COMAT The 10th International Conference on ADVANCED COMPOSITE MATERIALS ENGINEERING

Transilvania University of Brasov
FACULTY OF MECHANICAL ENGINEERING

22-23 October 2024

The evolution of the eigenvalues for a rectangular plate with different aspect ratios

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Abstract: *The paper presents the analytical results obtained for the eigenvalues of a thin rectangular plate simply supported on two opposite sides and clamped on the other two for different ratios of the sides of the plate a/b, from 1/3 to 3. The obtained eigenvalues are necessary for the calculation of the natural frequencies of the plate and for the representation of the modal shapes. Considering the sides are simply supported at x=0 and x=a, the results indicate a strong change in the eigenvalues for ratios a/b<1, respectively a slight change for a/b>1, changes that influence the natural frequencies and the modal shapes of the plate.*

Keywords: *rectangular plate, eigenvalues, dynamic behavior*

1. INTRODUCTION

Plates are of particular importance in mechanical engineering, as many structures are made up of plates of various shapes and sizes [1]. A plate is a solid body that has one of its dimensions (thickness, noted with h) smaller than the other two (length and width, noted with a and b) and can be thought of as the materialization of a surface, just as a bar is the materialization of a line [2]. According to the literature, for a plate to be considered thin, the condition must be met: min(a, or b)/h>5. If this ratio is smaller than 5, then the plate is thick [3].

The paper presents the evolution of the eigenvalues for a thin rectangular plate simply supported on two opposite sides and clamped on the other two, for different values of the a/b ratio.

2. ANALYTICAL MODEL

Let's consider a thin rectangular plate (fig. 1), simply supported (SS) along the y direction at: x=0 and x=a. The thickness of the plate is denoted by h, and the other two dimensions are denoted by a and b, along the x and y directions. The plate is fixed (C) along the x direction at: y=0 and y=b.

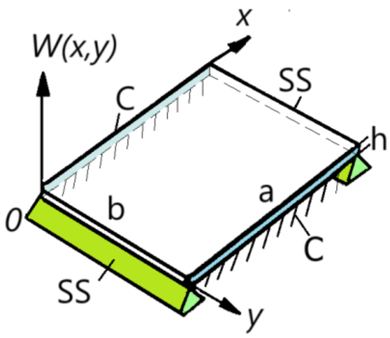

Figure 1: Rectangular plate simply supported (SS) along two opposing edges and clamped (C) on the other two.

In the case of flat plates, the equation of motion [1] is:

$$D\left(\frac{\partial^4 W(x,y)}{\partial x^4}+2\frac{\partial^4 W(x,y)}{\partial x^2\partial y^2}+\frac{\partial^4 W(x,y)}{\partial y^4}\right)-\rho h\omega^2 W(x,y)=0 \quad (1)$$

where,

W(x,y)=X(x)·Y(y) is the motion function along the x and y directions;

$D=\frac{Eh^3}{12(1-\nu^2)}$ [Nm] is the stiffness of the plate;

ν is Poisson's ratio;

E [N/m^2] is the longitudinal modulus of elasticity;

ρ [kg/m^3] is the density of the plate material;

ω [rad/s] is the pulsation;

X(x) and Y(y) are chosen as the fundamental modal shapes of the beams having the plate boundary conditions.

The boundary conditions for simply supported edges are:

$$W(0,y)=W(a,y)=\frac{\partial^2 W(0,y)}{\partial x^2}=\frac{\partial^2 W(a,y)}{\partial x^2}=0 \quad (2)$$

Solving the system of equations (2), it obtains the solution from the simply supported beam (3), respectively (4) for the plate [4]:

$$X(x)=sin\left(\frac{m\pi}{a}x\right) \quad (3)$$

$$W(x,y) = Y(y)\, sin\left(\frac{m\pi}{a}x\right) \tag{4}$$

Substituting (4) into relation (1), it can be written in the form:

$$\frac{d^4Y(y)}{dy^4} - 2\left(\frac{m\pi}{a}\right)^2 \frac{d^2Y(y)}{dy^2} + \left[\left(\frac{m\pi}{a}\right)^4 - \frac{\rho h}{D}\omega^2\right] Y(y) = 0 \tag{5}$$

We make the substitution:

$$Y(y) = \sum_{i=1}^{4} C_i e^{\lambda_i \frac{y}{b}} \tag{6}$$

and inserted into (5), the following relation is obtained:

$$\left(\frac{\lambda_i}{b}\right)^2 = \left(\frac{m\pi}{a}\right)^2 \pm \sqrt{\frac{\rho h}{D}\omega^2} = \left(\frac{m\pi}{a}\right)^2 \left[1 \pm \left(\frac{a}{m\pi}\right)^2 \sqrt{\frac{\rho h}{D}\omega^2}\right] = \left(\frac{m\pi}{a}\right)^2 [1 \pm K] \tag{7}$$

where,

$$K = \frac{\omega}{\left(\frac{m\pi}{a}\right)^2 \sqrt{\frac{D}{\rho h}}} \tag{8}$$

represent the eigenvalues of the modal shapes, respectively of the natural frequencies, and:

$$\lambda_1 = \frac{b}{a} m\pi\sqrt{K+1} \qquad \lambda_2 = \frac{b}{a} m\pi\sqrt{K-1} \tag{9}$$

represent the dimensionless wave numbers. Under these conditions, the function Y(y) can be written:

$$Y(y) = Acosh\left(\lambda_1 \frac{y}{b}\right) + Bsinh\left(\lambda_1 \frac{y}{b}\right) + Ccos\left(\lambda_2 \frac{y}{b}\right) + Dsin\left(\lambda_2 \frac{y}{b}\right) \tag{10}$$

The boundary conditions for clamped edges are [5]:

$$Y(0) = Y(b) = \frac{dY(0)}{dy} = \frac{dY(b)}{dy} = 0 \tag{11}$$

and by solving the system (11) we obtain the frequency equation (12) whose solutions give us the eigenvalues (8) and the dimensionless wavenumbers (9):

$$2\lambda_1\lambda_2[cos(\lambda_2)cosh(\lambda_1) - 1] + \left(\lambda_2{}^2 - \lambda_1{}^2\right)sin(\lambda_2)sinh(\lambda_1) = 0 \tag{12}$$

3. RESULTS

To obtain the eigenvalues and the dimensionless wave numbers, the following steps are performed:

- in relation (9), consider m=1, give values to K, calculate λ_1 and λ_2 so that they are solutions for the characteristic equation (12);
- the first value of K that provides a solution for the characteristic equation corresponds to the vibration mode m=1, n=1; the second value of K that gives the solution to the characteristic equation corresponds to the vibration mode m=1, n=2, and so on, up to the desired n value;

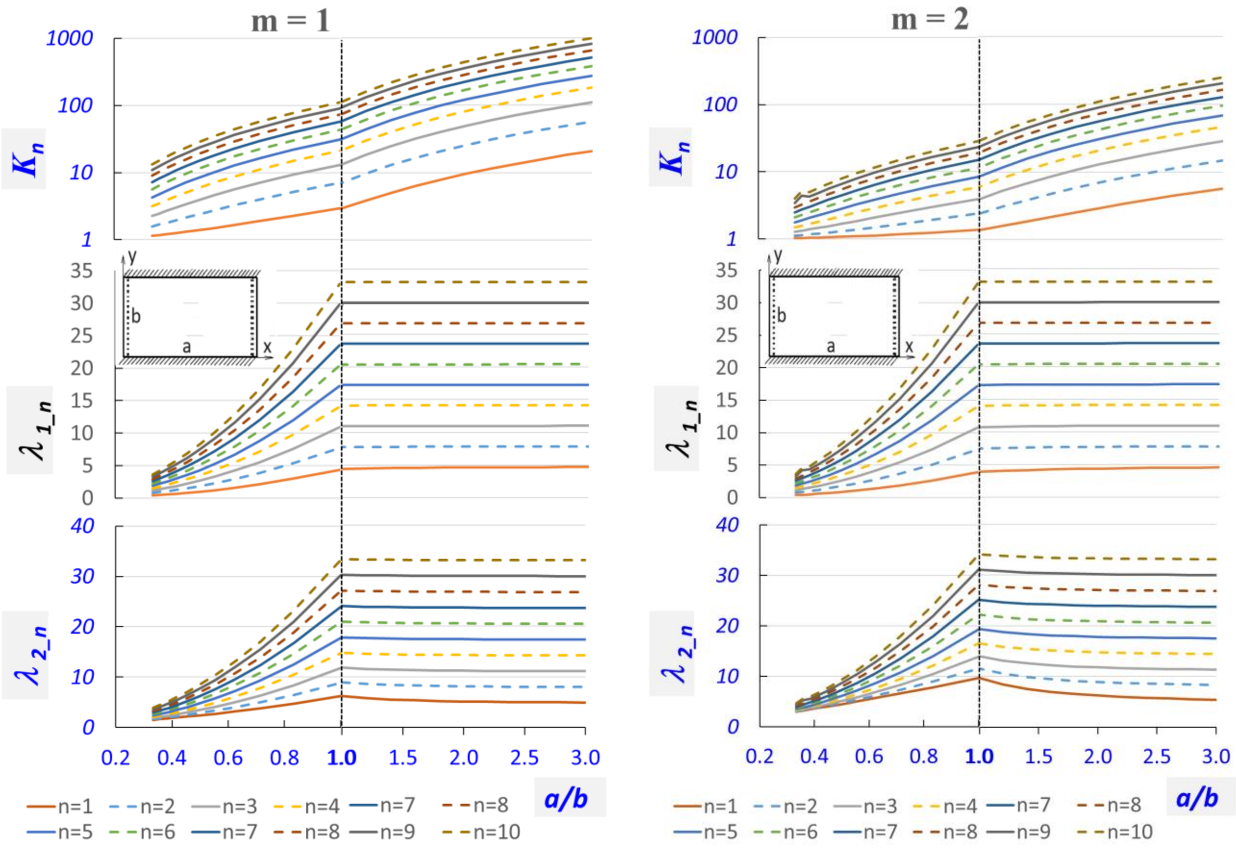

Figure 2: Eigenvalue and dimensionless wave numbers for m=1, m=2 and n=1, …, 10.

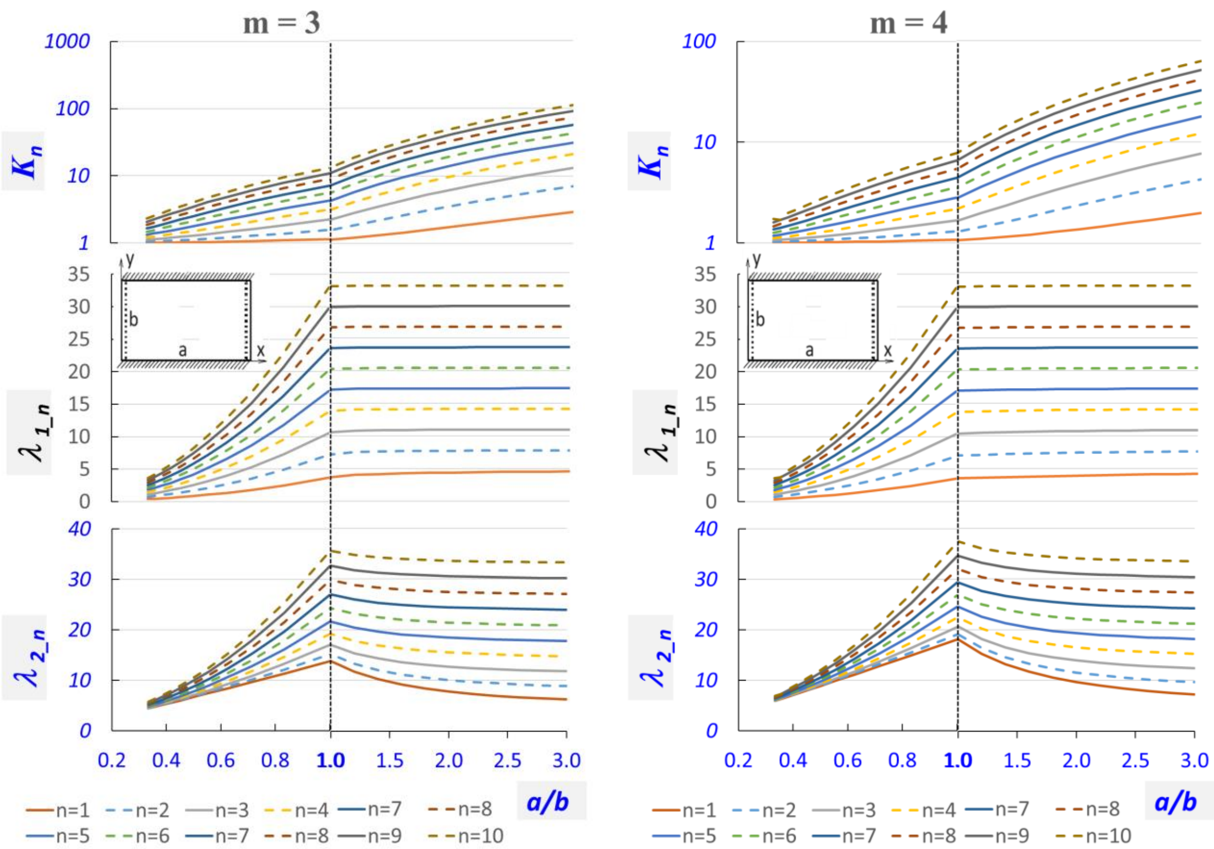

Figure 3: Eigenvalue and dimensionless wave numbers for m=3, m=4 and n=1, …, 10.

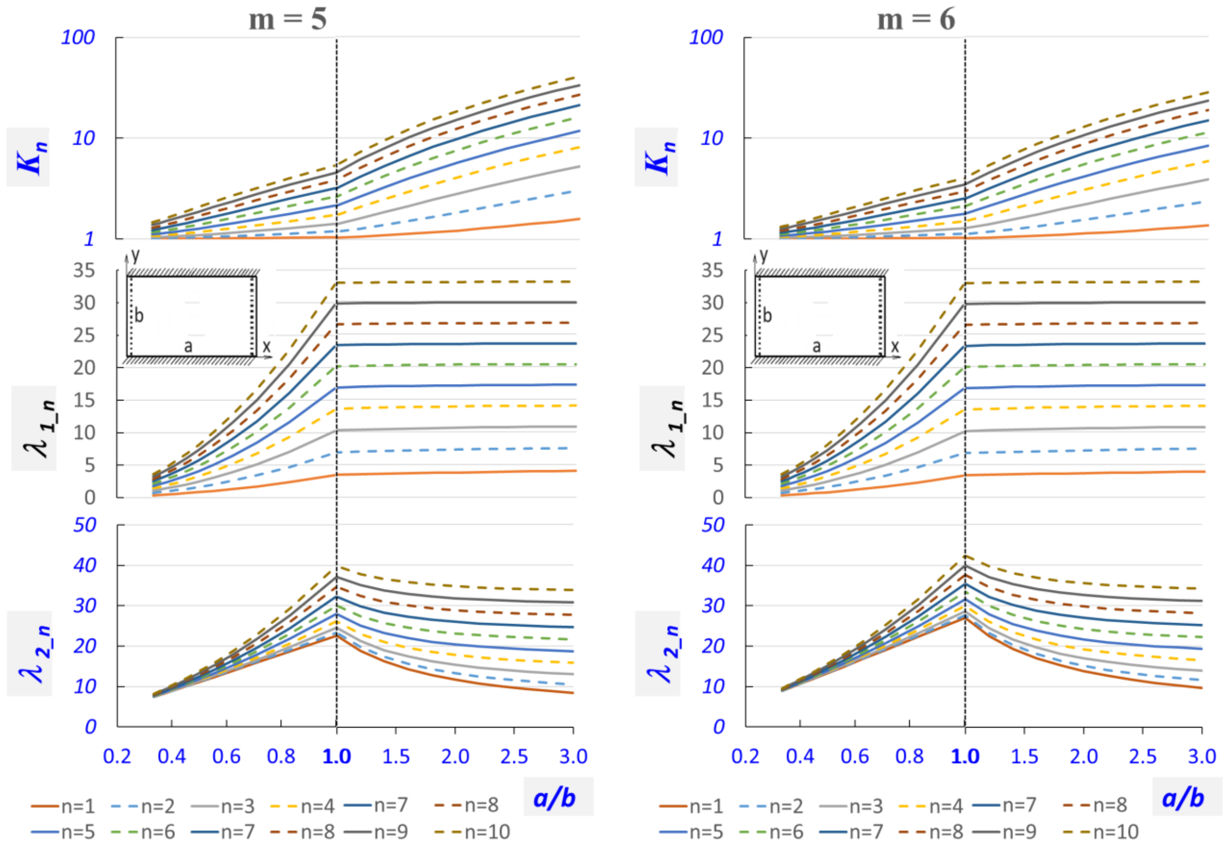

Figure 4: Eigenvalue and dimensionless wave numbers for m=5, m=6 and n=1, …, 10.

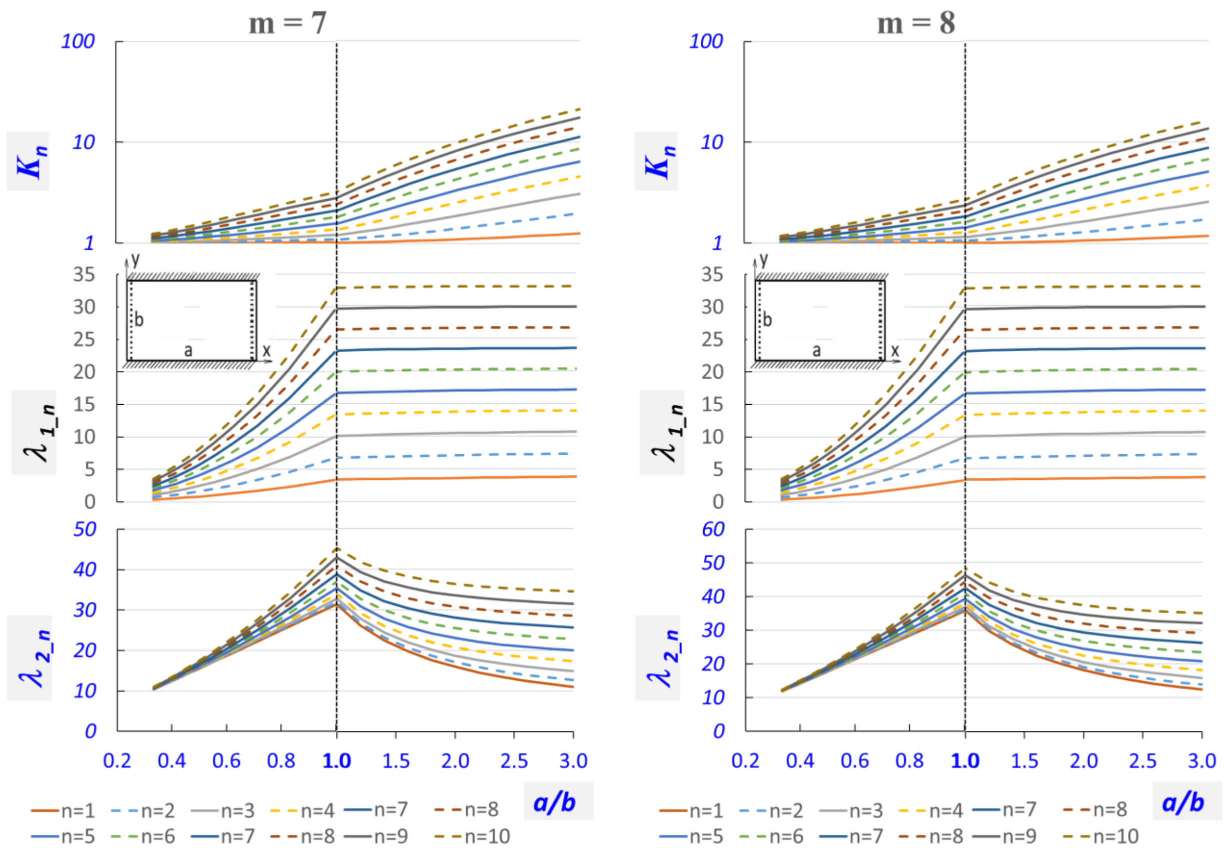

Figure 5: Eigenvalue and dimensionless wave numbers for m=7, m=8 and n=1, …, 10.

- in relation (9), consider m=2, give values to K, calculate λ_1 and λ_2 so that they are solutions for the characteristic equation (12) for vibration mode m=2, up to the desired vibration mode n;
- the process continues until the desired vibration mode m.

As can be seen from (9), the dimensionless wave numbers are a function of the aspect ratio of the rectangular plate, b/a. On this basis, the paper presents the evolution of the eigenvalues for the rectangular plate for a/b ratios from 3 to 1/3. On this basis, the paper presents the evolution of the eigenvalues for the rectangular plate for ratios a/b from 3 to 1/3, respectively for a/b=3, the edge a is clamped over a length three times greater than the simply supported side b, and for a/b=1/3, the edge a is clamped over a length three times smaller than the simply supported side b.

Figures 2 and 5 show the evolution of eigenvalues and dimensionless wavenumbers for vibration modes m=1, ..., 8 and n=1, ..., 10 for aspect ratios a/b from 1/3 to 3.

1. CONCLUSIONS

For the considered thin rectangular plate model, the following conclusions can be drawn from the analysis of the obtained results presented in the figures 2 to 5:

- eigenvalues and dimensionless wave numbers do not take into account the nature of the material from which the plate is made, nor its thickness, and only the aspect ratio, length and width;
- eigenvalues and dimensionless wave numbers values increase as the vibration modes n increase and as the vibration modes m increase, the eigenvalues $K_{n,m}$ decrease;
- regardless of the mode of vibration, for ratios a/b<1, the eigenvalues K_n show a small increase, while the dimensionless wave numbers show a pronounced increase, a phenomenon that reverses for a/b>1;
- it can be seen that the dimensionless wave numbers λ_2 have a maximum for a/b=1 (a square shaped plate), after which they start to decrease.

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