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METHOD OF RANDOM VIBRATION FOR THE DUFFING OSCILLATOR WITH NON-LINEAR ELASTIC CHARACTERISTIC AND EXTERNAL WHITE NOISE VEXCITATION

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Abstract: *The statistical linearization technique can also tackle a wide variety of problems and also provides approximate information on the frequency domain characteristics of the stochastic response. In this technique, a linear model, which optimally ts the original, nonlinear system (in some statistical sense), is constructed. The cause for this underestimation of the variance can be found by comparing accurate, simulated frequency domain characteristics with those determined using the linear model.This approximate representation of the system leads to estimates of the response spectrum that agree extremely well with those obtained by direct numerical simulation of the governing equation.*

Keyword: *statistical linearization, random vibration, power spectral density, response, gaussian white noise.*

1. INTRODUCTION

Nonlinear dynamic systems subject to random excitations are frequently met in engineering practice. The source of randomness can vary from surface randomness in vehicle motion and environmental changes, such as earthquakes or wind exciting high rise buildings or wave motions at sea exciting offshore structures or ships, to electric or acoustic noise exciting mechanical structures.
Given the large number of practical situations in which the teachings of the thesis find applicability, building vibrations when exposed to seismic movements, vibrations of the wheels of a car in traffic conditions, vibrations of the wing of an airplane. I expect that this thesis makes an important contribution and will trigger increased interest of various researchers in the field. The thesis can equally serve as a basis for further scientific endeavors, offering a strong basis for a researcher starting at the upper-graduate or post-graduate level.
While some particular exact solutions are available for specific systems under white noise excitations, many practical problems have been handled by approximate approaches such as linearization and closure assumptions. Even in these approximate procedures estimation of non-Gaussian characteristics of the responses, joint density functions and PSD functions pose difficulties. The most commonly applied and convenient procedure uses Yingfang, L. Zhao, Q. Chen.

2. SYSTEM MODEL

Consider the following oscillator with a nonlinear restoring force component. The results and the statistical parameters influences on response will be done based on computer running the corresponding programs.
The ordinary differential equation [1,2] of the motion can be written as:

$$m\ddot{\eta}(t) + c\dot{\eta}(t) + g(\eta(t)) = F(t) \tag{1}$$

where m is the mass, c is the viscous damping coefficient, $F(t)$ is the external excitation signal with zero mean and $\eta(t)$ is the displacement response of the system.
Dividing the equation by m , the equation of motion can be rewritten as:

$$\ddot{\eta}(t)+2\xi p\dot{\eta}(t)+h(\eta(t))=f(t)\ . \qquad (2)$$

where f(t) is a zero mean stationary Gaussian white noise excitation [3,4], i.e. a power spectral density $S'_0=\frac{S_F}{m^2}$ =1

We can always find a way to decompose the nonlinear restoring force [3,4,5] to one linear component plus a nonlinear component

$$h(\eta)=p^2(\eta+\frac{1}{\beta}G(\eta))\ , \qquad (3)$$

where β is the nonlinear factor to control the type and degree of nonlinearity in the system.

Obtain

$$\ddot{\eta}(t)+2\xi_e p_e\dot{\eta}(t)+p_e{}^2\eta(t)=f(t), \qquad (4)$$

where ξ_e is the critical damping factor

$$\xi_e=\frac{p}{p_e}\xi\ , \qquad (5)$$

and p_e is the undamped natural frequency.

The difference between the nonlinear stiffness and linear stiffness terms [6,7,8] is

$$e=h(\eta(t))-p^2{}_e\eta(t)\ . \qquad (6)$$

The value of p_e can be obtained by minimizing [6,7] the expectation of the square error

$$\frac{dE\{e^2\}}{dp_e{}^2}=0\ , \qquad (7)$$

Obtain

$$p_e{}^2=p^2(1+\frac{E\{\eta(t)G(\eta(t))\}}{\beta\sigma_\eta{}^2})\ . \qquad (8)$$

Obtains a solution for the stationary joint probability density function P [6,8,9] as:

$$P(\eta)=C_1\exp\left(\frac{-2\xi p}{\pi S'_0}\int_0^\eta h(u)du\right), \qquad (9)$$

where C_1 is a constant which normalises the density function.

The mean square value for the displacement of the system [9,10] is given by equation

$$\sigma_\eta^2=C_1\int_0^\infty \eta^2 e^{\frac{-2\xi p}{\pi S'_0}\int_0^\eta h(u)du}d\eta\ . \qquad (10)$$

The power spectral density of response [6,7,8] is

$$S_\eta(\omega)=\frac{S_F}{m^2}\frac{1}{\left(p^2+\frac{p^2E\{\eta G(\eta)\}}{C_1\beta\int_0^\infty\eta^2 e^{\frac{-2\xi p}{\pi S'_0}\int_0^\eta p^2(u+\frac{1}{\beta}G(u))du}d\eta}-\omega^2\right)^2+4\xi^2p^2\omega^2}\ . \qquad (11)$$

3. NUMERICAL RESULTS

To illustrate the procedure of equivalent linearization theory, let us consider the ordinary differential equation of the motion

$$\ddot{\eta}(t)+2\xi p\dot{\eta}(t)+p^2\eta(t)+\frac{1}{\beta}p^2 ch(\beta\eta(t))=f(t), \quad (12)$$

The difference between the nonlinear stiffness and linear stiffness terms is

$$e=p^2[\eta(t)+\frac{1}{\beta}ch(\beta\eta(t))]-p^2{}_e\eta(t). \quad (13)$$

Becouse

$$E\{\eta^2(t)\}=\sigma_\eta{}^2=\int_{-\infty}^{\infty}\eta^2(t)P(\eta(t))d\eta, \quad (14)$$

obtain for p_e

$$p_e{}^2=p^2\left(1+\alpha\frac{E\{\eta(t)ch(\beta t)\}}{\sigma_\eta{}^2}\right). \quad (15)$$

The density function of the sistem is

$$P(\eta)=C_1\exp\left(\frac{-2\xi p}{\pi S'_0}\int_0^\eta p^2[u+\frac{1}{\beta}ch(\beta u)]du\right), \quad (16)$$

or

$$P(\eta)=C_1 e^{\frac{-2\xi p}{\pi S'_0}\int_0^\eta p^2[u+\frac{1}{\beta}ch(\beta u)]du}=C_1 e^{-\frac{2\xi p^3}{\pi S'_0}\eta^2}. \quad (17)$$

We have

$$\int_{-\infty}^{\infty}P(\eta)d\eta=1. \quad (18)$$

For $m=1,25kg$, $k=38\frac{N}{m}$, $c=2,5\frac{Ns}{m}$, $\beta=6s^{-1}$, with $S_F=1N^2\cdot s$, we will find the statistical parameters of function.

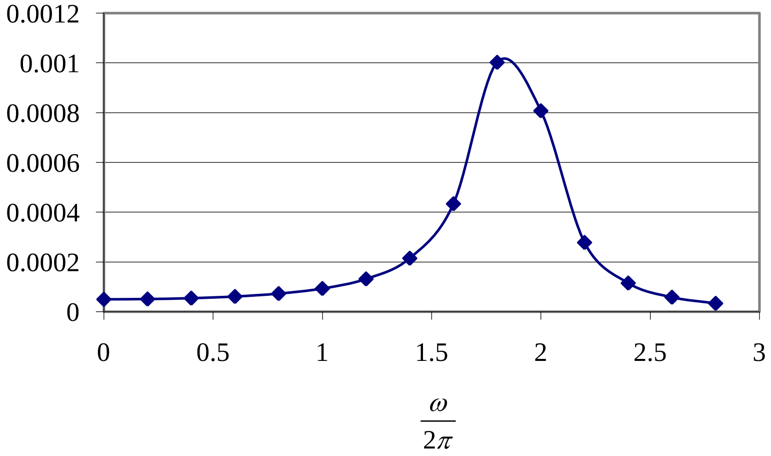

Fig. 5 The power spectral density $S_\eta\,[m^2\cdot s]$ for $m=1,25kg, k=48\frac{N}{m}, c=2,5\frac{Ns}{m}, \beta=6m^{-1}$.

The standard deviation of $\eta(t)$ is

$$\sigma_\eta^2=\int_{-\infty}^{\infty}\eta^2 P(\eta)d\eta=\frac{p}{\pi}\sqrt{\frac{2\xi p}{S'_0}}\int_{-\infty}^{\infty}\eta^2 e^{-\frac{2\xi p^3}{\pi S'_0}\eta^2}d\eta. \quad (19)$$

or

$$\sigma_\eta^2 = \frac{\pi S'_0}{4\xi p^3}. \tag{20}$$

The powe spectral density for thr system

$$S_\eta(\omega) = \frac{S'_0}{\left[p^2\left(1+e^{\frac{\pi S'_0 \beta^2}{8\xi p^3}}\right)-\omega^2\right]^2+4\xi^2 p^2\omega^2} = \frac{S_F}{m^2}\frac{1}{\left[p^2\left(1+e^{\frac{\pi S'_0 \beta^2}{8\xi p^3}}\right)-\omega^2\right]^2+4\xi^2 p^2\omega^2}. \tag{21}$$

3. CONCLUSION

The statistical linearization technique can also tackle a wide variety of problems and also provides approximate information on the frequency domain characteristics of the stochastic response. The cause for this underestimation of the variance can be found by comparing accurate, simulated frequency domain characteristics with those determined using the linear model. This method is approximate equation of motion of an equation equivalent to this, but will be the linear, and able to find out the statistical characteristics of the response. It can be applied in the case of non-linear spring characteristic, and the damping characteristic is linear. The resonant peak is described very satisfactorily by the approximate solution.

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