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ASSESSMENT OF DAYLIGHT TRANSMISSION VIA LIGHT-PIPES INTO INTERIOR SPACES

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Abstract: One of the main directions for reducing the carbon emissions into the atmosphere consists in the use of advanced daylighting technologies capable of transmitting daylight into the depth of buildings. The possibility of using daylight throughout the day for illuminating the interiors of a building would help to significantly reduce the global energy consumption of that building and to cut down the carbon emissions. Lighting is responsible for 40-60% of the total energy consumption within commercial buildings and the use of daylight instead of electric lighting would lead to large energy savings. This paper assesses the performance of daylight transmission through light-pipes using the SUNPIPE systems installed in the Laboratory of Lighting Systems at the Faculty of Civil Engineering of Brasov. There are two light-pipes used in this study: one horizontal pipe of 300mm diameter and 3m length, collecting daylight on the facade of the building, and one vertical pipe of 300mm diameter and 1m length, collecting daylight on the roof of the building. The light-pipes are similar and use 98% reflectance pure silver base mirror-finish aluminium tubes. Illuminance values are measured inside the pipes and on the working plane for different external scenarios and an evaluation module is proposed by the use of nonlinear regressions.

Keywords: Daylighting, Light-pipes, Sidelighting, Toplighting

1. INTRODUCTION

The light-pipe is a secondary light source which transmits light from the primary (natural or artificial) source to a specific target or on specific reflective or transmitting surface within interior spaces [1]. Light transmission is achieved at the end of the light-pipe, where light is distributed and directed depending on task particularities, or by side transfer towards specific targets. Light-pipes transmit light radiation through total internal reflection.

The light-pipe is perhaps the most technologically exciting among the innovative daylighting systems because of the long distances over which it can operate. In principle, light-pipes collect, direct, and channel daylight into virtually any area of a building. The system consists of three main components: heliostat or light dome (collecting and concentrating unit), transport system (reflective conducts) and emitter (distributing light into the targeted space). The use of light-pipes can increase energy savings, but generally system efficiency is low because of light losses within ramification or direction changing [8].

There are specific light-pipe systems for roof applications, known as solar tubes. These systems maximize the concept of renewable energy by reflecting and intensifying sunlight and even normal daylight, down through a highly reflective silver mirror-finish aluminium tube. As compared to heliostat collecting units, the solar tubes have the advantage of collecting sunlight and skylight by means of fixed, passive collecting domes. These domes collect daylight for any sun position in the sky, without consuming energy for lens rotations.

Other systems are based on the micro-prismatic film as the element which performs the total internal reflection and have 0.98 reflectance. The internal reflection is produced within the structure of the 0.5 mm thick optic film, made of transparent acryl or polycarbonate [7].

2. SUNPIPE NATURAL LIGHTING SYSTEM

This system developed by the British manufacturer Monodraught Ltd. is a revolutionary new way to pipe natural daylight from the rooftop into the building to brighten areas from dawn to dusk where daylight from windows cannot reach, even on overcast days [4].

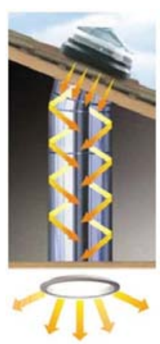

Fig. 1 How the SUNPIPE works

The diamond dome specially designed to maximise the capture of sunlight collects both direct sunlight and diffused daylight. The faceted top surface catches sunlight from any angle and the vertical prisms at the base of the diamond dome capture low level light, i.e. early in the morning and late in the afternoon. Global daylight is piped down into the desired room by means of silver-coated aluminium pipes with a mirrored surface internally. At ceiling level, the diffuser has the ability to distribute the light in every direction, giving an even spread of light throughout the interior space (see figure 1).

A SUNPIPE can be almost any length that is needed, but loses 6% of light for every metre. There are different SUNPIPE diameters, from 230mm to 1000mm, typically lighting up areas of 6 to 70 sq. metres. SUNPIPE systems have a 98% reflectance, which means that there is 2% loss on every bounce, so the longer the light-pipe, the greater the loss is and in addition, there are losses through the roof dome or light collector and the ceiling diffuser. Nevertheless, the performance of a light-pipe is remarkable and typically, the 300mm diameter SUNPIPE can light up an area of 10 sq. metres to a normal daylight level, that is, without the need for electric lighting during normal daylight hours [5]. Larger light-pipes, of 450mm and 530mm diameter, are used in larger offices and buildings with higher ceilings.

Tests carried out by the BRE (Building Research Establishment) in the UK showed a 68% increase [5] of the lighting performance on the 300mm diameter for the new Super Silver SUNPIPE as compared to the original anodised aluminium SUNPIPE.

3. MEASUREMENT OF DAYLIGHT TRANSFER THROUGH SUNPIPE SYSTEMS

For the purpose of this paper, one horizontal 2.4m long 300mm diameter SUNPIPE was used in order to collect daylight on the façade at the top level of the window and to pipe daylight inside the depth of the room. Two 45 degrees elbows are used to simulate a descent of the SUNPIPE at a false ceiling, as it can be seen in figures 2 and 3.

Fig. 2 Daylight dome collector of the horizontal SUNPIPE installed on the façade

Fig. 3 Horizontal SUNPIPE ending with two 45 degrees elbows above imaginary false ceiling

A second vertical 1.2m long 300mm diameter SUNPIPE was used to bring in daylight collected at roof level. This light-pipe is the daylighting component of a hybrid system used both for daylighting and natural ventilation, installed in the laboratory.

The illuminance was measured inside the two light-pipes at specific points along their axis: the vertical axial illuminance $E_{v,l}$ was taken into consideration for the horizontal light-pipe and the horizontal axial illuminance $E_{h,l}$ was taken into consideration for the vertical light-pipe. The daylight transport factor *DTF* was introduced as the ratio between the measured values of the axial illuminance and the external horizontal illuminance E_0 for each light-pipe. These values were thereafter used as criterion of comparison between the two light-pipes and to develop modelling equations for characterizing the daylight transport capacity of the two light-pipes.

The following equations (1) and (2) show the expressions for $DTF_{h,l}$ and $DTF_{v,l}$ which are the horizontal and vertical daylight transport factors of the horizontal light-pipe and of the vertical light-pipe respectively, specific to the vertical and horizontal illuminance respectively at distance l from the dome level:

$$DTF_{h,l} = \frac{E_{v,l}}{E_0} \cdot 100\% \quad (1) \text{ and } \quad DTF_{v,l} = \frac{E_{h,l}}{E_0} \cdot 100\% \quad (2)$$

Table 1. Measured values of the axial illuminance and daylight transport factors

Horizontal 2.4m long 300mm diameter light-pipe / E_0 = 4800lx		
l [cm]	$E_{v,l}$ [lx]	$DTF_{h,l}$ [%]
0	4600	95.83
60	2900	60.42
120	2600	54.17
180	2550	53.13
240	2300	47.92
Vertical 1.2m long 300mm diameter light-pipe / E_0 = 13200lx		
l [cm]	$E_{h,l}$ [lx]	$DTF_{v,l}$ [%]
0	10500	79.55
30	8500	64.39
60	7900	59.85
90	6500	49.24
120	6000	45.45

Basing on the measured values, modelling equations were created using the Levenberg-Marquardt method to solve nonlinear regressions in order to model the daylight transport capacity of the two light-pipes.

4. THE LEVENBERG-MARQUARDT METHOD

This method combines the steepest-descent method and a Taylor series based method to obtain a fast, reliable technique for nonlinear optimization [2]. Neither of the above optimization methods are ideal all of the time; the steepest descent method works best far away from the minimum, and the Taylor series method works best close to the minimum. The Levenberg-Marquardt (LM) algorithm allows for a smooth transition between these two methods as the iteration proceeds.

In general, the data modelling equation (with one independent variable) can be written as follows:

$$y = y(x;\vec{a}) \tag{3}$$

The above expression simply states that the dependent variable *y* can be expressed as a function of the independent variable *x* and vector of parameters *a* of arbitrary length. Note that using the LM method, any nonlinear equation with an arbitrary number of parameters can be used as the data modelling equation. Then, the "merit function" we are trying to minimize is

$$\chi^2(\vec{a}) = \sum_{i=1}^{N}\left(\frac{y_i - y(x_i;\vec{a})}{\sigma_i}\right)^2 \tag{4}$$

where N is the number of data points, x_i denotes the x data points, y_i denotes the y data points, σ_i is the standard deviation (uncertainty) at point i, and $y(x_i,a)$ is an arbitrary nonlinear model evaluated at the i^{th} data point.

This merit function simply measures the agreement between the data points and the parametric model; a smaller value for the merit function denotes better agreement. Commonly, this merit function is called the chi-square.

From the area of pure optimization, two basic ways of finding a function minimum are a Taylor series based method and the steepest-descent method. The Taylor series method states that sufficiently close to the minimum, the function can be approximated as a quadratic. A step from the current parameters a to the best parameters a_{min} can be written as

$$\vec{a}_{\min} = \vec{a}_{cur} + H^{-1}\cdot\left[-\nabla\chi^2(\vec{a}_{cur})\right] \tag{5}$$

where H is the Hessian matrix (a matrix of second derivatives). If the approximation of the function as a quadratic is a poor one, then we might instead use the steepest-descent method, where a step to the best parameters from the current parameters is

$$\vec{a}_{\min} = \vec{a}_{cur} - c\nabla\chi^2(\vec{a}_{cur}) \tag{6}$$

This equation simply states that the next guess for the parameters is a step down the gradient of the merit function. The constant c is forced to be small enough that a small step is taken and the gradient is accurate in the region that the step is taken. Since we know the chi-square function, we can directly differentiate to obtain the gradient vector and the Hessian matrix. Taking the partial derivatives of the merit function with respect to a gives

$$\frac{\partial\chi^2}{\partial a_k} = -2\sum_{i=1}^{N}\frac{y_i - y(x_i;\vec{a})}{\sigma_i^2}\frac{\partial y(x_i;\vec{a})}{\partial a_k} \tag{7}$$

To obtain the Hessian matrix, take the gradient of the gradient above (so that we have a matrix of partial second derivatives):

$$\frac{\partial^2 \chi^2}{\partial a_k \partial a_l} = -2\sum_{i=1}^{N}\left[\frac{1}{\sigma_i^2}\frac{\partial y(x_i;\vec{a})}{\partial a_k}\frac{\partial y(x_i;\vec{a})}{\partial a_l} - \right.$$
$$\left. - \frac{y_i - y(x_i;\vec{a})}{\sigma_i^2}\frac{\partial^2 y(x_i;\vec{a})}{\partial a_l \partial a_k}\right] \tag{8}$$

Now, for convenience, define the gradient vector and the curvature matrix as

$$G_k = -\frac{1}{2}\frac{\partial \chi^2}{\partial a_k} = \sum_{i=1}^{N}\frac{y_i - y(x_i;\vec{a})}{\sigma_i^2}\frac{\partial y(x_i;\vec{a})}{\partial a_k} \tag{9}$$

$$C_{kl} = \frac{\partial^2 \chi^2}{\partial a_k \partial a_l} = \sum_{i=1}^{N}\left[\frac{1}{\sigma_i^2}\frac{\partial y(x_i;\vec{a})}{\partial a_k}\frac{\partial y(x_i;\vec{a})}{\partial a_l}\right] \tag{10}$$

Note that the second derivative term in C will be ignored because of two reasons: it tends to be small because it is multiplied by (y-y_i), and it tends to destabilize the algorithm for badly fitting models or data sets contaminated with outliers.

This action in no way affects the minimum found by the algorithm; it only affects the route in getting there. So, the Taylor series method (inverse Hessian method) can be written as the following set of linear equations:

$$\sum_{k=1}^{NP} C_{kl}\delta a_l = G_k \tag{11}$$

where NP is the number of parameters in the model that is being optimized.

This linear matrix will be the workhorse for this method after some modification; it can be solved for the increments δa that, when added to the current approximation for the parameters, gives the next approximation. Likewise, the convenient definitions can be substituted into the steepest descent formula to obtain

$$\delta a_l = cG_l \tag{12}$$

Neither of the aforementioned optimization methods are ideal all of the time; the steepest descent method works best far away from the minimum, and the Taylor series method works best close to the minimum. The Levenberg-Marquardt (LM) algorithm allows for a smooth transition between these two methods as the iteration proceeds.

The first issue in deriving the LM method is to attach some sort of scale to the constant c in the steepest-gradient method - equation (12).

Typically, there is no obvious way to determine this number, even within an order of magnitude. However, in this case, we have access to the Hessian matrix; examining its members, we see that the scale on this constant must be $1/C_{ll}$. But, that still may be too large, so let's divide that scale by a non-dimensional factor (λ) and plan on setting this much larger than one so that the step will be reduced (for safety and stability).

The second issue to formulate the LM method is noting that the steepest-descent and Taylor series methods may be combined if we define a new matrix M_{ij} by the following:

$$\begin{cases} M_{ii} = C_{ii}(1+\lambda) \\ M_{ij} = C_{ij},\ i \neq j \end{cases} \tag{13}$$

This matrix combines equations (11) and (12) into a convenient and compact form. So finally, we have a means of calculating the step δa in the parameters by the following system of linear equations:

$$\sum_{k=1}^{NP} M_{kl}\delta a_l = G_k \tag{14}$$

When λ is large, the matrix M is forced to be diagonally dominant; consequently, the above equation is equivalent to the steepest descent method - equation (12). Conversely, when the parameter λ goes to zero, the above equation is equivalent to the Taylor series method - equation (11).

Therefore, we vary λ to switch between the two methods, continually calculating a parameter correction δa that we apply to the most recent guess for the parameter vector.

The steps that are taken in the LM algorithm are as follows:

1. Compute $\chi^2(a)$
2. Pick a conservative value for λ
3. Solve the linear equations for δa
4. Evaluate $\chi^2(a+\delta a)$
5. If $\chi^2(a+\delta a) >= \chi^2(a)$, increase λ by a factor and go back to step 3
6. If $\chi^2(a+\delta a) < \chi^2(a)$, decrease λ by a factor, correct the parameter vector by $a=a+\delta a$, and go back to step 3

Iteration is stopped when $\left|\chi^2(a+\delta a)-\chi^2(a)\right|$ < tolerance

5. THE MODELLING EQUATIONS

The goal was to find a recurrence law for the light transmission. Therefore, the horizontal illuminance within the axis of the vertical light-pipe and the vertical illuminance within the axis of the horizontal light-pipe were measured at certain distances from the diamond domes.

Based on the values of *DTF* factors determined previously, recurrence laws were determined to predict the axial illuminance at the end of a light-pipe of specific length for a certain value of the external horizontal illuminance at roof level.

If E_0 is the external horizontal illuminance at roof level, then the horizontal illuminance $E_{h,l}$ within the axis of the vertical light-pipe at distance *l* [cm] from the diamond dome will be

$$E_{h,l} = E_0 \cdot DTF_v(l) \tag{15}$$

where $DTF_v(l)$ is the recurrence function expressing the daylight transport capacity of a light-pipe of length *l* measured in cm. This function was determined using the CurveExpert v1.4 software based on the Levenberg-Marquardt method of nonlinear regression.

The same applies for the vertical illuminance $E_{v,l}$ within the axis of the horizontal light-pipe, which at distance *l* [cm] from the diamond dome will be

$$E_{v,l} = E_0 \cdot DTF_h(l) \tag{16}$$

where the recurrence function $DTF_h(l)$ was determined using the same method as above.

For the measured values, the CurveExpert v1.4 software generated the recurrence functions using the Levenberg-Marquardt method of nonlinear regression for 23 regression models.

The most accurate form determined for the recurrence function of the daylight transport capacity of the horizontal light-pipe was:

$$DTF_h(l) = \frac{1}{a + b \cdot l^c} \tag{17}$$

if using the Harris regression model with 1.4965 standard error and 0.9985 correlation coefficient, where:

a = 1.0436 x 10^{-2} b = 1.4251 x 10^{-3} c = 3.5527 x 10^{-1}

For the daylight transport capacity of the vertical light-pipe, the following recurrence function was proposed:

$$DTF_v(l) = \frac{1}{a \cdot l + b} \tag{18}$$

if using the reciprocal model model with 2.0356 standard error and 0.9914 correlation coefficient, where:

$a = 7.9140 \times 10^{-5}$ $\qquad b = 1.2668 \times 10^{-2}$

Figures 4 and 5 below show the curves for the recurrence functions determined above.

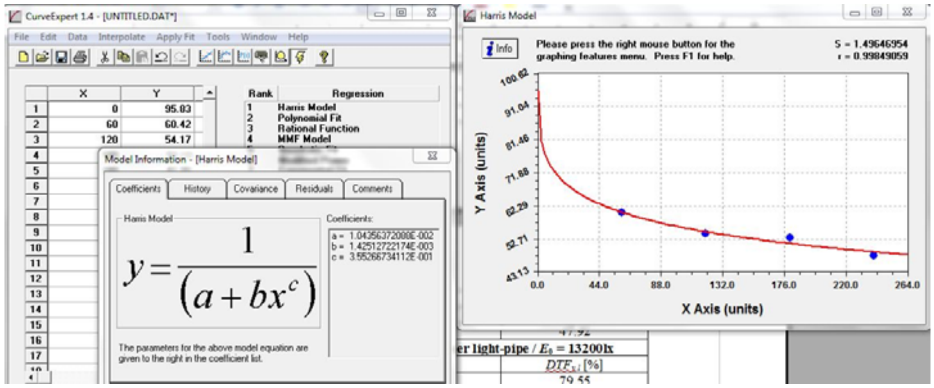

Fig. 4 Modelling equation for the daylight transport capacity of the horizontal light-pipe

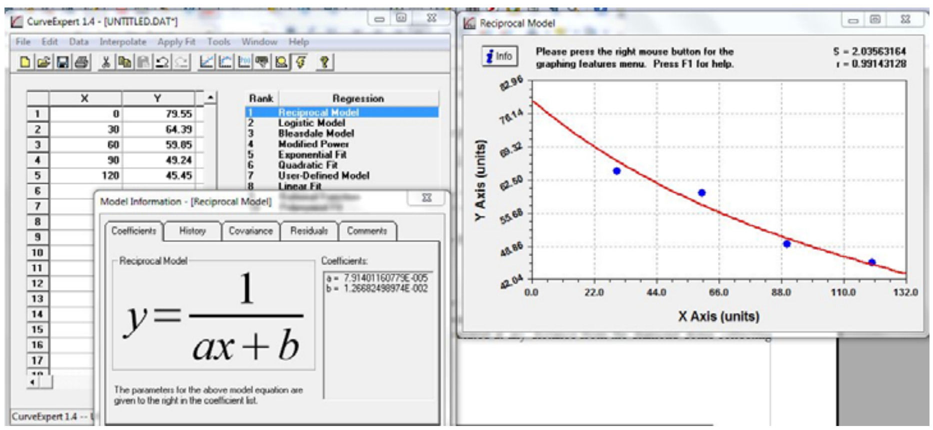

Fig. 5 Modelling equation for the daylight transport capacity of the vertical light-pipe

On the other hand, the daylight transport capacities of the two light-pipes can be compared by means of the daylight transport factors measured and modelled as above. Considering the equations determined above, *DTF* values can be calculated at any distance from the diamond dome collecting unit of the light-pipes.

Table 2. Daylight transport factors for the horizontal and vertical light-pipes

l [cm]	DTF_h [%]			DTF_v [%]			Difference [%]	
	Measured	Calculated	Error	Measured	Calculated	Error	Measured	Calculated
0	95.83	95.82	0.01	79.55	78.94	0.61	16.28	16.88
30	n/a	65.76	n/a	64.39	66.48	2.09	n/a	-0.72
60	60.42	60.46	0.04	59.45	57.42	2.03	0.97	3.04
90	n/a	57.19	n/a	49.24	50.53	1.29	n/a	6.66
120	54.17	54.81	0.64	45.45	45.12	0.33	8.72	9.69
150	n/a	52.94	n/a	n/a	40.75	n/a	n/a	12.19
180	53.13	51.41	1.72	n/a	37.16	n/a	n/a	14.25
210	n/a	50.10	n/a	n/a	34.14	n/a	n/a	15.96
240	47.92	48.96	1.04	n/a	31.58	n/a	n/a	17.38

Figure 7 below shows the difference between the daylight transport factors of the two light-pipes, which proves that a vertical light-pipe has indeed a higher transport capacity as compared to a horizontal light-pipe of similar length and diameter. The average calculated difference between the daylight transport factors is 10.59% in favour of the vertical light-pipe.

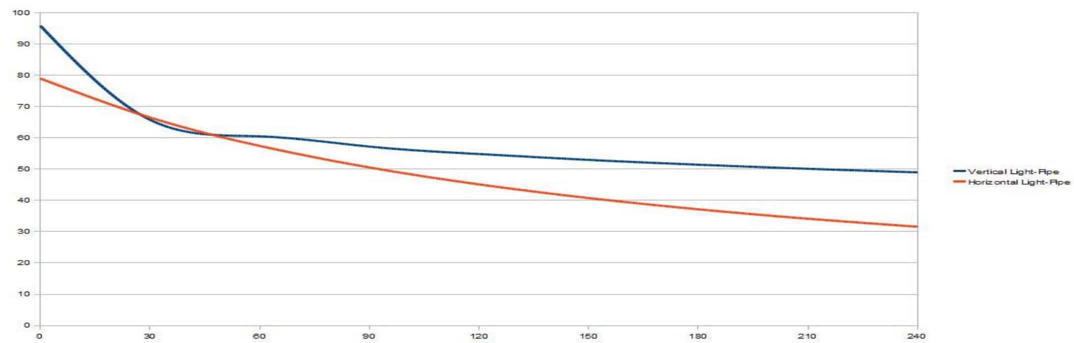

Fig. 7 Comparison of the daylight transport capacities of the vertical and horizontal light-pipes

6. CONCLUSIONS

The experimental research undertaken by the authors shows an important potential of light transmission towards interior spaces by means of light-pipes. It is essential to emphasize that the 300mm diameter used for the SUNPIPE systems used for this research is only the second one from the range of SUNPIPE diameters. The 450mm diameter would double the light transmission potential, while the 530mm diameter would triple it. Therefore, the quantity of light which may be transported by such light-pipes would grow significantly.

Although the vertical light-pipe proved to be more efficient in terms of daylight transport as compared to the horizontal light-pipes, it can be concluded that the difference of 10.59% is not so high. This may entitle us to suggest that horizontal light-pipes can be a relatively efficient solution for side daylighting, in order to collect daylight on the façade of buildings and to transport it towards blind interior spaces. It seems however that the daylight transport capacity of an horizontal light-pipe decreases faster with its length as compared to a vertical light-pipe. This fact would make it acceptable for relatively short distances of less than 3m from the façade to the blind interior space. Future work will be carried out by the authors to improve the modelling equations so that accurate estimates can be made for any length and diameter of the SUNPIPE light-pipes.

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