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ON THE MODELING AND SIMULATION OF AN ACOUSTIC CLOAK

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Abstract: *Transformation acoustics opens a new avenue towards the modeling and simulation of acoustic cloaks. The design of acoustic cloaks is based on the property of Helmoltz equations to be invariant under a geometric transformation. In this paper, a spherical shell cloak made of auxetic material (material with negative Poisson's ratio) is discussed. The original domain consists of spheres made from conventional foam with positive Poisson's ratio. The spatial compression leads to an equivalent domain filled with auxetic material.*
Keywords: *Helmoltz equation, transformation acoustics, acoustic cloak, auxetic material*

1. FORMULATION OF THE PROBLEM

Transformation acoustics is the key for the design of acoustic cloaks. Recent works show that such materials could cloak regions of space, making them invisible to sound [1-3]. We refer to acoustic cloaking which occurs when a medium contains a region in which noisy objects can be acoustically hidden. It is easy to imagine an object invisible to sound by building a box around it to prevent the wave from reaching the object.
The principle how to cloak a region of space to make its contents invisible or transparent to waves was discussed in [4, 5]. The geometric invariance of Helmoltz equation shows how a region of the space can be made inaccessible to acoustic waves by surrounding it with a suitable shield. As an alternative to a box made from a metamaterial, sonic composites exhibit the full band-gaps, where the sound is not allowed to propagate due to complete reflections [6-9].

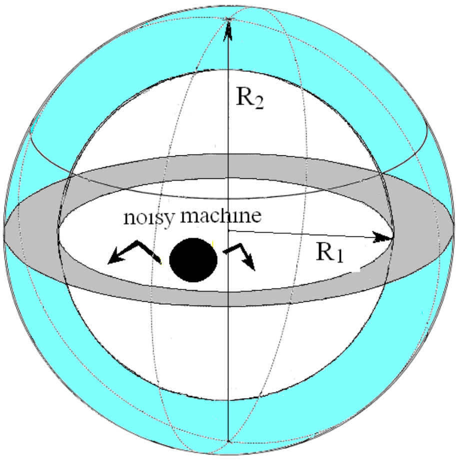

Figure 1: Sketch of the spherical cloak surrounding a noisy machine.

In this paper, we apply the 3D concave-down transformation to design a spherical cloak which surrounds a noisy machine, see Figure 1. The original domain is a sphere of radius R_2, consisting of a traditional foam with positive Poisson's ratio. After the transformation, the cloak contains a region $r < R_1$ which contains the noisy source, filled with air, while the shell $R_1 < r < R_2$ is filled by auxetic material.

2. TRASFORMATION ACOUSTICS

A finite size object surrounded by a coating consisting of a specially designed material would become invisible for acoustic waves at any frequency. In acoustics, the idea of the invisibility cloak is that the sound sees the space differently. For the sound, the concept of distance is modified by the acoustic properties of the regions through which the sound travels. In geometrical acoustics, we are used to the idea of the acoustical path; when travelling an infinitesimal distance $\mathrm{d}s$, the corresponding acoustical path length is $c^{-1}\mathrm{d}s$, where $c^{-1} = \sqrt{\rho/\kappa}$ with ρ is the fluid density and κ is the compression modulus of the fluid. To understand the problem, we consider the 3D acoustic equation for the pressure waves propagating in a bounded fluid region $\Omega \subset \mathrm{R}^3$

$$\nabla \cdot (\underline{\underline{\rho}}^{-1} \nabla p) + \frac{\omega^2}{\kappa} p = 0 , \tag{1}$$

where p is the pressure, $\underline{\underline{\rho}}$ is the rank-2 tensor of the fluid density, κ is the compression modulus of the fluid, and ω is the wave frequency. Let us consider the geometric transformation from the coordinate system (x', y', z') of the compressed space to the original coordinate system (x, y, z), given by $x(x', y', z')$, $y(x', y', z')$ and $z(x', y', z')$. The change of coordinates is characterized by the transformation of the differentials through the Jacobian $J_{xx'}$ of this transformation, i.e.

$$\begin{pmatrix} \mathrm{d}x \\ \mathrm{d}y \\ \mathrm{d}z \end{pmatrix} = J_{xx'} \begin{pmatrix} \mathrm{d}x' \\ \mathrm{d}y' \\ \mathrm{d}z' \end{pmatrix}, \quad J_{xx'} = \frac{\partial(x, y, z)}{\partial(x', y', z')} . \tag{2}$$

From the geometrical point of view, the change of coordinates implies that, in the transformed region, one can work with an associated metric tensor [10]

$$T = \frac{J_{xx'}^{\mathrm{T}} J_{xx'}}{\det(J_{xx'})} . \tag{3}$$

In terms of the acoustic parameters, one can replace the material from the original domain (homogeneous and isotropic) by an equivalent compressed one that is inhomogeneous (its characteristics depend on the spherical (r', θ', ϕ') coordinates) and anisotropic (described by a tensor), and whose properties, in terms of $J_{x'x}$, are given by

$$\underline{\underline{\rho}}' = J_{x'x}^{-\mathrm{T}} \cdot \rho \cdot J_{x'x}^{-1} \cdot \det(J_{x'x}) , \quad \kappa' = \kappa \det(J_{x'x}) , \tag{4}$$

or, equivalently, in terms of $J_{xx'}$

$$\underline{\underline{\rho}}' = \frac{J_{xx'}^{\mathrm{T}} \cdot \rho \cdot J_{x'x}}{\det(J_{xx'})} , \ \kappa' = \frac{\kappa}{\det(J_{xx'})} . \tag{5}$$

Here, $\underline{\underline{\rho}}'$ is a second order tensor. When the Jacobian matrix is diagonal, (4) and (5) can be more easily written.

The geometric transformation may be linear or nonlinear. Qiu [11] classified the geometric transformation functions in terms of the negative (i.e., concave-down) or positive (i.e., concave-up) sign of the second order derivative of this function. All transformations, i.e. linear, concave-up and concave-down transformations, are perfect cloaks for the exact inhomogeneous design.

The concave-down nonlinear transformation compresses a sphere of radius R_2 in the original space Ω into a shell region $R_1 < r' < R_2$ in the compressed space Ω' as

$$r(\beta) = \frac{R_2^{\beta+1}}{R_2^{\beta} - R_1^{\beta}} \left(1 - \left(\frac{R_1}{r'} \right)^{\beta} \right) , \tag{6}$$

where β denotes the degree of the nonlinearity in the transformation. By taking $\beta \to 0$ in (6), the linear case is obtained, namely

$$r(\beta) = \frac{R_2 \mathrm{Ln}(r'/R_1)}{\mathrm{Ln}(R_2/R_1)}. \tag{7}$$

All curves belonging to (6) have negative second order derivative with respect to the physical space r'. This class of transformations is termed as the *concave-down* transformation. The transformation function (6) depends on the radial component r' in the spherical coordinate system (r', θ', ϕ').

The concave-up nonlinear transformation compresses a sphere of the radius R_2 in the original space Ω into a shell region $R_1 < r' < R_2$ in the compressed space Ω' as

$$r(\beta) = \frac{R_2 R_1^\beta}{R_2^\beta - R_1^\beta}\left(\left(\frac{r'}{R_1}\right)^\beta - 1\right). \tag{8}$$

As $\beta \to 0$, one obtains again the linear case (7). This class of transformations is termed as the *concave-up* transformation because (8) has positive second order derivatives.

All curves belonging to (6) have negative second order derivative with respect to the physical space r'. This class of transformations is termed as the *concave-down* transformation. The nonlinear transformation function in (6) only depends on the radial component r' in the spherical coordinate system (r', θ', ϕ'). The cloak properties in the both transformed coordinates are given by (4) and (5) where $J_{r'r} = \partial r'/\partial r$.

Indeed, the equations governing the propagation of elastodynamic waves with a time harmonic dependence are written, in a weak sense, as

$$\nabla \cdot C : \nabla u + \rho\omega^2 u = 0, \tag{9}$$

where ρ is the scalar density of an isotropic heterogeneous elastic medium, C is the fourth-order elasticity tensor, ω is the wave angular frequency, and $u(x_1, x_2, x_3, t) = u(x_1, x_2, x_3)\exp(-\mathrm{i}\omega t)$ is the vector displacement. It is easy to show that under a change of coordinates (x', y', z') to (x, y, z) such that $u'(x') = J_{x'x}^{-\mathrm{T}} u(x)$, $J_{x'x} = \dfrac{\partial(x', y', z')}{\partial(x, y, z)}$, Eq. (9) takes the form

$$\nabla'' \cdot (C' + S') : \nabla' u' + \underline{\underline{\rho}}'\omega^2 u' = D' : \nabla' u', \tag{10}$$

which preserves the symmetry of the new elasticity tensor $C' + S'$. Equation (10) contains two third-order symmetric tensors S' and D' with $D'_{pqr} = S'_{qrp}$, and a second-order tensor ρ'_{pq}.

3. ACOUSTIC CLOAK

We write the Helmholtz equation which governs the behavior of the sphere filled with traditional foam in the coordinate system (x_1, x_2, x_3) as

$$\nabla \cdot (\zeta^{-1}\nabla\Theta) + \omega^2 \Lambda^{-1}\Theta = 0. \tag{11}$$

where $\zeta = (2k(1-2\nu))^{-1/2}$, k is the bulk modulus of the foam, and $\nu > 0$ is the Poisson's ratio of the foam, ρ its effective density, $\Lambda = \rho^{-1}$ and ω the frequency. Let us apply the concave-down transformation (6) to (11), which compresses the original domain Ω occupied by a sphere of radius R_2 into a shell region $R_1 < r' < R_2$ in the compressed space Ω', characterized by

$$\underline{\underline{\zeta}}'^{-1}_{p,e}(r') = J^{\mathrm{T}}_{rr'}\zeta^{-1}_{p,r}(r)J_{rr'}/\det(J_{rr'}),\ \underline{\underline{\Lambda}}'^{-1}(r') = J^{\mathrm{T}}_{rr'}\Lambda^{-1}(r)J_{rr'}/\det(J_{rr'}),\ J_{rr'} = \partial r/\partial r', \tag{12}$$

In the new coordinates, the transformed equation (11) now reads as

$$\nabla \cdot \underline{\underline{\zeta}}^{-1}_{p,e}\nabla(\Delta_{33}\nabla \cdot \underline{\underline{\zeta}}^{-1}_{p,e}\nabla\Theta') - \Lambda^{-1}_{33}\gamma_0^4\Theta' = 0, \tag{13}$$

where $\underline{\underline{\zeta}}_{p,e}^{-1}$ is the upper diagonal part of the inverse of $\underline{\underline{\zeta}}$ and Λ_{33}^{-1} is the third diagonal entry of $\underline{\underline{\Lambda}}^{-1}$, and $\nu < 0$.
The cloak has the inner radius $R_1 = 0.5\text{m}$ and outer radius $R_2 = 1\text{m}$.
We must say that the condition of $-1 < \nu < 0.5$ corresponds to the usual range of properties for stability of the material. The Poisson's ratio $\nu = \nu_{yx}$ (for tensile and compressive tests) was calculated as the negative ratio between the radial and longitudinal strains using a best fit to the strain-strain graph $\nu_{yx} = -\dfrac{\varepsilon_x}{\varepsilon_y}$. The most important physical parameter to dominate the negative Poisson's ratio transformation is the compression ratio $\vartheta = \dfrac{(R_2'^2 - R_1'^2)l'}{R_2^2 l}$, where prime denotes the final parameters. Figure 2 shows the transformed annulus domains for $\vartheta = 0.25$, 0.26, 0.3 and 0.4. . We observe that the conventional foam becomes auxetic ($-0.15 \le v < 0$) for $0.55 \le 1-\vartheta \le 0.77$, or $0.23 \le \vartheta \le 0.45$. It is very interesting to see that the auxetic foam is changing the sign for its Poisson's ratio for $0.46 \ge 1-\vartheta$. It is of interest to underline that the results provides an overall agreement with the experimental values for the auxetic foam [11].

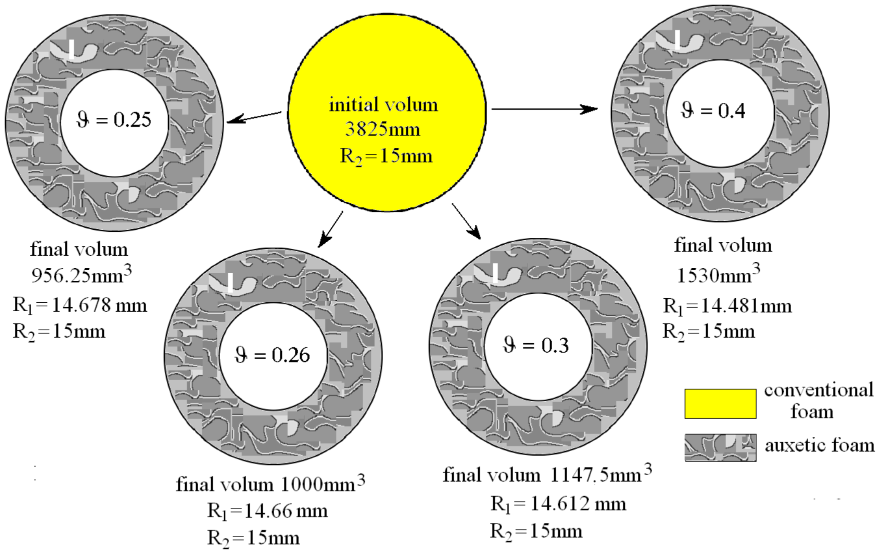

Figure 2: Transformed domains.

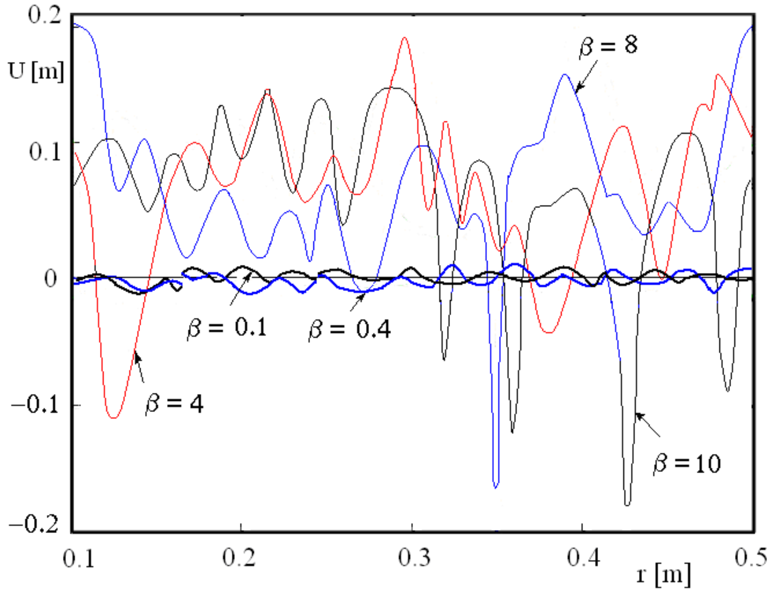

Figure 3: Variation of the displacement amplitude with respect to β in the region $r \le R_1$.

The concave-down transformation presents an overlapping for all mapping curves for $\beta < 0.1$, which means the same results in applications. The effect of β on the amplitude of displacements, which vary from $-U$ to U ($U = \sqrt{u_1^2 + u_2^2 + u_3^2}$) inside the cloak $r \le R_1$, is illustrated in Figure 3. It can be seen that when β increases, the amplitude increases significantly inside the region $r \le R_1$ of the cloak. This is due to the fact that more energy is guided towards the inner boundary $r = R_1$, which in turn makes the cloaked object more *acoustically visible* to external incidences. For $\beta = 0.1$ and 0.4, the acoustically invisibility is good. The effect of β on the amplitude of displacements in the shell region $R_1 < r < R_2$ is illustrated in Figure 4. In a similar manner, when β increases, the amplitude increases significantly in the shell region of the cloak.

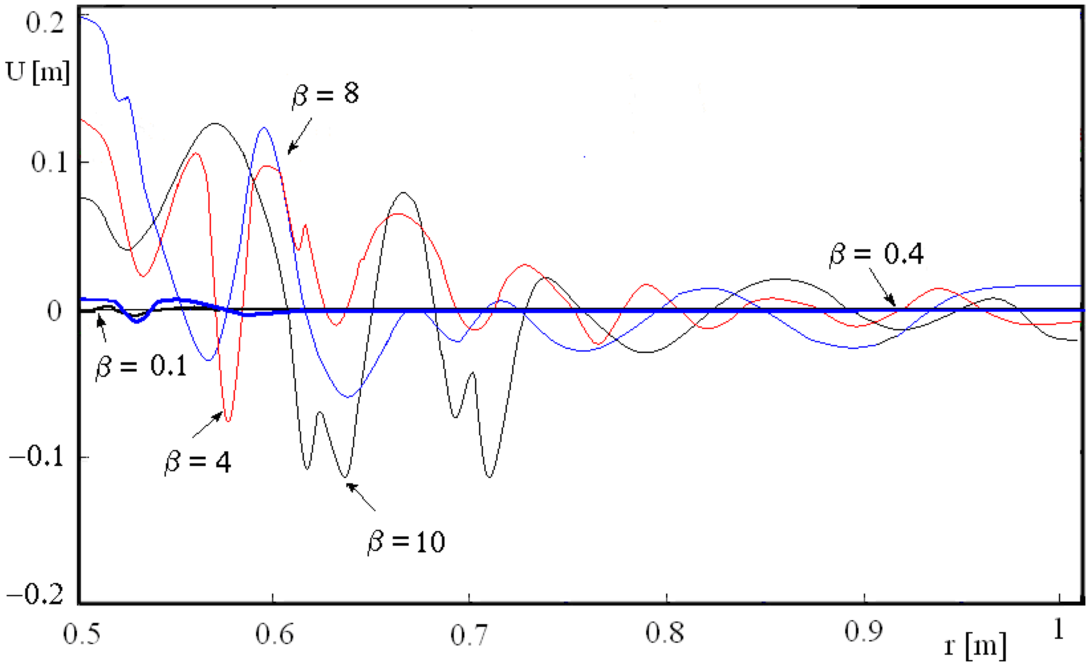

Figure 4: Variation of the displacement amplitude with respect to β in the region $R_1 < r < R_2$.

The absence of the scattering of waves generated by an external source outside the cloak is observed in Figure 5 for $\beta = 0.1$ and $\nu = -0.15$. The waves are smoothly bent around the central region inside the cloak. The results reported in Figure 5 show that the wave field inside the cloak, i.e. the inner region of radius R_1 which surrounds the noisy machine, is completely isolated from the region situated outside the cloak. The waves generated by a noisy source are smoothly confined inside the inner region of the cloak, and the sound invisibility detected from the observer is proportional to β. The inner region is acoustically isolated and the sound is not detectable by an exterior observer because the amplitudes on the boundary vanish. The domain $r < R_1$ is an acoustic invisible domain for exterior observers. The waves generated by the exterior source outside the cloak do not interact with the interior field of waves. A possible interaction or coupling between the internal and external wave fields is cancelled out by the presence of the shell region $R_1 < r < R_2$ filled with auxetic material.

Hence we can conclude that for the concave-down spherical cloaks, smaller values for β lead to a smaller disturbance in the acoustic fields in both the inner and the outer spaces $r < R_2$ and $r > R_2$, respectively

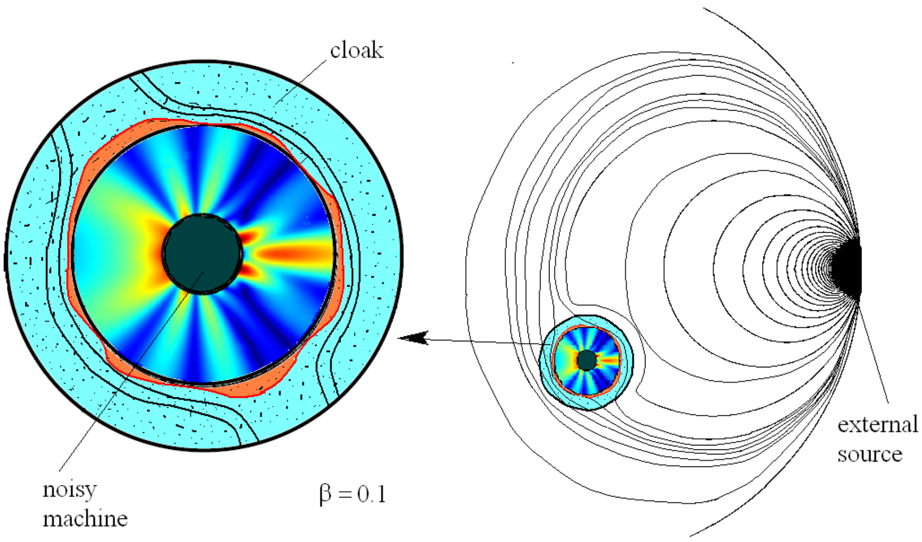

Figure 5: The wave fields inside and outside the cloak for $\beta = 0.1$ and $\nu = -0.15$.

4. CONCLUSION

In this paper, we have identified new aspects in the 3D spherical cloaking related to new reflectionless solutions which may exist for cloaking systems that are not isomorphic to electromagnetism. The original domain consists of traditional foam. The spatial compression obtained by applying the concave-down transformation has led to an equivalent domain of auxetic material. However, the present study represents an application of the aforementioned analytical results, in the sense that a numerical implementation, which treated a new kind of material that might be useful in the design of elastic cloaking devices, was developed.

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