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WAVE VELOCITY ESTIMATION BY APPLICATION OF THE INTRINSIC TRANSFER MATRIX

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Abstract

The work presents experimental data of wave velocity measurements in wood samples, obtained by using intrinsic transfer method. The method is based on the properties of the behavior of the eigenvalues of the transfer matrix in resonance cases, this means the method is a particular modal approach of a resonance method. To find the wave velocity in a sample, respective sample is built-in an embedded system containing gauge material and the sample under tests. A numerical analysis applied on the analithycal expression of the eigenvalue permits the wave velocity estimation.

Keywords: *transfer matrix; eigenmodes; elastic wave velocity in wood*

1.INTRODUCTION

If we consider a simple solid homogeneous elastic rod with the length l, much larger than its diameter, and characteristic impedance $Z = {}^{\circ}c$, placed between two semiinfinite media with characteristic impedances Z_{in} and Z_{out}, which propagates a longitudinal plan wave, TM which connects the amplitudes of the Fourier components of the incident and reflected displacements in the sample $A(\mu)$ and $B(\mu)$ has the expression:

$$\begin{pmatrix} A_{out}(\mu,l) \\ B_{out}(\mu,l) \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1+\frac{Z}{Z_{out}} & 1-\frac{Z}{Z_{out}} \\ 1-\frac{Z}{Z_{out}} & 1+\frac{Z}{Z_{out}} \end{pmatrix}\begin{pmatrix} e^{- \ l}e^{i\frac{\mu}{c}l} & 0 \\ 0 & e^{\ l}e^{-i\frac{\mu}{c}l} \end{pmatrix}\begin{pmatrix} 1+\frac{Z_{in}}{Z} & 1-\frac{Z_{in}}{Z} \\ 1-\frac{Z_{in}}{Z} & 1+\frac{Z_{in}}{Z} \end{pmatrix}\begin{pmatrix} A_{in}(\mu,0) \\ B_{in}(\mu,0) \end{pmatrix} \tag{1}$$

with attenuation factor related to amplitude, μ angular frequency and c the speed of the wave. In the stationary case, when the stationary wave is confined inside the sample, only the intrinsic part of the TM is involved, i.e.

$$TM(\ ,\mu)=\begin{pmatrix} e^{-\ l}e^{i\frac{\mu}{c}l} & 0 \\ 0 & e^{\ l}e^{-i\frac{\mu}{c}l} \end{pmatrix} \tag{2}$$

Intrinsic part of the TM has the eigenvalues:

$$a_{1,2}=\left[cos\left(\frac{\mu l}{c}\right)\cdot cosh(\ l)-i\cdot sin\left(\frac{\mu l}{c}\right)\cdot sinh(\ l)\right]\pm \sqrt{\left[cos\left(\frac{\mu l}{c}\right)\cdot cosh(\ l)-i\cdot sin\left(\frac{\mu l}{c}\right)\cdot sinh(\ l)\right]^2-1} \quad `(3)$$

The eigenmodes of the rod, which correspond to the real values of the eigenvalues, are given by: $sin\left(\frac{\mu l}{c}\right)=0,$ and therefore $l=\frac{n^{a}w}{2},\quad n=1,2,...$

2. TRANSFER MATRIX IN THE CASE OF A TERNARY SYSTEM

Consider a ternary system which contain three layers having the thicknesses l_1,l_2,l_3 and characteristic impedances Z_1,Z_2,Z_3 which propagates longitudinally waves with wavenumbers k_1,k_2,k_3, placed between two semiinfine elastic media, with characteristic impedances Z_{in} and Z_{out}. The spectral amplitudes of the waves at input and output connected by a TM are given below[1][2]:

$$\begin{pmatrix} A_{out}(\mu) \\ B_{out}(\mu)\end{pmatrix}=\frac{1}{16}\begin{pmatrix} 1+\frac{Z}{Z_{out}} & 1-\frac{Z}{Z_{out}} \\ 1-\frac{Z}{Z_{out}} & 1+\frac{Z}{Z_{out}}\end{pmatrix}\begin{pmatrix} e^{ik_3l_3} & 0 \\ 0 & e^{-ik_3l_3}\end{pmatrix}\cdot\begin{pmatrix} 1+\frac{Z_2}{Z_3} & 1-\frac{Z_2}{Z_3} \\ 1-\frac{Z_2}{Z_3} & 1+\frac{Z_2}{Z_3}\end{pmatrix}\cdot\begin{pmatrix} e^{ik_2l_2} & 0 \\ 0 & e^{-ik_2l_2}\end{pmatrix}\cdot$$
$$\cdot\begin{pmatrix} 1+\frac{Z_1}{Z_2} & 1-\frac{Z_1}{Z_2} \\ 1-\frac{Z_1}{Z_2} & 1+\frac{Z_1}{Z_2}\end{pmatrix}\cdot\begin{pmatrix} e^{ik_1l_1} & 0 \\ 0 & e^{-ik_1l_1}\end{pmatrix}\begin{pmatrix} 1+\frac{Z_{in}}{Z} & 1-\frac{Z_{in}}{Z} \\ 1-\frac{Z_{in}}{Z} & 1+\frac{Z_{in}}{Z}\end{pmatrix}\begin{pmatrix} A_{in}(\mu) \\ B_{in}(\mu)\end{pmatrix} \tag{4}$$

The intrinsic part of the TM, taking into consideration attenuation is:

$$TM(\mu)=\frac{1}{4}\cdot\begin{pmatrix} e^{i\frac{\mu}{c_3}l_3-\ _3l_3} & 0 \\ 0 & e^{-i\frac{\mu}{c_3}l_3+\ _3l_3}\end{pmatrix}\cdot\begin{pmatrix} 1+\frac{Z_2}{Z_3} & 1-\frac{Z_2}{Z_3} \\ 1-\frac{Z_2}{Z_3} & 1+\frac{Z_2}{Z_3}\end{pmatrix}\cdot\begin{pmatrix} e^{i\frac{\mu}{c_2}l_2-\ _2l_2} & 0 \\ 0 & e^{-i\frac{\mu}{c_2}l_2+\ _2l_2}\end{pmatrix}\cdot$$
$$\cdot\begin{pmatrix} 1+\frac{Z_1}{Z_2} & 1-\frac{Z_1}{Z_2} \\ 1-\frac{Z_1}{Z_2} & 1+\frac{Z_1}{Z_2}\end{pmatrix}\cdot\begin{pmatrix} e^{i\frac{\mu}{c_1}l_1-\ _1l_1} & 0 \\ 0 & e^{-i\frac{\mu}{c_1}l_1+\ _1l_1}\end{pmatrix} \tag{5}$$

The eigenvalues of $TM(\mu)$ with identical materials of the layers 1 and 3 are:

$$
{}^{a}{}_{1,2}(\mu)=\left[\left(\frac{Z_1+Z_2}{2\sqrt{Z_1Z_2}}\right)^2\left[\begin{matrix} cos\left(\frac{\mu}{c_1}l_1+\frac{\mu}{c_2}l_2+\frac{\mu}{c_1}l_3\right)\cdot cosh\left({}_1l_1+{}_2l_2+{}_1l_3\right)- \\ i\cdot sin\left(\frac{\mu}{c_1}l_1+\frac{\mu}{c_2}l_2+\frac{\mu}{c_1}l_3\right)\cdot sinh\left({}_1l_1+{}_2l_2+{}_1l_3\right)\end{matrix}\right]\right]-
$$

$$
\left[\left(\frac{Z_1-Z_2}{2\sqrt{Z_1Z_2}}\right)^2\left[\begin{matrix} cos\left(\frac{\mu}{c_1}l_1-\frac{\mu}{c_2}l_2+\frac{\mu}{c_1}l_3\right)\cdot cosh\left({}_1l_1-{}_2l_2+{}_1l_3\right)- \\ i\cdot sin\left(\frac{\mu}{c_1}l_1-\frac{\mu}{c_2}l_2+\frac{\mu}{c_3}l_3\right)\cdot sinh\left({}_1l_1-{}_2l_2+{}_1l_3\right)\end{matrix}\right]\right]\pm\sqrt{F(\mu)} \qquad (6)
$$

where:

$$
F(\mu)=\left\{\begin{matrix}\left[\left(\frac{Z_1+Z_2}{2\sqrt{Z_1Z_2}}\right)^2\left[\begin{matrix} cos\left(\frac{\mu}{c_1}l_1+\frac{\mu}{c_2}l_2+\frac{\mu}{c_1}l_3\right)\cdot cosh\left({}_1l_1+{}_2l_2+{}_1l_3\right)- \\ i\cdot sin\left(\frac{\mu}{c_1}l_1+\frac{\mu}{c_2}l_2+\frac{\mu}{c_1}l_3\right)\cdot sinh\left({}_1l_1+{}_2l_2+{}_1l_3\right)\end{matrix}\right]\right]- \\ \left[\left(\frac{Z_1-Z_2}{2\sqrt{Z_1Z_2}}\right)^2\left[\begin{matrix} cos\left(\frac{\mu}{c_1}l_1-\frac{\mu}{c_2}l_2+\frac{\mu}{c_1}l_3\right)\cdot cosh\left({}_1l_1-{}_2l_2+{}_1l_3\right)- \\ i\cdot sin\left(\frac{\mu}{c_1}l_1-\frac{\mu}{c_2}l_2+\frac{\mu}{c_3}l_3\right)\cdot sinh\left({}_1l_1-{}_2l_2+{}_1l_3\right)\end{matrix}\right]\right]\end{matrix}\right\}^2-1 \qquad (7)
$$

A modal analysis combined with a numerical method able to study the behavior of the eigenvalues can be proposed to find elastic constants of the solid materials. Such a method can be applied to study simple embedded systems containig gauge materials and the sample under test. Such simple systems are characterized in the case of a longitudinal plan wave propagation by a simple distribution of the eigenmodes and also by a simple and convenient analytical expression TM. Examples of such simple systems are binary or ternary built-in systems, which contain gauge materials and materials for investigation[3].

3. APPLICATION OF THE METHOD FOR SOME SPECIAL MATERIALS

A good application of the method is to characterize the elastic properties of the wood samples[4] Because the elastic and mechanical properties of wood (elastic module, mass density, and Poisson's ratios) are random variables that vary significantly for the same wood species [5], the intrinsic transfer method offers a fast and convenient method to characterize such materials. Instead of huge poles with the ends connected to emitters and receivers [6], the samples used in the transfer matrix method are much smaller. The wood samples consisting of small cylinders were built-in ternary systems brass-wood-brass, taking the brass as gauge material. Moreover, the small cylinders can be cut so as to comply the cylindrical geometry used in characterization of the orthotropic behavior of wood. Tables 1 and 2 express the configuration of the experimental setup, samples sizes and the obtained values of sound velocity along the fiber c_l and perpendicular to it c_r. ¬ represents the value of the frequency of the eigenmode taken into consideration for numerical analysis.

Table 1. Elastic wave velocity along the fiber estimated by intrinsic transfer method

No.	Species of wood	l_2 (mm)	Diameter (mm)	° (Kg/m3)	l_1 (brass) (mm)	l_3 (brass) (mm)	¬ (Hz)	c_l along (m/s)
1	Fir tree	29.19	10	473.14	160.2	93.57	3300.78	4563
2	Oak	29.98	9.95	724.85	199	129.16	3164	4161
3	Beech	29.90	9.98	703.77	123.4	123.4	4355	4905
4	Spruce	29.45	9.47	450.73	160	93.39	3808	5437
5	Ash	29.5	9.52	801.11	198.4	129.15	3398.44	4253

Table 2. Elastic wave velocity transversal to the fiber estimated by intrinsic transfer method

No	Species of wood	l_2 (mm)	Diameter (mm)	° (Kg/m3)	l_1 (mm)	l_3 (mm)	¬ (Hz)	c_r Radial (m/s}
1	Fir tree	31.37	10.02	473.14	160.2	93.57	1269	1512
2	Oak	31.07	10.0	724.85	199	129.16	1562.5	1707
3	Beech	31.15	10.1	703.77	123.4	123.4	1738.28	1692
4	Spruce	16.03	9.8	450.73	159.64	92.72	1171.88	1019
5	Ash	22.14	9.93	801.11	198.07	129.42	1972.66	1850

4. CONCLUSIONS

The work proposes a resonance method based on the properties of the eigenvalues of the wave transfer matrix combined with a numerical method, in order to find the velocity of elastic waves in solid elastic samples. The study also considers the attenuation and shows that attenuation affects the frequency of eigenmodes for an embedded system. The ternary system is preferred because a such system preserves much better the longitudinally plan wave, special case for which the transfer matrix has a simple mathematical form. Although the theory is valid for plane longitudinal waves, we consider that the experimental model consisting of three solid rods connected in line by adhesion, with extreme rods made from identical materials and the sample of interest placed between these gauge materials, approaches quite well the theoretical assumption. All experiments were done using noncontact methods based on Doppler interferomety

REFERENCES

[1] Song, B. H., Bolton, J. S., A transfer matrix approach for estimation the characteristic impedance and wave number of limp and rigid porous materials, Journal of the Acoustical Society of America, Volume 107, Issues 3, Pages 1131-1152, 2000

[2] Nayfeh, A. H., The general problem of elastic wave propagation in multilayered anisotropic media, Journal of the Acostical Society of America, Volume 89, Issue4, pages 1521-1531,1997

[3] Cretu, N., Nita, G., A simplified modal analysis based on the propeties of the transfer matrix, Mechanics of Materials, Volume 60, pages 121-128, 2013

[4]Green, D.W., Winandy, J.E., Kretschmann, D.E. Mechanical properties of wood Wood Handbook. Wood As An Engineering Material. General Technical Report FPL GTR 113, pp. 1-45, 1999.
[5] Tallavo, F., Cascante, G., Pandey, M. D., Estimation of the Probability Distribution of Wave Velocity in Wood Poles, Journal of Materials in Civil Engineering, Volume 23, Issue 9, Pages 1272-1280, 2011
[6] Bucur , V., Feeney, F., Attenuation of ultrasound in solid wood, Ultrasonics, Volume 30, Issue 2, Pages 76-81,1992