



## USING SENSITIVITY ANALYSIS FOR MODELING VEHICLE DYNAMICS

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**Abstract :** *The objective of the paper is to analyze experimental data and to identify the relationship between different measured parameters using sensitivity analysis. Sensitivity analysis is a technique used to determine how different values of an independent variable will impact a particular dependent variable under a given set of assumptions. It is useful for testing the robustness of the results of a model or system in the presence of uncertainty, increasing understanding of the relationships between input and output variables in a system or model, searching for errors in the model by encountering unexpected relationships between inputs and outputs, model simplification by fixing model inputs that have no effect on the outputs. In our study, data were obtained by measuring the dynamic parameters of vehicles and engines. Testing program aimed to capture a large range of operating regimes. The study noted the need to perform sensitivity analysis before developing mathematical models of vehicle dynamic*

**Keywords :** *sensitivity analysis, vehicle dynamics, statistics, modeling, experimental data*

### 1. INTRODUCTION

Experimental research followed the execution of a large number of tests, more than 100, on different runways, pavement mosaic tiles or asphalt; in various weather conditions, sunny and warm weather or rain or sleet and snow; adopting both, normal and sport style of driving, using 13<sup>th</sup> Nubira vehicles with mileages from 13500 to 115000 km. Most of the experimental data were acquired and stored in the tester "SCAN - 100", intended to diagnose cars Daewoo Nubira. Two parameters, torque and consumption, could not be measured using onboard system. They were computed using static characteristics of the engine that had been obtained on the test bench. At the end, there were selected 64<sup>th</sup> test with 15<sup>th</sup> parameters and saved as a tensor.[2]

Mathematical models are used to approximate highly complex physical or engineering phenomena. Model development consists of several logical steps. One of this is the determination of parameters which is most influential on model results. In this respect, sensitivity analysis of these parameters serves to guide future research efforts. Some reasons to made a sensitivity analysis are: the need to determine which parameters are insignificant and can be eliminated from the final model or the need to determine which inputs contribute most to output variability.[1]

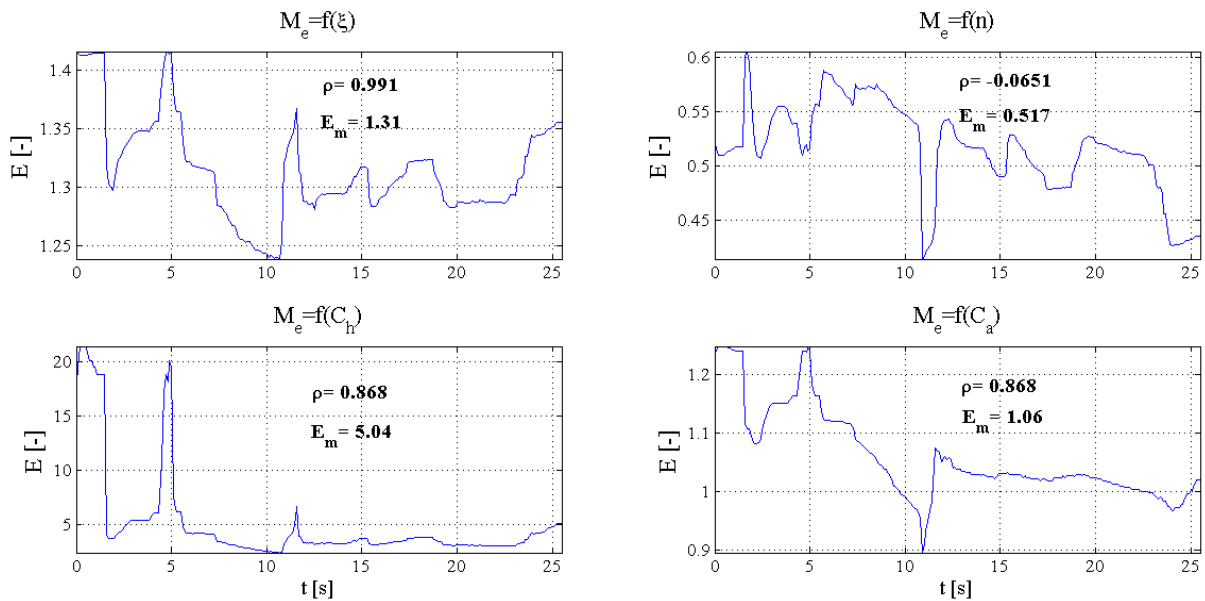
### 2. SIMPLE SENSITIVITY ANALISIYS OF VEHICLE DYNAMICS

More than a dozen methods have been developed to perform sensitive analysis for various modeling situations. Only few of these are intended for highly complex or very large models. The one we choose is known as direct method or differential sensitivity analysis [1].

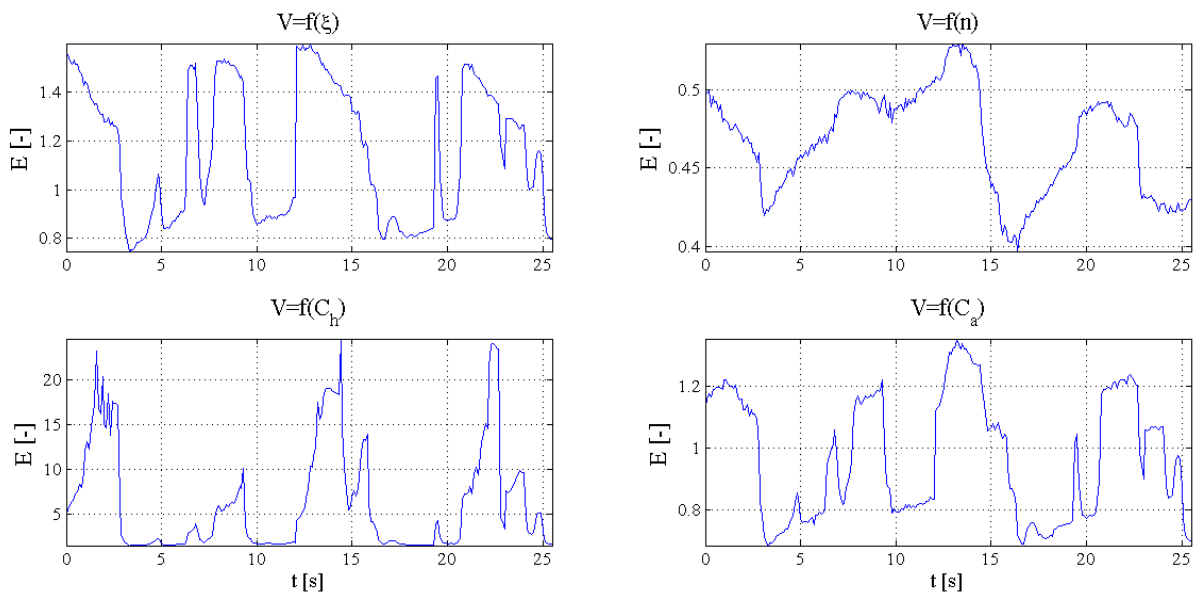
Differential analysis of parameter sensitivity is based on partial differentiation of a model in a aggregated form. Simple sensitivity function  $E_{y;x}$  is defined by equation (1):

$$E_{y;x} = \frac{dy}{y} : \frac{dx}{x} \quad (1)$$

where  $y$  is the resultant parameter and  $x$  is one factorial parameter. It is equal with the rate of relatively change of the resultant parameter-  $y$  produced by a unit of relatively change of one factorial parameter-  $x$ . [2] A sensitivity coefficient is basically the ratio of the change in output to the change in input while all parameters remain constant.[3]



**Figure 1:** Engine torque simple sensitivity functions of different models, test I24n



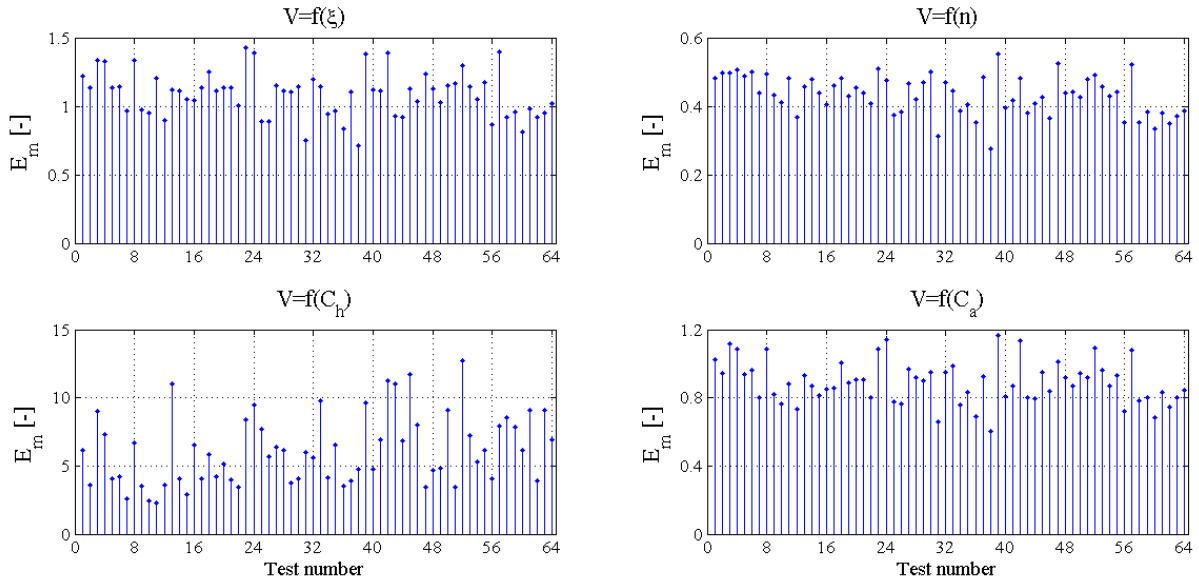
**Figure 2:** Vehicle speed simple sensitivity functions of different models, test I27n

Figure 1 presents simple sensitivity functions of four models of vehicle dynamics: engine torque- $M_e$  as resultant parameter and engine angular speed- $n$ , hourly fuel consumption- $C_h$ , and air consumption- $C_a$  as factorial parameters in case of test I24n. The graphs show a high sensitive dependency between engine torque- $M_e$  and hourly fuel consumption- $C_h$ , which is a normal appearance considering that extra fuel sent into the combustion chamber should produce more torque. Also, that proves the engine is in good working order.

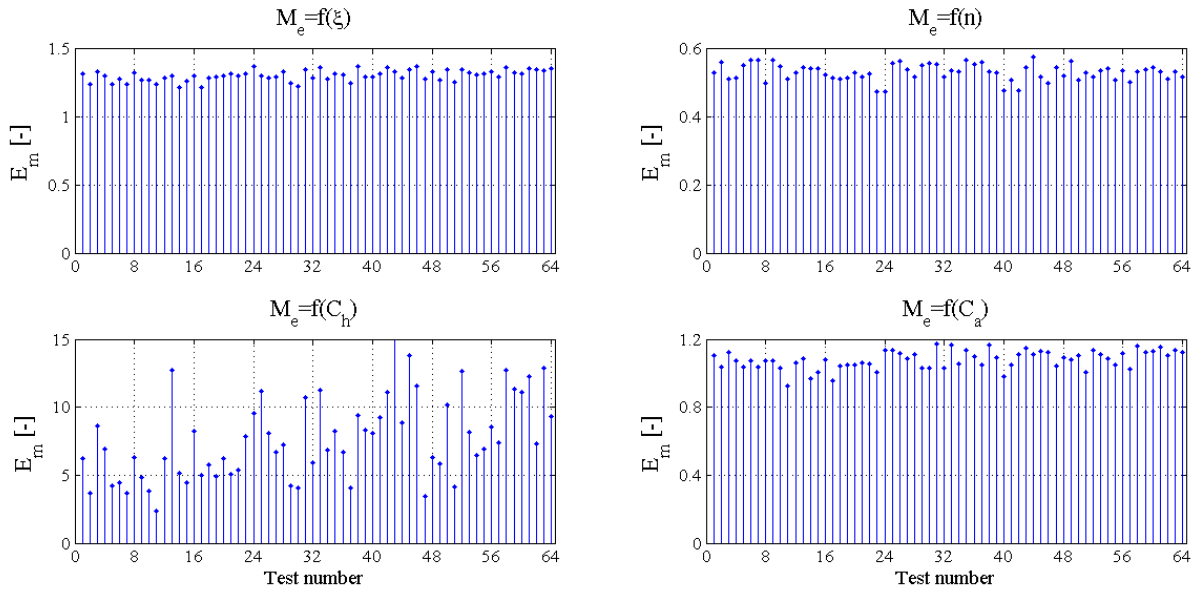
On each graph were mentioned the correlation coefficients and the mean value of sensitivity function of each model. As can be seen, a good correlation between two parameters does not cause a high sensitivity between them.

Sensitivity functions of vehicle speed as resultant parameter and engine angular speed- $n$ , hourly fuel consumption- $C_h$ , and air consumption- $C_a$  as factorial parameters in case of test I27n are presented in Figure 2. In this case, vehicle speed is high sensible of engine's load- $\xi$  and hourly fuel consumption- $C_h$ . Dynamic series

shows strong variations, so is difficult to conduct a good sensitive analyze for all parameters and tests. A good way to solve this problem is to use mean values of sensitive functions for each model and test. In Figure 3 is shown the mean values of sensitivity functions for all tests, for vehicle speed- $V$  as resultant parameter and engine angular speed- $n$ , hourly fuel consumption- $C_h$ , and air consumption- $C_a$  as factorial parameters.



**Figure 3:** Mean values for sensitivity functions for different vehicle speed models and all tests



**Figure 4:** Mean values for sensitivity functions for different engine torque models and all tests

Figure 4 presents the mean values of sensitivity functions for all tests, for engine torque- $M_e$  as resultant parameter and engine's load- $\zeta$ , engine angular speed- $n$ , hourly fuel consumption- $C_h$ , and air consumption- $C_a$  as factorial parameters. Both figures confirm that the highest sensitivity is between vehicle speed- $V$  and engine torque- $M_e$  as resultants parameters and hourly fuel consumption- $C_h$  as factorial factor.

### 3. MULTIPLE SENSITIVITY ANALISIYS OF VEHICLE DYNAMICS

Multiple sensitivity model is defined by one resultant parameter and two or more factorial parameters [4]. The multiple sensitivity coefficient is computed with [2]:

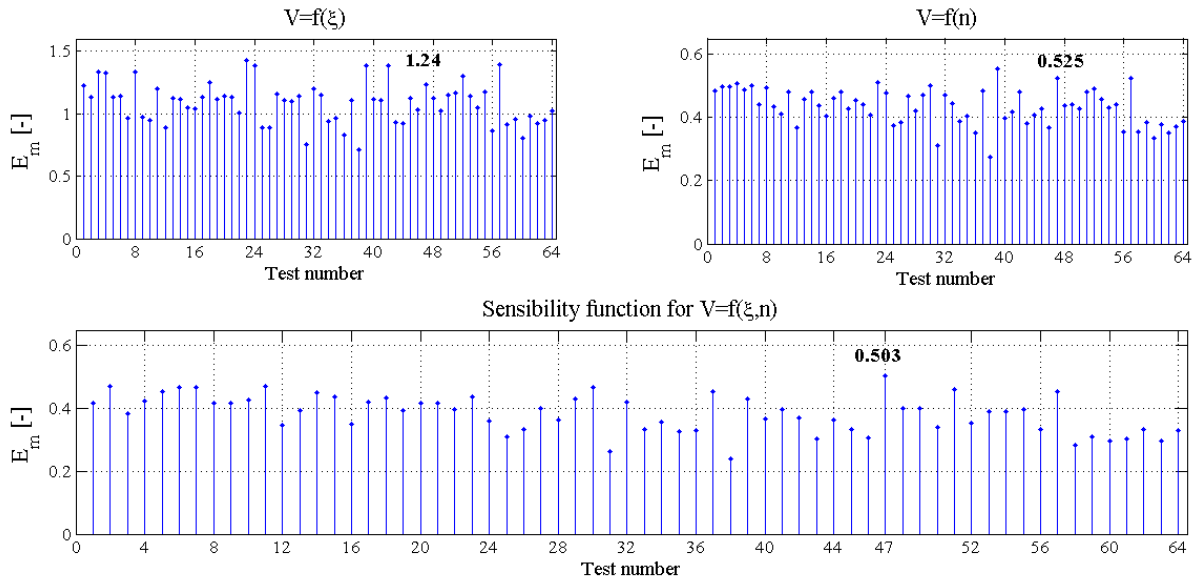
$$E_{y;x_1;x_2} = \sqrt{\frac{E_{y;x_1}^2 + E_{y;x_2}^2 - 2E_{y;x_1}E_{y;x_2}E_{x_1;x_2}}{1 - E_{x_1;x_2}^2}} \quad (2)$$

$$E_{y;x_1} = \frac{dy}{y} \cdot \frac{dx_1}{x_1} \quad (3)$$

$$E_{y;x_2} = \frac{dy}{y} \cdot \frac{dx_2}{x_2} \quad (4)$$

$$E_{x_1;x_2} = \frac{dx_1}{x_1} \cdot \frac{dx_2}{x_2} \quad (5)$$

where  $E_{y;x_1}$ ,  $E_{y;x_2}$  and  $E_{x_1;x_2}$  are simple sensitivity coefficients of  $y=f(x_1)$ ,  $y=f(x_2)$  and  $x_1=f(x_2)$  models.



**Figure 5:** Mean values for sensitivity functions for vehicle speed for all tests

Figure 5 depicts a comparison of two single sensitivity models and a multiple sensitivity model for vehicle speed as resultant parameter, and engine's load- $\zeta$  and engine angular speed- $n$  as factorial parameters. The maximum of the mean values of multiple sensitivity function is reach during the test I47n. Its value is lower than the corresponding values of simple sensitivity models for test I47n. As stated in Figure 5, taking into account two influence factors the multiple sensitivity changes as compared with simple sensitivity.

#### 4. CONCLUSION

Sensitivity, like correlation should be studied as both influence the accuracy of the linear mathematical models adopted for statistical vehicle's dynamics.

#### REFERENCES

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