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# Changing the normalized natural frequencies for a rectangular plate with a damage

**Zeno-Iosif Praisach[1], Gigi Atinge*[2], Patric Timotei Stan[3], Cristian Tufiși[4]**

1. Babeș-Bolyai University, Cluj-Napoca, Faculty of Engineering Reșița Romania, zeno.praisach@ubbcluj.ro
2. Babeș-Bolyai University, Cluj-Napoca, Faculty of Engineering Reșița Romania, gigi.atinge@stud.ubbcluj.ro
   *Corresponding author: nicoleta.bacescu@stud.ubbcluj.ro
3. Babeș-Bolyai University, Cluj-Napoca, Faculty of Engineering Reșița Romania, patric.stan@ubbcluj@ubbcluj.ro
4. Babeș-Bolyai University, Cluj-Napoca, Faculty of Engineering Reșița Romania, cristian.tufiși@ubbcluj.ro

***Abstract:*** *The paper presents the analytical results obtained for a thin rectangular plate with a damage from the point of view of the dynamic behavior. The damage can occupy any position on the plate surface, and the change in natural frequencies is presented for the normalized values by the ratio between the natural frequency of the damaged plate and the natural frequency of the healthy plate. The paper presents two cases of analysis: the rectangular plate simply supported on all sides and the plate simply supported on two opposite sides and clamped on the other two. Changes in natural frequencies are illustrated as 3D surfaces and provide an overview of the dynamic behavior of the plate when the damage can occupy any position on the plate.*

***Keywords:*** *rectangular plate, damage, natural frequencies, mode shape*

## 1. INTRODUCTION

Damage detection is a major problem that concerns many researchers, who use analytical and experimental methods [1]. Structure monitoring methods can be global or local. Global methods evaluate the conditions of the entire structure simultaneously, while local methods provide information about a relatively small region of the structure [2].

The paper presents a global analytical method for the detection of damages in thin rectangular plates by comparing the normalized strain energy with the ratio of natural frequencies for the structure with the damage and the healthy structure.

The paper presents two cases of support of an equal-sides, namely: the plate simply supported on two opposite sides and clamped on the other two (denoted as SS-C-SS-C, and the plate simply supported on all sides (denoted SS-SS-SS-SS).

## 2. SUPPORTED RECTANGULAR PLATE TYPE: SS-C-SS-C

Let's consider a thin rectangular plate (fig. 1), simply supported (SS) along the y direction at: x=0 and x=a.

The thickness of the plate is denoted by h, and the other two dimensions are denoted by a and b, along the x and y directions. The plate is fixed (C) along the x direction at: y=0 and y=b. The ratio a/b=1, i.e. the plate is square.

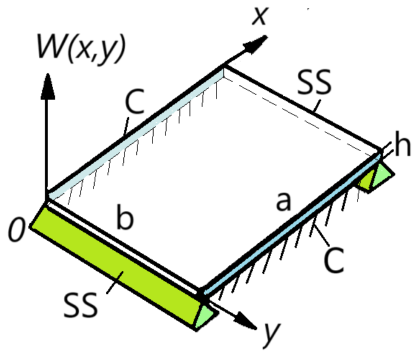


**Figure 1:** Rectangular plate simply supported (SS) along two opposing edges and clamped (C) on the other two.

In the case of flat plates, the normalized modal shape function is [3, 4]:

$$W_{m,n}(x,y) = \left\{ cos\left(\rho_{2n}\frac{y}{b}\right) - cosh\left(\rho_{1n}\frac{y}{b}\right) - \frac{cos(\rho_{2n}) - cosh(\rho_{1n})}{\frac{\rho_{1n}}{\rho_{2n}}sin(\rho_{2n}) - sinh(\rho_{1n})} \cdot \left[\frac{\rho_{1n}}{\rho_{2n}}sin\left(\rho_{2n}\frac{y}{b}\right) - sinh\left(\rho_{1n}\frac{y}{b}\right)\right] \right\} sin\left(\frac{m\pi x}{a}\right) \tag{1}$$

where,

$\rho_1$, $\rho_2$ are the dimensionless wave numbers;

m, n are the vibration modes numbers along the x and y directions.

Considering the relationship [5, 6] deduced from the beams that allows us to calculate the natural frequencies for the beam with a damage ($f_D$) as a function of the natural frequencies of the healthy beam ($f_U$) and the normalized strain energy:

$$f_D = f_U\left[1-\gamma\left(\frac{\partial^2 W(x)}{\partial x^2}\right)^2\right]; \tag{2}$$

where, $\gamma$ is the severity of the damage.

For the considered case, it can be deduced that the ratio of these frequencies, considering that the severity $\gamma$=1, and taking into account the strain energy of the thin plate, gives:

$$\frac{f_D}{f_U} = 1-\gamma\left\{\left(\frac{\partial^2 W_{m,n}(x)}{\partial x^2}+\frac{\partial^2 W_{m,n}(y)}{\partial y^2}\right)^2 - 2(1-\nu)\left[\frac{\partial^2 W_{m,n}(x)}{\partial x^2}\cdot\frac{\partial^2 W_{m,n}(y)}{\partial y^2}-\left(\frac{\partial^2 W_{m,n}(x,y)}{\partial x\partial y}\right)^2\right]\right\}^2 \in [0,\ldots,1]; \tag{3}$$

where, $\nu$ is the Poisson coefficient.

## 3. SUPPORTED RECTANGULAR PLATE TYPE: SS-SS-SS-SS

Let's consider a thin rectangular plate simply supported (SS-SS-SS-SS) all around.

The normalized modal shape function [3, 4] is:

$$\frac{\partial^4 W_{m,n}(x,y)}{\partial x^2\partial y^2} = \pi^4\left(\frac{mn}{ab}\right)^2\cdot sin\left(\frac{m\pi x}{a}\right)\cdot sin\left(\frac{n\pi y}{b}\right) \tag{4}$$

The ratio between natural frequencies for the rectangular plate with a damage and the natural frequencies of the healthy rectangular plate considering that the severity $\gamma$=1, is given by relationship (3).

## 4. RESULTS

Based on relations (1), (3) and (4), figures 2-7 show the modal shapes and natural frequency changes for the rectangular plate when a damage appears on the plate regardless of the location of the damage.

Figures 2, 3 and 4 show on the left side the modal shapes for vibration modes m=1, n=1; m=1, n=2; m=3, n=1 for the rectangular plate with the aspect ratio a/b=1, simply supported on two opposite sides and clamped on the other two (SS-C-SS-C), and on the right side of the figures, are illustrated the evolution of the normalized natural frequencies by the ratio $f_D/f_U$, according to the relationship (3), regardless by the location of the defect.

For the same vibration modes (m=1, n=1; m=1, n=2; m=3, n=1), for comparison, figures 5, 6 and 7, show on the left side the modal shapes for the rectangular plate with the aspect ratio a/b=1, simply supported on all sides (SS-SS-SS-SS), and on the right side of the figures, are illustrated the evolution of the normalized natural frequencies by the ratio $f_D/f_U$, according to the relationship (3), regardless by the location of the defect.

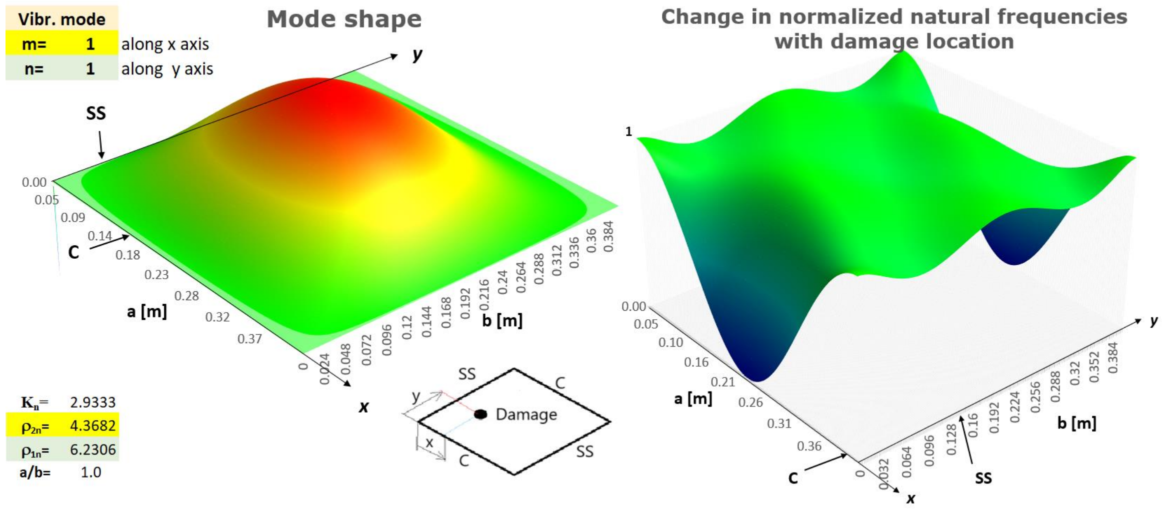


**Figure 2:** Vibration mode shape (left) for m=1, n=1 of the rectangular plate SS-C-SS-C and the changes in the normalized natural frequencies with the location of the damage (right).

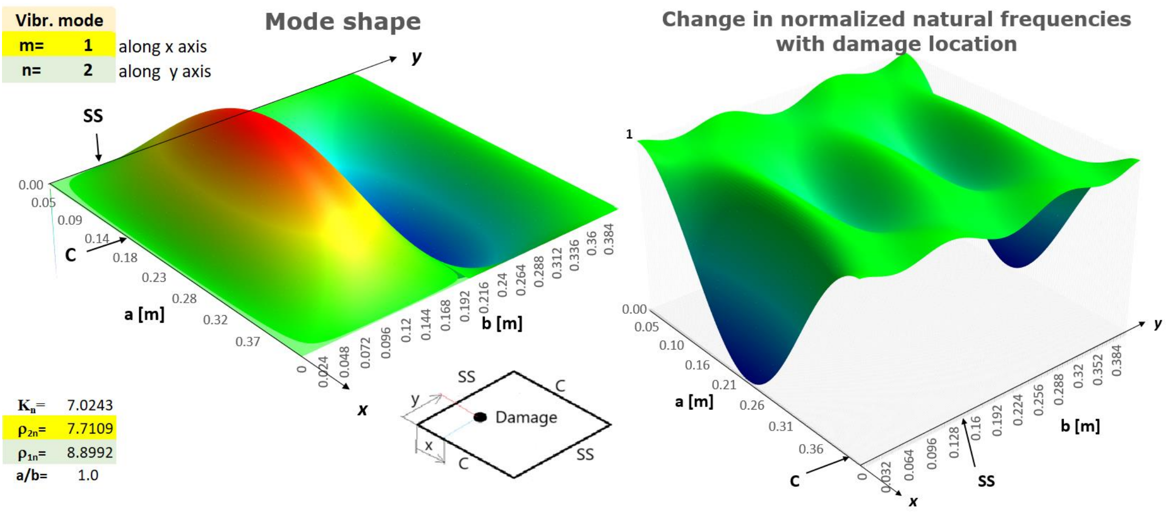


**Figure 3:** Vibration mode shape (left) for m=1, n=2 of the rectangular plate SS-C-SS-C and the changes in the normalized natural frequencies with the location of the damage (right).

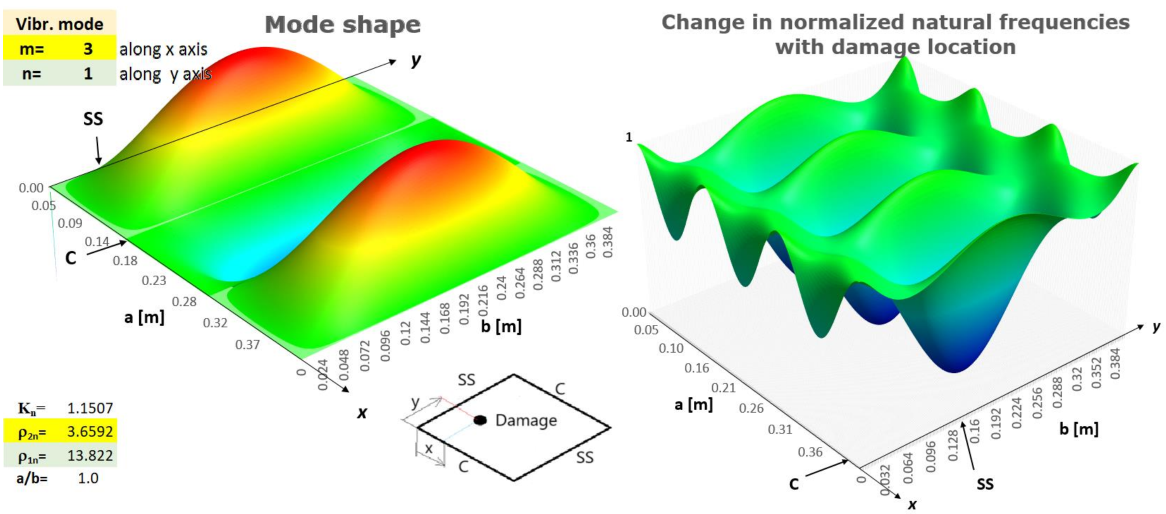


**Figure 4:** Vibration mode shape (left) for m=3, n=1 of the rectangular plate SS-C-SS-C and the changes in the normalized natural frequencies with the location of the damage (right).

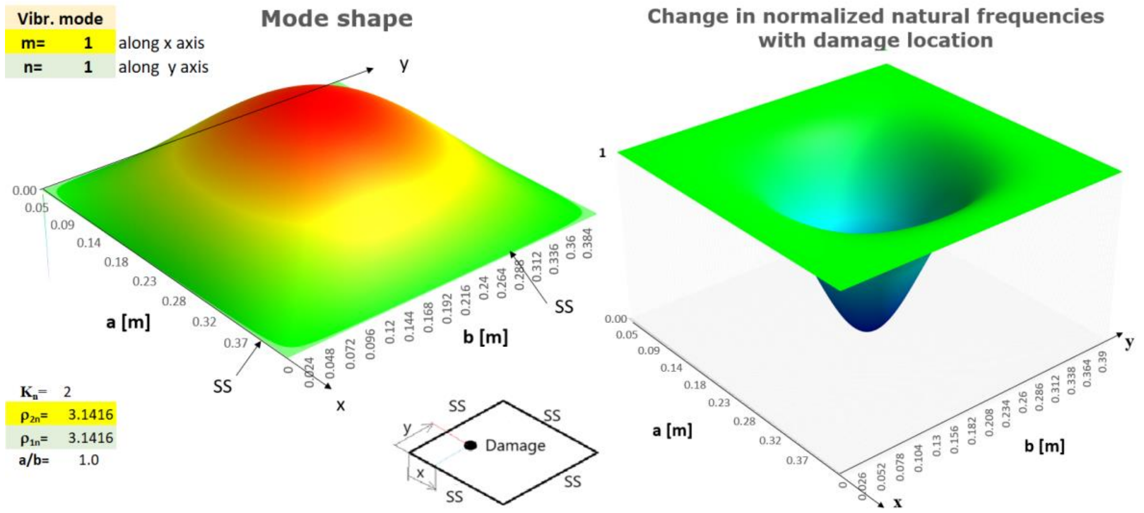


**Figure 5:** Vibration mode shape (left) for m=1, n=1 of the rectangular plate SS-SS-SS-SS and the changes in the normalized natural frequencies with the location of the damage (right).

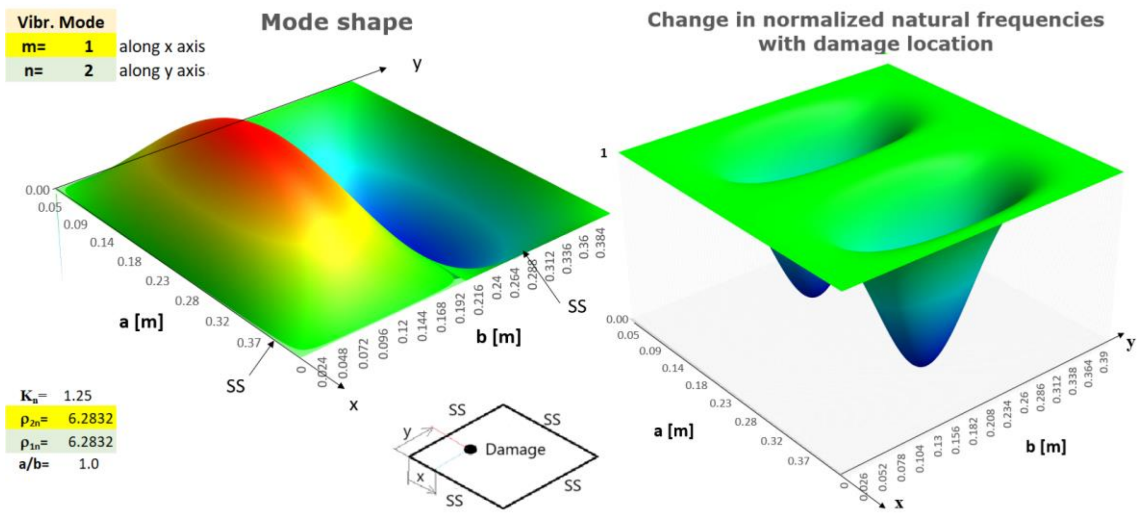


**Figure 6:** Vibration mode shape (left) for m=1, n=2 of the rectangular plate SS-SS-SS-SS and the changes in the normalized natural frequencies with the location of the damage (right).

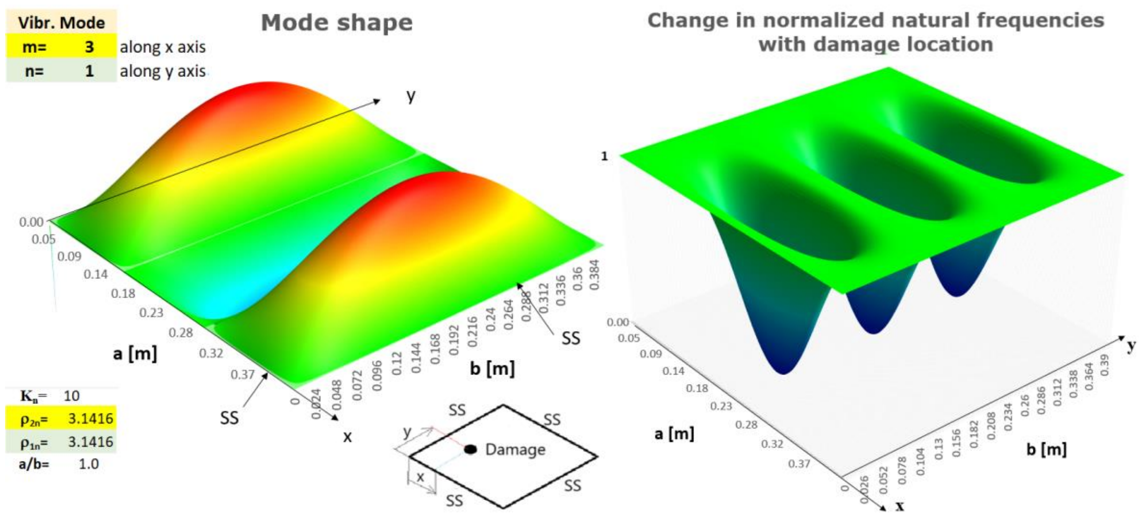


**Figure 7:** Vibration mode shape (left) for m=3, n=1 of the rectangular plate SS-SS-SS-SS and the changes in the normalized natural frequencies with the location of the damage (right).

## 5. CONCLUSIONS

For the considered thin rectangular plate model, with the ratio a/b=1 of the sides, the following conclusions can be drawn from the modal analysis of the obtained results, it can be concluded that:

- The normalized modal functions (1) and (4) for rectangular plates represent the product of the modal functions from the beams in the two directions x and y for which the same boundary conditions are imposed;
- For the two types of supports of the rectangular plate, the modal shapes illustrated in figures 2 – 7, do not show significant differences in terms of appearance, although the modal functions (1) and (4) are totally different;
- Relation (3) presents a general form for obtaining the natural frequencies when a plate is damaged depending on the natural frequency of the healthy plate, regardless of the location of the damage;
- For the two types of supports of the rectangular plate, the changing of the natural frequencies of the damaged plate shows a significant difference in terms of appearance for the same vibration mode, although the relationship (3) is the same.
- From the point of view of the dynamic behavior, at the occurrence of a damage on the plate, the influence of the clamped sides (figures 2 – 4) for the SS-C-SS-C plate is much more pronounced compared to (figures 5 – 7) the case of the plate simply supported on all sides (SS-SS-SS-SS).

The paper presents an analytical solution for detecting and localization of a damage in a rectangular thin plate by using a global method considering the dynamic behavior of the structure.

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