young' modulus value of the laminated plate. The material constants $\tilde{C} = \{\lambda, \mu, \kappa, \alpha, \beta, \gamma, C_1, C_2, C_3\}$ for this laminated composite are periodic functions of x, $\tilde{C}(x + P) = \tilde{C}(x)$, where P is the period equal to the length of the basic cell for the composite $P = 2l$, where $2l$ is the period represented by the length of the basic cell for the composite.

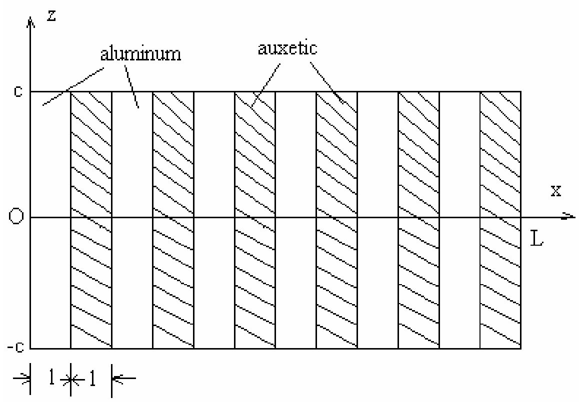

Figure 2: Sketch of the composite plate.

We find

$$E = F(\tilde{C}') + E',\ E' = \frac{(2\mu' + \kappa')(3\lambda' + 2\mu' + \kappa_{aux})}{(2\lambda' + 2\mu' + \kappa_{aux})} + \frac{1}{2}\tilde{p}^2,\quad \tilde{p}^2 = \frac{2\kappa_{aux}}{(K_0'^2 - 1)}, \tag{14}$$

$$K_0'^2 = 1 + \frac{(C_{1aux} + C_{2aux} + C_{3aux})^2}{(\lambda' + 2\mu' + \kappa_{aux})(\alpha_{aux} + \beta_{aux} + \gamma_{aux})}, \tag{15}$$

where $\tilde{C}_{al}$ are the aluminum constants and $\tilde{C}_{aux}$, the auxetic constants. The function $F(\tilde{C}')$ is numerically determined only.

Figure 3 represents the variation of the homogenized Young's modulus with respect to the volume fraction θ of aluminum and Poisson's ratio ν of the auxetic material, for $N = 15$ alternating aluminum and auxetic layers. In this simulation, the Young's moduli are 109 GPa for aluminum and 1.55 GPa for auxetic material, respectively.

To continue the curves in the region $\nu > 0$, an *equivalent* material with positive Poisson's ratio is introduced to replace the auxetic material. The curve $\theta \to 1$ corresponds to a laminated plate made of 15 alternating aluminum and equivalent material layers. This curve is situated below the curve $\theta = 0.8$. According to this variation, two values for Poisson's ratio correspond to the Young's modulus of 109 GPa, namely $\nu = -0.23$ and $\nu = 0.34$. The last is the Poisson's ratio for aluminum.

We observe that Young's modulus is increasing with respect to θ from 2 GPa, to about 150 GPa, having a maximum value for $\theta = 0.8$, $\nu \in [-1, -0.3) \cup (-0.05, 0.4]$ and $\theta = 0.3$, $-0.3 \le \nu \le -0.05$.

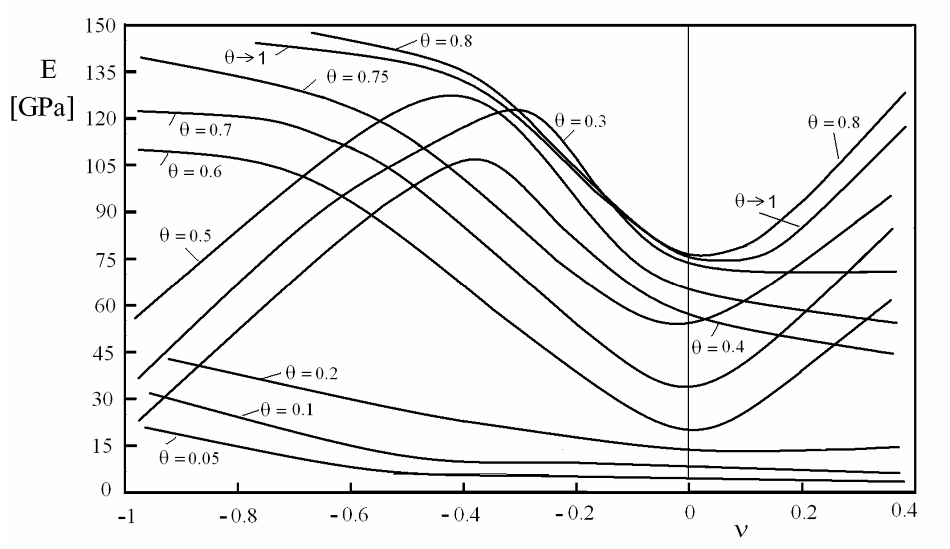

Figure 3: The homogenized Young's modulus variation with respect to Poisson's ratio of the auxetic material and an equivalent material with positive Poisson's ratio.

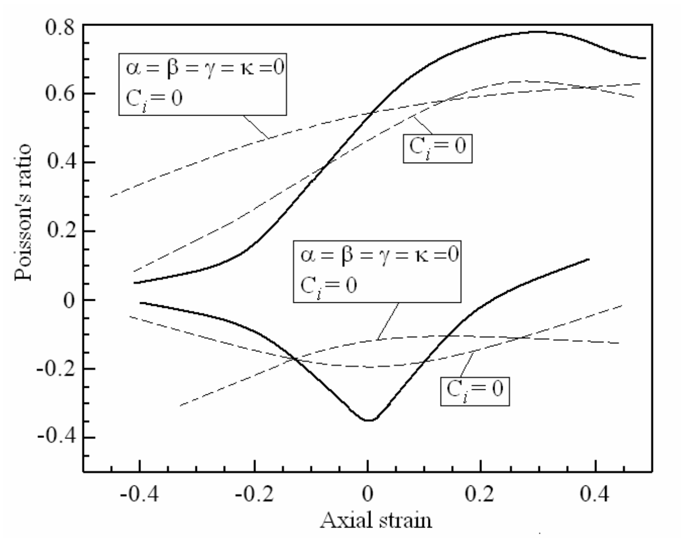

Figure 4: Poisson's ratio versus axial strain.

The Poisson's ratio depends of strain. The variation of the Poisson's ratio with respect to axial deformation in tension and compression is plotted in figure 4. The results shown in this diagram by solid lines are qualitatively similar to the experimental results reported in [18]. In this figure, dashed lines refer to the cases of classical isotropic linear elastic solid ($\alpha = \beta = \gamma = \kappa = 0$, $C_1 = C_2 = C_3 = 0$).

3. CONCLUSION

In this paper, the re-entrant cell structures are modeled with chiral Cosserat elasticity which admits degrees of freedom not present in classical elasticity. Negative Poisson's ratio materials easily undergo volume changes but resist shape changes and may thus be viewed as the opposite of rubbery materials, or antirubbers.

The estimation of Young's modulus for a laminated periodic structure made up of alternating aluminum and auxetic layers by Laplace and Fourier techniques, is the main aim of this paper. The behaviour of such materials is interpreted in the light of Cosserat elasticity, by considering the rotation of points in the material, and a couple per unit area or couple stress. As a result, the auxetic material implies a stiffening effect leading to increased Young's elastic moduli.

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