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RESONANCE FREQUENCIES OF AN OPTIMAL ULTRASONIC HORN CONCENTRATOR

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Abstract: *An ultrasonic horn concentrator designed by variational calculus theory was simulated and its resonance frequencies were obtained. Simulations employed 3 methods: a transfer matrix method, a coupled oscillators model and a 3D finite elements method. The working frequency of the horn was fixed by adding end couplings.*
Keywords: *ultrasonic horns, variational calculus, transfer matrix, finite element method*

1. INTRODUCTION

Ultrasonic horns are used to amplify the ultrasonic signal by increasing the wave density of energy. Ultrasonic equipments used in manufacturing processes consist of four main parts: ultrasonic generator, transducer, ultrasonic solid horn (USH) and the tool. USH carries the acoustic energy from a transducer to the tool attached at the output end with high efficiency (Figure 1).

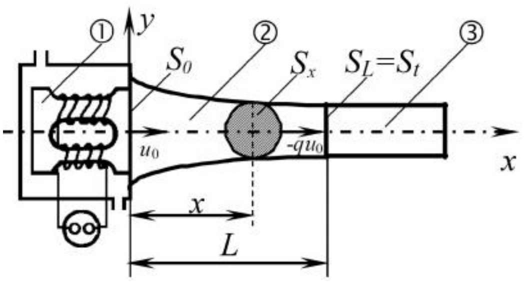

Figure 1: Ultrasonic system: 1 - electromechanical transducer; 2 – ultrasonic horn; 3 – tool

A wave with amplitude u_0 is emitted by the transducer, then it is amplified by the horn with an amplification magnitude q. The amplification is ensured by the USH varying cross-section, from an initial value S_0 at input end to a minimum value S_L at the output (Figure 1). The gain of USH is defined as $q = \left| u_{out} / u_0 \right|$. The transducer, USH and tool form an embedded mechanical system, and the operating frequency depends on the dimensions and elastic properties of both components. The system's performance can be improved by optimizing the USH.

2. THEORETICAL CONSIDERATIONS

The USH with a variable cross-section is described by Webster's horn equation (see [1]):

$$\frac{\partial^2 u(x,t)}{\partial x^2} + \frac{\partial u(x,t)}{\partial x}\frac{\partial}{\partial x}\left(\ln S_x\right) = \frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2}, \tag{1}$$

where $u(x,t)$ = displacement of ultrasonic signal, S_x = cross-section area at distance x, and c = sound velocity in the horn's material. By separation of variables [2] $u(x,t) = u(x)\,u(t) = u_x u_t$ and the following equations are obtained:

$$\ddot{u}_t + \mu^2 u_t = 0 \tag{2}$$

$$u''_x + \frac{S'_x}{S_x}u'_x + k^2 u_x = 0, \tag{3}$$

where μ = ultrasound angular frequency $[rad/s]$ and $k = \mu/c$ = wave number.

By imposing the condition that the maximum acoustic pressure $p_{\max}$ is equal to the normal stress $\pm_L$ at the end of the horn [3] and considering Hooke's law, a variational formulation of equation (3) with minimisation of concentrator volume can be obtained:

$$I = \int_0^L \left[S_x + \text{ª}_x\left(S_x u''_x + S'_x u'_x + k^2 S_x u_x\right)\right]dx - \text{ª}_1\left(\frac{\pm^2}{2E} - \frac{\pm_L u'_L}{2}\right), \tag{4}$$

where $\text{ª}_x = \text{ª}(x)$ and ª_1 are Lagrange multipliers, and u'_L is the strain value at the end of the horn [4]. The function (4) has an extremum value if $\text{¢} I = 0$, with the condition $\text{¢}u = \text{¢}u' = 0$ at both ends of the horn [4]. These conditions reduce to solving the system of equations:

$$\begin{cases} \text{ª}_x u'_x \text{¢} S_x\big|_0^L + \text{ª}_x S_x \text{¢} u'_x\big|_0^L - \frac{\text{ª}_1}{2}\left[u'_L \text{¢} \pm_L + \pm_L \text{¢} u'_L\right] = 0, \\ \left[\left(\text{ª}'_x S_x\right)' + \text{ª}_x k^2 S_x\right] = 0, \\ \left(1 + k^2 \text{ª}_x u_x - \text{ª}'_x u'_x\right) = 0. \end{cases} \tag{5}$$

A corresponding solution is obtained:

$$u_x = -\frac{A}{4k^2 B}e^{kx} + Be^{-kx}, \tag{6}$$

where A and B are two integration constants, which can be found from the boundary conditions. The imposed initial conditions are: $u(0) = u_0$ and $u(L) = u_L = -qu_0$ with $q = 5$. Moreover, it is assumed that there are extremum values of $u(x)$ only at the end points of the horn [2] such that $u'_{x=0} = u'_{x=L} = 0$. The following expressions for A and B are obtained:

$$\begin{cases} A = 4k^2 B\left(B - u_0\right) \\ B = \dfrac{u_0\left(q + e^{kL}\right)}{e^{kL} - e^{-kL}} \end{cases} \tag{7}$$

The spatial component of the wave is then $u_x = \left(u_0 - B\right)e^{kx} + Be^{-kx}$. The position of the nodal point x_n, which is chosen as the fastening point of the horn, can be obtained by considering $u(x) = 0$:

$$x_n = \frac{1}{2k} \ln\left(\frac{B}{B - u_0} \right). \tag{8}$$

From Hooke's elasticity theory it follows that the maximum strain and maximum stress happen at the nodal point. Also $u''_x = k^2 u_x$. Label $h(x) = h_x = \left[(u_0 - B)e^{kx} - Be^{-kx}\right]$. The optimal horn section is, from (3):

$$S_x = S_L \left(\frac{h_L}{h_x} \right)^2, \tag{9}$$

with L the horn's length. It follows that the maximum cross-section is situated at the nodal point, which is an important advantage for practical applications.

Taking into account the optimization procedure described above, we designed an USH made of steel, with an end diameter $d_L = 6\,mm$, an initial amplitude $u_0 = 2 \cdot 10^{-7}\,m$, and a gain coefficient $q = 5$. The nodal point is located at $x_n = 34.50\,mm$, where the diameter is $d_{\max} = 29.70\,mm$.

2. COMPUTER SIMULATIONS

Computer simulations were carried in order to determine the USH resonance frequencies by using 3 methods: the transfer matrix method (TMM), a coupled oscillators (CO) model and a 3D Finite Element Method (FEM).
In the transfer matrix method (TMM) the sound wave propagation is modeled by a transfer matrix $\mathbf{T}$ [5-9] in relation to a progressive and a regressive wave of complex-valued amplitudes $A = A(f)$ and $B = B(f)$ respectively, at frequency f:

$$\begin{pmatrix} A_{out} \\ B_{out} \end{pmatrix} = \mathbf{T} \begin{pmatrix} A_{in} \\ B_{in} \end{pmatrix}, \tag{10}$$

with the horn's input end in and the output end out. The USH was divided into N slices $j = 1,2,...,N$ with transversal surfaces S_j as in (9), thickness Δx and acoustic impedance $Z_j = °\, cS_j$, where $°$ = bulk density of concentrator's material. The sound wave propagation along each strip j is characterized by a propagation matrix $\mathbf{P}_j$ and propagation from layer j to layer $j+1$ is characterised by a discontinuity matrix $\mathbf{D}_{j,j+1}$; total reflection at the ends is characterised by a reflection matrix $\mathbf{R}$:

$$\mathbf{P}_j = \begin{pmatrix} \exp\left(i \frac{\mu}{c} \Delta x \right) & 0 \\ 0 & \exp\left(-i \frac{\mu}{c} \Delta x \right) \end{pmatrix}, \quad \mathbf{D}_{j,j+1} = \frac{1}{2} \begin{pmatrix} 1 + \frac{Z_j}{Z_{j+1}} & 1 - \frac{Z_j}{Z_{j+1}} \\ 1 - \frac{Z_j}{Z_{j+1}} & 1 + \frac{Z_j}{Z_{j+1}} \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \tag{11}$$

The transfer matrix from input to output is then:

$$\mathbf{T}_{in,out} = \mathbf{P}_N \mathbf{D}_{N-1,N} \mathbf{P}_{N-1} ... \mathbf{P}_3 \mathbf{D}_{2,3} \mathbf{P}_2 \mathbf{D}_{1,2} \mathbf{P}_1. \tag{12}$$

Propagation from input to output and back is modeled by the the matrix: $\mathbf{T}_{total} = \mathbf{T}_{out,in} \mathbf{R} \mathbf{T}_{in,out}$, such that:

$$\begin{pmatrix} A'_{in} \\ B'_{in} \end{pmatrix} = \mathbf{T}_{total} \begin{pmatrix} A_{in} \\ B_{in} \end{pmatrix}. \tag{13}$$

With $A_{in} = 1$ the resonance condition is $\left| A'_{in} - 1 \right| = 0$.

The concentrator was also modeled as a linear arrangement of coupled oscillators [10], set at equal distances and connected with springs. Parameters were computed from Hooke's law. The displacements $u_j(t)$ of the oscillators were obtained by numerically integrating the equations of motion

$$m_j \ddot{u}_j = k_{j-1,j}(u_{j-1} - u_j) + k_{j,j+1}(u_{j+1} - u_j) \quad (14)$$

with a 4th-order Runge-Kutta integration method [11]. The Fourier spectrum at the horn's ends was used to determine the resonance frequencies.

The 3D Finite Element simulation was done with the Elmer FEM software and the resonance modes were obtained. Those with a longitudinal behaviour were taken into consideration.

The USH was fitted with constant section input and output couplings that model the transducer and tool. Their lengths were chosen as $L_1 = 38.88$ mm for the input coupling and $L_2 = 32.13$ mm for the output coupling , such that the resonance frequency of the whole system is set at $f_{res} = 19900\ Hz$ and the position of the nodal point is fixed at $x_n = 34.72$ mm into the USH body.

Table 1: Resonance frequency values (Hz) of the concentrator with end couplings obtained with different methods.

TM	1D CO	3D FEM
19900	19836.73	19729.10
25802	25787.75	25937.11
38144	38147.55	37874.84
51955	52033.26	51325.97

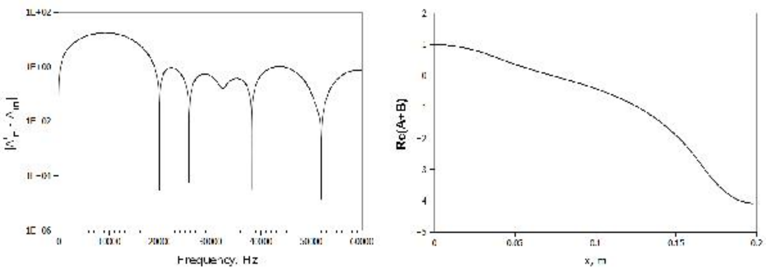

Figure 2: The frequency response of the USH with end couplings by using the TMM. Left: the resonance condition plot. Right: the wave amplitude along the system at $f = 19900\ Hz$.

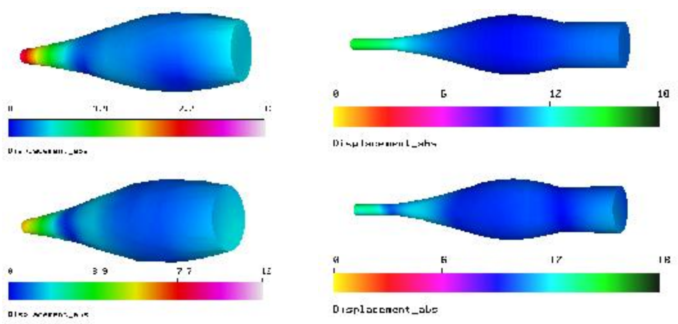

Figure 3: A few longitudinal eigenmodes of the USH obtained with FEM for the concentrator without (left) and with (right) end couplings.

3. CONCLUSION

An acoustic horn concentrator was designed by optimization with variational calculus. Simulations were carried in order to obtain the resonance frequencies [12]. Next coupling ends were added to the concentrator in order to set its working frequency at $f = 19900\ Hz$ for a total amplification factor $q = 5$. The concentrator is useful in mechanical manufacturing processes involving ultrasound, in which high-amplitude ultrasound is injected into a working piece. The dimensions of both the piece and the ultrasound generator establish the frequency at which optimal transmission occurs. Thus, it is important to take the 2 devices into account. The current approach allows for optimization of the embedded system formed from the acoustic horn concentrator and connected devices.

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