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OPTIMIZATION OF A "SANDWICH" STRUCTURE FOR THE INSULATION OF A PREFABRICATED WOODEN HOUSE

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Abstract: This paper aims to determine an optimal solution in terms of cost to the problem of determining a structure composed of layers of different material which ensures a required heat transfer from inside a building to outside them (a suitable insulation at a minimum price) . The problem is placed in a zone of interference between the heat exchange and construction research domains. A theoretical study on the method of optimizing the objective function cost price considering heat transfer by laminated flat surfaces was carried out, and experimental measurements were made in order to validate the results. The literature reveals the existence of a corrective coefficient for each insulation material but not for structures of the type we studied. Knowledge of these factors allows the design of dimensionally and thermally constructions having similar composition as those in the study and allows optimizing the performance in terms of costs.
Keywords: insulation material, structures, heat transfer.

1. INTRODUCTION

In order to achieve resistance elements and structures with outstanding performance, industry experts are increasingly concerned about the many possibilities of combining wood with other materials and products made from wood or other materials, with different properties. The paper proposes the use of an optimization method for the determination of a solution to combine several types of boards so as to obtain a material with good thermal insulating properties. [1]-[6]
There have been theoretical and experimental research on a large number of "sandwich" structures that can be widely used in the construction of prefabricated wooden houses. The method we adopted was following on the possibility of combining aspects of common insulation materials, cheap, domestic (mineral wool, polystyrene, PAL) to form a sandwich structure thermophysical properties that are suitable for use as wall panels for prefabricated houses.
In practice, this kind of panels are developed without scientifically motivating a number of issues such as: the possibility of combining different materials (with various physical and mechanical, elastic and strength characteristics) of large diversity, the influence the dimensions (width, length thickness, etc.) on the characteristics of rigidity and stability of panels of thermal and sound absorbing properties. Therefore, a consistent methodology according to objective criteria in conformity with the related problems of choosing materials to achieve, in terms of minimum cost, of insulating panels for construction is required,.
These panels can be classified as composite materials with a special structure - designed to capitalize on the higher characteristics of each material that is included in them, in order to be able to cope with a variety of operating conditions characterized by variations in humidity, temperature, mechanical, static and dynamic stresses.

2. OPTIMIZATION METHOD

The general formulation of a single-criterion linear programming problem is the following:

$$\text{minimize} \quad f(x) \quad \text{with} \quad x = (x_1, x_2, \dots, x_N)$$

subjected to: $g_j(x) \leq 0$ where $j=1,2,....,P$
$h_i(x) = 0$ where $i=P+1,...,P+M$

$f(x)$ is a linear function, $g_j(x)$ and $h_i(x)$ are inequality and equality constraints, respectively.

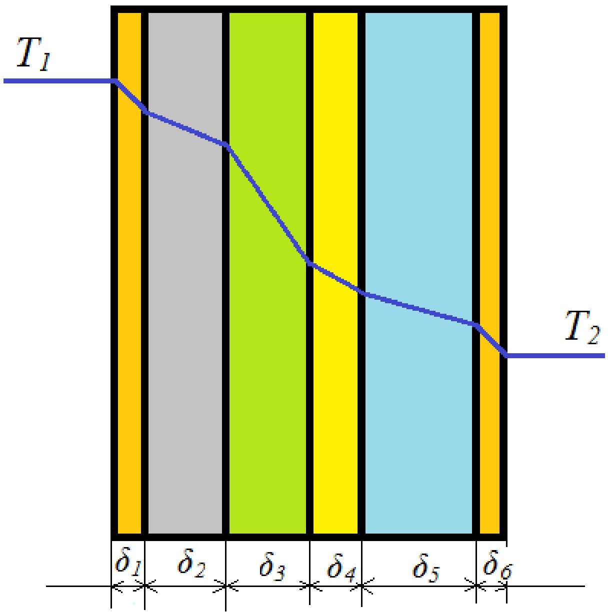


Figure 1. Multilayered construction material

The cost function f(x) can be written using a linear expression:

$$f(x) = \alpha_1\delta_1 + \alpha_2\delta_2 + ... \quad . + \alpha_N\delta_N$$

where α_1, α_2 ... ., α_N are cost coefficients and $\delta_1, \delta_2, ...$, δ_N represent, respectively, the width of the layers 1, 2,... ., N.

The cost calculation is founded on material costs and fabrication costs, which have direct effect on the dimensions and geometry of the structure. Generally, the cost function includes the cost of material, assembly, painting, cutting, forming the shell but for our purposes we will consider the costs of the materials used considering, for the begining, the other costs being the same for all solutions. Informations considering different fabrication costs can be found in [7]-[12].

3. EXPERIMENTAL DATA

Based on the principle of minimizing the cost price by imposing conditions on the total wall resistance and maximum wall thickness, we are considering solutions resulting from the calculation and measure the heat transfer coefficient variation for assemblies in the various combinations of these materials.The conditions that were considered in choosing the types of structures made were:

- to be common (most commonly used in the construction of wooden houses) ;
- to allow drawing conclusions by comparing the values obtained from tests that are practical recommendations for users.Other conditions were imposed in the development, not only in the choice of these structures :
- the insulating layer to be formed only of mineral wool and polystyrene , so the outside is chipboard ;
- for multilayered structures, the polystyrene layer to be on the warm side
- for three-layer structures, the insulating layer to be formed by combining two symmetrical insulation materials.

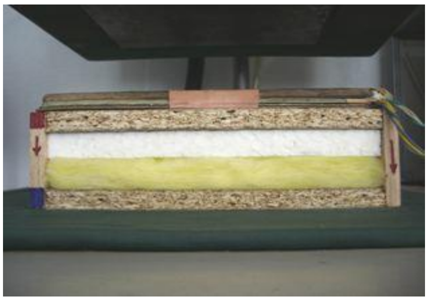
For ease of tracking, processing and interpretation of experimental results we have developed a code for each specimen, using the notation: P - PA;L p - polystyrene; v - wool, followed by figures representing the layer thickness. We have carried out experiments on three types of structures:

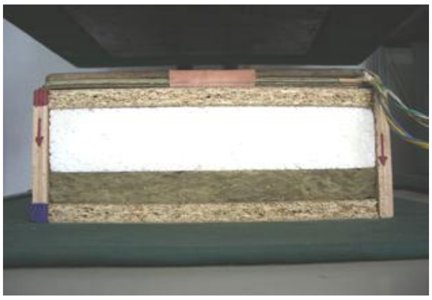
I.monolayer
a) mineral wool insulation layer
b) polystyrene insulating layer

a) b)

Figure 2. Monolayer

II.dual-layer
a) the insulating materials are mineral wool (varies) and polystyrene (constant)
b) the insulating materials are mineral wool (constant) and polystyrene (varies)





a) b)

Figure 3. Dual-layer

We determined the thermal conductivity coefficient equivalent to the material resulting from the composition of different types of materials. Knowledge of these factors determines a correction factor (denoted with $c = \frac{\lambda_e}{\lambda_t}$ where λ_e is the coefficient of thermal conductivity experimentally determined and λ_t is the coefficient of thermal conductivity determined by calculation).

Based on the Fourier's law the heat flux is proportional to the local temperature gradient. For a three layer the heat transfer rate through the first layer is:

$$q_1 = \frac{\lambda_1 A}{t_1}(T_0 - T_1) = \frac{(T_0 - T_1)}{R_1} \quad (1)$$

while the heat rates through the second and the third layer are:

$$q_2 = \frac{\lambda_2 A}{t_2}(T_1 - T_2) = \frac{(T_1 - T_2)}{R_2} \quad (2)$$

$$q_3 = \frac{\lambda_3 A}{t_3}(T_2 - T_3) = \frac{(T_2 - T_3)}{R_3} \quad (3)$$

As a steady-state analysis is considered in this paper, the heat flows passing each layer are equal while no internal heat is generated:

$$q_1 = q_2 = q_3 = q \quad (4)$$

Therefore by substituting equations (1), (2) and (3), the heat transfer rate thorough the layered composite is:

$$q = \frac{T_0 - T_3}{R_1 + R_2 + R_3} = \frac{T_0 - T_3}{\sum R_i} \quad (5)$$

where R_i is the conductive thermal resistance of layer *i*:

$$R_i = \frac{t_i}{\lambda_i A} \quad (6)$$

The global heat transfer coefficient is:

$$\lambda_e = \frac{\delta}{\sum \frac{\delta_i}{\lambda_i}}.$$

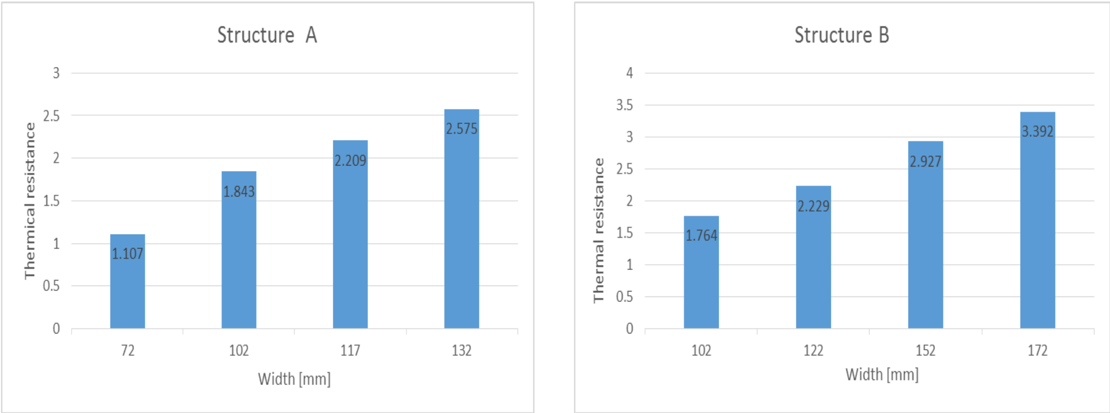
The results of measurements and theoretical calculations are summarized in tabelar form corresponding to each category of structures in order to be analyzed and interpreted graphically. Table 2 is a complete data on monolayer structures in two variants:

a) mineral wool insulation layer
b) polystyrene insulating layer

Table 2. Table of cumulative data for monolayer samples

Sample	Total width (mm)	The coefficient of thermal conductivity experimentally determined λ_e (W/mK)	The coefficient of thermal conductivity theoretically determined λ_t (W/mK)	Theoretically calculated thermal resistance R (m^2K/W)	The correction coefficient $c = \frac{\lambda_e}{\lambda_t}$
PvvP 16,20,20,16	72	0,051	0,065	1,104	0,784
PvvP 16,35,35,16	102	0,049	0,055	1,843	0,890
PvvP 16,35,50,16	117	0,047	0,053	2,209	0,886
PvvP 16,50,50,16	132	0,040	0,051	2,575	0,784
PppP 16,40,30,16	102	0,048	0,058	1,764	0,827
PppP 16,40,50,16	122	0,047	0,055	2,229	0,854
PpppP 16,50,20,50,16	152	0,043	0,052	2,927	0,826
PpppP 16,50,40,50,16	172	0,040	0,051	3,392	0,784

Having all the necessary data, a graphic was made for the dependence between the coefficient of thermal conductivity experimentally determined (λ_e) and the theoretically determined one (λ_t), taking into considerration the mineral wool insulation thickness (fig 1.)



a. b.

Figure 4
a. thermal resistance variation for the (PvvP) variant
b. thermal resistance variation for the (PppP) variant

Fig. 5 Shows the temperature variation through the composite layups considered.

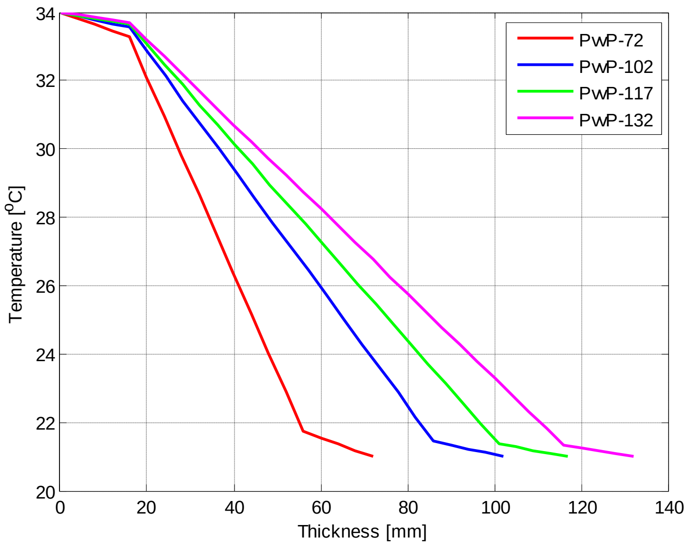


Figure 5. Temperature variation through the layered composite PvvP

An almost identical decrease for the two heat transfer coefficients can be very well seen and, therefore, an increased equivalent thermal resistance compared to using mineral wool, reinforcing further the findings from the literature regarding the influence on the heat transfer of moisture and uniformity of the material.

A planar steady state heat transfer finite element analysis has been performed for all the composite layups presented in the previous section. Fig. 6 shows the 2D finite element model and the corresponding primary output data represented by the temperature variation through the layered composite PvvP-72.

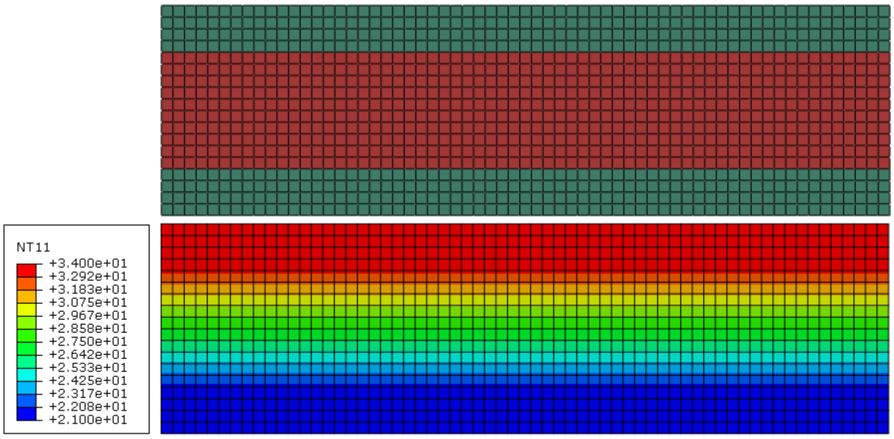


Figure 6. Temperature variation through the layered composite PvvP-72

Fig. 7 shows the temperature variation through the composite layups considered.

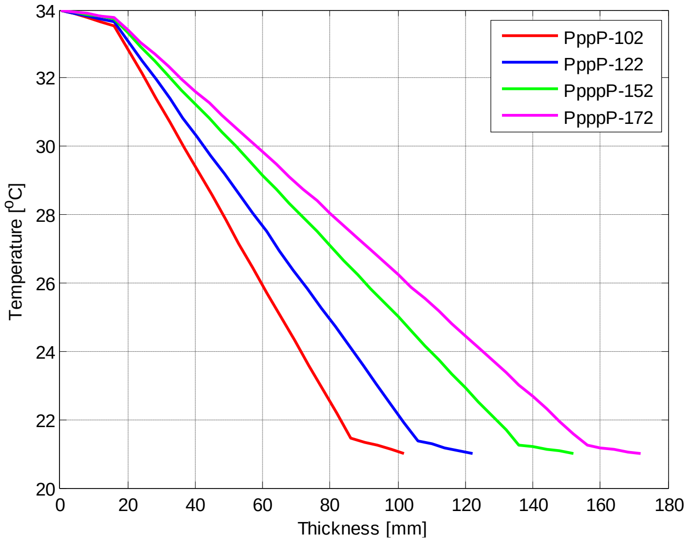
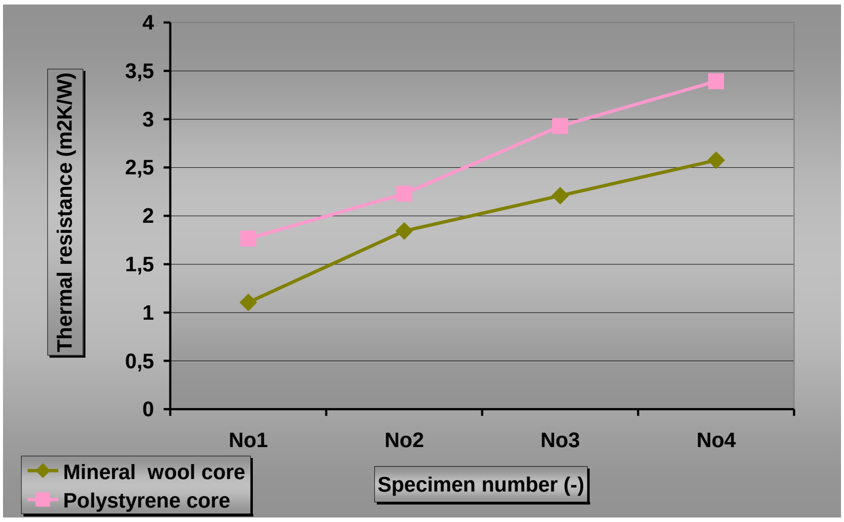


Figure 7. Temperature variation through the layered composite PppP

From the study of this representation , an almost linear decrease of the two heat transfer coefficient can be seen, depending on the insulation thickness. The somewhat larger differences that occur in specimens of thickness 72 mm and 132 mm are explained by the fact that both specimens have some moisture at the surface and in the interior and the materials used were not the same for each sample. No material was changed from one sample to another, using commercially available materials that vary even from batch to batch and, also, the mineral wool is not as compact as polystyrene.

The decrease transfer coefficient theory should involve a linear increase of thermal resistance, which is well highlighted in Figure 8.



Mineral wool core - POLYSTYRENE CORE

Figure 8. Thermal resistance variation for PvvP , PppP

Analyzing the chart above we can draw the conclusion that, with the increasing thickness of both thread and polyester wool, an increase of the value of thermal resistance of the sample can be observed and, at the same time, involving a decrease in the value of heat transfer coefficient (both the experimentally and the theoretically determined) . This linear increase in the thermal resistance happens in concordance with the linear decrease of the value of the heat transfer coefficient, being inversely proportional to the thickness of the insulating layer. Experimental and theoretical research data for dual-layered structures are summarized in Table 3 in order to be able to be interpreted.

Table 3. Table of cumulative data for dual-layer samples

Epruveta	Total width (mm)	The coefficient of thermal conductivity experimentally determined λ_e (W/mK)	The coefficient of thermal conductivity theoretically determined λ_t (W/mK)	Theoretically calculated thermal resistance R (m^2K/W)	The correction coefficient $c = \frac{\lambda_e}{\lambda_t}$
PpvP 16,20,20,16	72	0,060	0,066	1,089	0,909
PpvP 16,20,30,16	82	0,058	0,062	1,333	0,935
PpvP 16,20,50,16	102	0,050	0,056	1,821	0,892
PpvP 16,20,80,16	132	0,044	0,052	2,552	0,846
PpvP 16,20,20,16	72	0,060	0,066	1,089	0,909
PpvP 16,30,20,16	82	0,057	0,062	1,322	0,919
PpvP 16,50,20,16	102	0,050	0,057	1,787	0,877
PpvP 16,80,20,16	132	0,046	0,053	2,484	0,867

For these types of structures, in the case when the mineral wool insulation layer varies and the polystyrene remains constant, based on data in Table 3, we were able to represent the variation of the two heat transfer coefficients, λ_e and λ_t (Fig.7) and also we were able to show the change in resistance heating of the specimen (Fig. 8). Both graphs are based on the thickness of the specimen, thus the thickness of mineral wool.

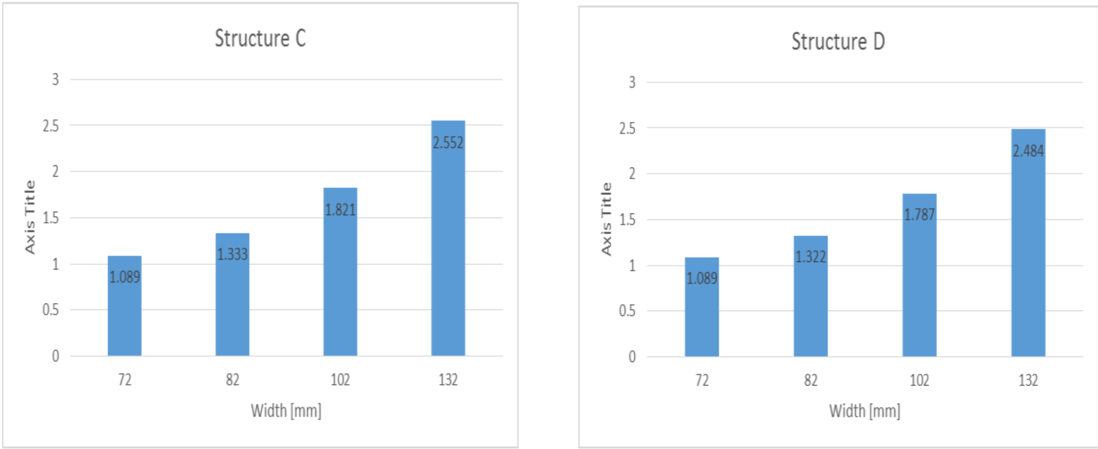


Figure 9.
a. thermal resistance variation for the (PpvP) variant
b. thermal resistance variation for the (PvpP) variant

A quite close variation of the two factors can be seen, the differences being due to the fact that the mineral wool insulation layer is not compact and has air gaps in the structure, factors which have influence on the value of the heat transfer coefficient.

This time, considering only the increase in the polystyrene layer, which is more compact than the wool, the most important aspect is the almost constant difference between the results obtained experimentally and those which were theoretically determined.

For these types of dual-layered structures, with a simultaneous increase of both the layer of mineral wool and the polystyrene layer so that the overall insulation thickness remains constant, we started to study on the same graph of the variation of heat transfer coefficient determined experimentally (Fig. 8) in the two situations.

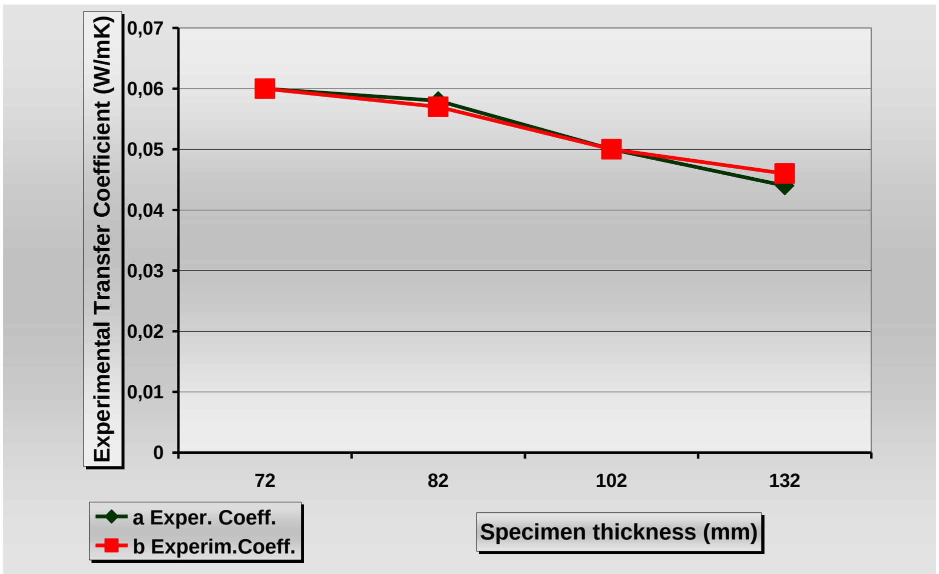


Figure 10. The heat transfer coefficient in the dual-layered structure options (a- the polystyrene-layer remains constant, b- the wool layer remains constant)

It can be seen from the superposition of the two graphs that the two solutions provides almost identical results. The order of layers in achieving the final solution is not important.

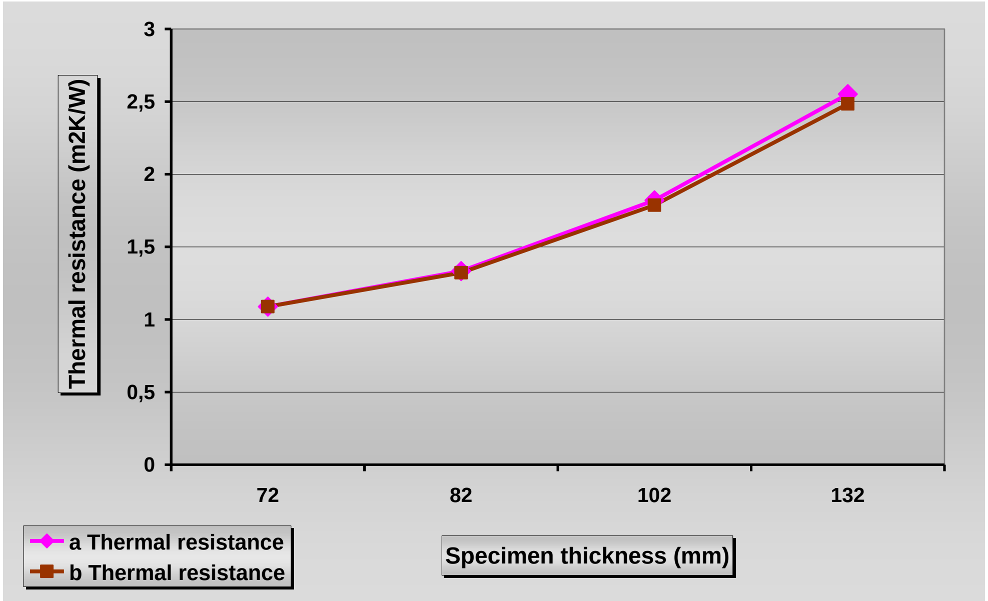


Figure 11. Thermal resistances for the two cases of dual-layered structures
(a - the polystyrene layer remains constant, b - the wool layer remains constant)

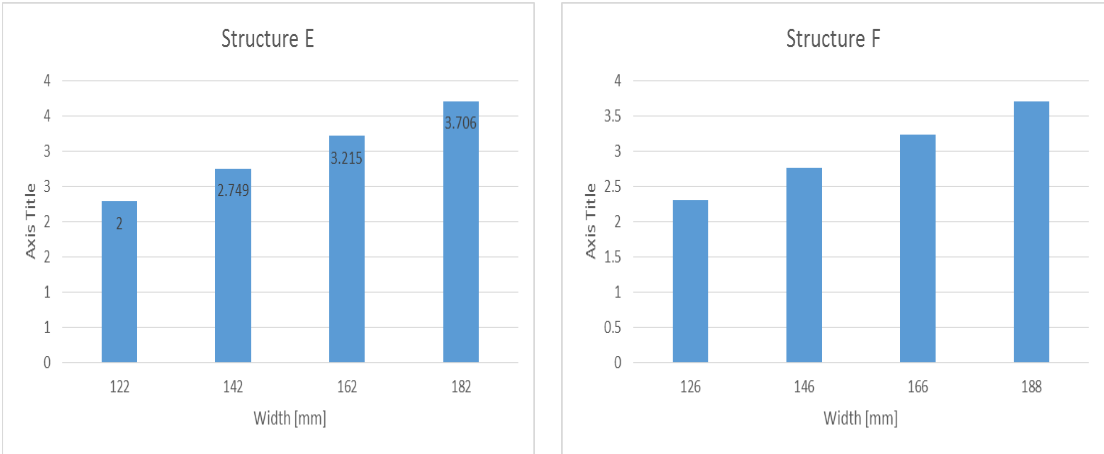


Figure 12. The thermal resistances for the two cases of triple-layered structures for two studied variants

4. CONCLUSIONS

The diagrams resulted from the experiments on the presented structures show the change of layout for the analyzed characteristics and can eventually be used as guidance for thicknesses greater than those analyzed in the paper , with relatively small errors by extrapolation. This observation is supported by the small differences that occur between the values of theoretical and experimental data

Analyzing the results obtained, we could appreciate against the influence of types of insulation material, and the thickness thereof, for the "sandwich " structure . We can draw the following conclusions:

• a good insulating material - with a low thermal conductivity - has a certain porosity , pores containing gases that contribute to the reduction of thermal conductivity and volume density.

• the lowering of the heat transfer coefficient with increasing insulation thickness - but this also depends on the sequence of layers . A large increase in the insulating layer is not recommended , it's better to follow a very good correlation between the insulation thickness and heat transfer coefficient in order to simultaneously solve the problem of the weight of the construction and the low heat transfer

A few different types of constraints can be formulated in order to optimize the performance, such as:

• determined values for the heat transfer coefficient are imposed. An optimum value for the total heat transfer coefficient 0.3…0.5 W/m2K ;

• a given temperature is required on the outer surface or the inner wall ;

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