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OPTIMIZATION OF A SPATIAL SYSTEM OF BARS AT WHICH ONE ADDS AN EXTRA BAR

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Abstract: *In a previous paper we have studied some conditions for optimizing a spatial system of spherical articulated bars at both ends and having a common end. Optimizations have been studied in the case of adding two bars. In this work, only one bar is added, the optimizations referring to the minimum displacement of the common point of the bars or the minimization of tension in a certain bar.*

Keywords: *spatial system of bars, numerical simulation, deformations, tensions*

1. INTRODUCTION

In our previous paper [1] we discuss the case of a spatial system of bars at which one adds two bars and wants to obtain a certain optimization based on a criterion. The bars were spherically jointed at their ends. In the present paper we add only one bar at the existent system of spatial bars and again we want to optimize some parameters.

The references [225-] were described in [1] and they will not be presented again here. The study is performed using the screw coordinates.

The working hypotheses are also presented in [1].

2. MATHEMATICAL MODEL

The mathematical model is captured in Fig. 1. Only one bar of the system is presented. The angles between the straight bar OB_i and the three axes of coordinates are α_i, β_i, and γ_i, respectively. The common point of the bars is point O and at this point the force $\mathbf{F}$ acts (the components of this force being F_x, F_y, and F_z on the three axes). Under the action of the force $\mathbf{F}$ the point O

suffers a spatial displacement of components Δx, Δy, and Δz, respectively. The nominal length of the bar OB_i is l_i, but the bar may present an error of fabrication so there exists a deviation δ_i^* of its nominal length. The modulus of elasticity of the bar is E_i, while the area of the cross-section of the bar is A_i.

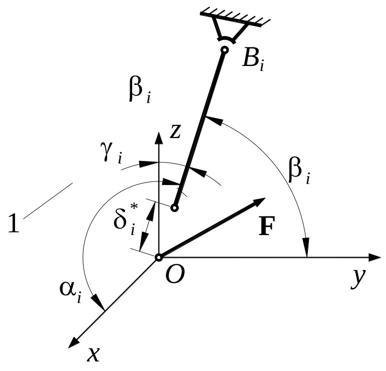

Figure 1: Mathematical model

Proceeding as in [1] one may establish the following system of equations of equilibrium

$$\left(\sum_{i=1}^{n} k_i \cos^2\alpha_i + k_{n+1}\cos^2\alpha_{n+1}\right)\Delta x + \left(\sum_{i=1}^{n} k_i \cos\alpha_i \cos\beta_i + k_{n+1}\cos\alpha_{n+1}\cos\beta_{n+1}\right)\Delta y$$
$$+\left(\sum_{i=1}^{n} k_i \cos\alpha_i \cos\gamma_i + k_{n+1}\cos\alpha_{n+1}\cos\gamma_{n+1}\right)\Delta z = F_x + \sum_{i=1}^{n} k_i\delta_i^* \cos\alpha_i + k_{n+1}\delta_{n+1}^*\cos\alpha_{n+1}, \tag{1a}$$

$$\left(\sum_{i=1}^{n} k_i \cos\beta_i \cos\alpha_i + k_{n+1}\cos\beta_{n+1}\cos\alpha_{n+1}\right)\Delta x + \left(\sum_{i=1}^{n} k_i \cos^2\beta_i + k_{n+1}\cos^2\beta_{n+1}\right)\Delta y$$
$$+\left(\sum_{i=1}^{n} k_i \cos\beta_i \cos\gamma_i + k_{n+1}\cos\beta_{n+1}\cos\gamma_{n+1}\right)\Delta z = F_y + \sum_{i=1}^{n} k_i\delta_i^* \cos\beta_i + k_{n+1}\delta_{n+1}^*\cos\beta_{n+1}, \tag{1b}$$

$$\left(\sum_{i=1}^{n} k_i \cos\gamma_i \cos\alpha_i + k_{n+1}\cos\gamma_{n+1}\cos\alpha_{n+1}\right)\Delta x + \left(\sum_{i=1}^{n} k_i \cos\gamma_i \cos\beta_i + k_{n+1}\cos\gamma_{n+1}\cos\beta_{n+1}\right)\Delta y$$
$$+\left(\sum_{i=1}^{n} k_i \cos^2\gamma_i + k_{n+1}\cos^2\gamma_{n+1}\right)\Delta z = F_z + \sum_{i=1}^{n} k_i\delta_i^* \cos\gamma_i + k_{n+1}\delta_{n+1}^*\cos\gamma_{n+1}, \tag{1c}$$

where we considered that the extra bar is denoted by $n+1$, while

$$k_i = \frac{E_i A_i}{l_i},\ i = \overline{1, n+1}. \tag{2}$$

3. NUMERICAL SPATIAL SIMULATIONS

In the simulations we will consider that the original system has four bars, while the fifth bar is the added one.

In the first case of spatial system of bars the following values are selected: (modulii of elasticity of the five bars) $E_1 = 2.11\times10^{11}\left[\mathrm{N/m^2}\right]$, $E_2 = 2.11\times10^{11}\left[\mathrm{N/m^2}\right]$,

$E_3 = 10^{11}[\mathrm{N/m^2}]$, $E_4 = 1.5\times10^{11}[\mathrm{N/m^2}]$, $E_5 = 1.3\times10^{11}[\mathrm{N/m^2}]$, (the components of the force $\mathbf{F}$) $F_x = 10^6[\mathrm{N}]$, $F_y = 1.3\times10^6[\mathrm{N}]$, $F_z = -1.5\times10^6[\mathrm{N}]$, (the variations of the lengths of the five bars with respect to their nominal lengths) $\delta_1^* = 2\times10^{-3}[\mathrm{m}]$, $\delta_2^* = 0[\mathrm{m}]$, $\delta_3^* = -1\times10^{-3}[\mathrm{m}]$, $\delta_4^* = 0[\mathrm{m}]$, $\delta_5^* = 1.1\times10^{-3}[\mathrm{m}]$, (the diameters of the cross section of the five bars) $d_1 = 20\times10^{-3}[\mathrm{m}]$, $d_2 = 25\times10^{-3}[\mathrm{m}]$, $d_3 = 40\times10^{-3}[\mathrm{m}]$, $d_4 = 45\times10^{-3}[\mathrm{m}]$, $d_5 = 35\times10^{-3}[\mathrm{m}]$, (the nominal lengths of the first four bars) $l_1 = 1[\mathrm{m}]$, $l_2 = 0.9[\mathrm{m}]$, $l_3 = 1.2[\mathrm{m}]$, $l_4 = 0.75[\mathrm{m}]$, (the angles between the first four bars and the axes of coordinates) $\alpha_1 = \frac{\pi}{6}[\mathrm{rad}]$, $\beta_1 = \frac{\pi}{2}[\mathrm{rad}]$, $\gamma_1 = \frac{\pi}{3}[\mathrm{rad}]$, $\alpha_2 = \frac{5\pi}{3}[\mathrm{rad}]$, $\beta_2 = \frac{2\pi}{3}[\mathrm{rad}]$, $\gamma_2 = \frac{\pi}{4}[\mathrm{rad}]$, $\alpha_3 = \frac{7\pi}{5}[\mathrm{rad}]$, $\beta_3 = \frac{2\pi}{3}[\mathrm{rad}]$, $\gamma_3 = 1.1668221882[\mathrm{rad}]$, $\alpha_4 = \frac{\pi}{4}[\mathrm{rad}]$, $\beta_4 = -\frac{\pi}{3}[\mathrm{rad}]$, $\gamma_4 = \frac{\pi}{3}[\mathrm{rad}]$, (the elastic constants of the bars, the formula for the fifth one is similar but its length is not known at the beginning) $k_i = \frac{\pi d_i^2}{4 l_i} E_i$, $i = \overline{1,4}$, $x_{5\min} = 0.1[\mathrm{m}]$, $x_{5\max} = 0.5[\mathrm{m}]$, $y_{5\min} = 0.3[\mathrm{m}]$, $y_{5\max} = 0.6[\mathrm{m}]$, $z_{5\min} = 1.2[\mathrm{m}]$, $z_{5\max} = 1.3[\mathrm{m}]$ (the minimum and maximum values along the three axes inside which the point B_5 can be situated), $\mathrm{d}x = 10^{-3}[\mathrm{m}]$, $\mathrm{d}y = 10^{-3}[\mathrm{m}]$, $\mathrm{d}z = 10^{-3}[\mathrm{m}]$ (the incremental steps along the three axes).

The second case of simulation is characterized by: $E_1 = 2.11\times10^{11}[\mathrm{N/m^2}]$, $E_2 = 2.11\times10^{11}[\mathrm{N/m^2}]$, $E_3 = 10^{11}[\mathrm{N/m^2}]$, $E_4 = 1.5\times10^{11}[\mathrm{N/m^2}]$, $E_5 = 1.3\times10^{11}[\mathrm{N/m^2}]$, $F_x = -1.3\times10^6[\mathrm{N}]$, $F_y = 2\times10^6[\mathrm{N}]$, $F_z = 2.5\times10^6[\mathrm{N}]$, $\delta_1^* = 2\times10^{-3}[\mathrm{m}]$, $\delta_2^* = 0[\mathrm{m}]$, $\delta_3^* = -1\times10^{-3}[\mathrm{m}]$, $\delta_4^* = 0[\mathrm{m}]$, $\delta_5^* = 1.1\times10^{-3}[\mathrm{m}]$, $d_1 = 20\times10^{-3}[\mathrm{m}]$, $d_2 = 25\times10^{-3}[\mathrm{m}]$, $d_3 = 40\times10^{-3}[\mathrm{m}]$, $d_4 = 45\times10^{-3}[\mathrm{m}]$, $d_5 = 35\times10^{-3}[\mathrm{m}]$, $l_1 = 1[\mathrm{m}]$, $l_2 = 0.9[\mathrm{m}]$, $l_3 = 1.2[\mathrm{m}]$, $l_4 = 0.75[\mathrm{m}]$, $\alpha_1 = \frac{\pi}{6}[\mathrm{rad}]$, $\beta_1 = \frac{\pi}{2}[\mathrm{rad}]$, $\gamma_1 = \frac{\pi}{3}[\mathrm{rad}]$, $\alpha_2 = \frac{5\pi}{3}[\mathrm{rad}]$, $\beta_2 = \frac{2\pi}{3}[\mathrm{rad}]$, $\gamma_2 = \frac{\pi}{4}[\mathrm{rad}]$, $\alpha_3 = \frac{7\pi}{5}[\mathrm{rad}]$, $\beta_3 = \frac{2\pi}{3}[\mathrm{rad}]$, $\gamma_3 = 1.1668221882[\mathrm{rad}]$, $\alpha_4 = \frac{\pi}{4}[\mathrm{rad}]$, $\beta_4 = -\frac{\pi}{3}[\mathrm{rad}]$, $\gamma_4 = \frac{\pi}{3}[\mathrm{rad}]$, $k_i = \frac{\pi d_i^2}{4 l_i} E_i$, $i = \overline{1,4}$, $x_{5\min} = 0.3[\mathrm{m}]$, $x_{5\max} = 0.7[\mathrm{m}]$, $y_{5\min} = 0.2[\mathrm{m}]$, $y_{5\max} = 0.8[\mathrm{m}]$, $z_{5\min} = 1.1[\mathrm{m}]$, $\mathrm{d}x = 10^{-3}[\mathrm{m}]$, $\mathrm{d}y = 10^{-3}[\mathrm{m}]$, $\mathrm{d}z = 10^{-3}[\mathrm{m}]$.

In the first case the answers are as follows: $|\Delta x|_{\min} = 0.000042[\mathrm{m}]$ for $\alpha_5 = 1.490467[\mathrm{rad}]$, $\beta_5 = 1.310279[\mathrm{rad}]$, and $\gamma_5 = 0.273175[\mathrm{rad}]$; $|\Delta y|_{\min} = 0.000060[\mathrm{m}]$ for $\alpha_5 = 1.490417[\mathrm{rad}]$, $\beta_5 = 1.312606[\mathrm{rad}]$, and $\gamma_5 = 0.270962[\mathrm{rad}]$; $|\Delta z|_{\min} = 0.0000000095[\mathrm{m}]$ for $\alpha_5 = 1.485617[\mathrm{rad}]$, $\beta_5 = 1.312711[\mathrm{rad}]$, and $\gamma_5 = 0.272392[\mathrm{rad}]$; $\left(\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}\right)_{\min} = 0.002778[\mathrm{m}]$ for $\alpha_5 = 1.214063[\mathrm{rad}]$, $\beta_5 = 1.138389[\mathrm{rad}]$, and $\gamma_5 = 0.576975[\mathrm{rad}]$; $|N_1|_{\min} = 0.047432[\mathrm{N}]$ for $\alpha_5 = 1.350652[\mathrm{rad}]$, $\beta_5 = 1.279145[\mathrm{rad}]$, and $\gamma_5 = 0.369401[\mathrm{rad}]$; $|N_2|_{\min} = 0.163784[\mathrm{N}]$ for $\alpha_5 = 1.358946[\mathrm{rad}]$, $\beta_5 = 1.150587[\mathrm{rad}]$, and

$\gamma_5 = 0.476816[\mathrm{rad}]$; $|N_3|_{\min} = 975.8196[\mathrm{N}]$ for $\alpha_5 = 1.490843[\mathrm{rad}]$, $\beta_5 = 1.293300[\mathrm{rad}]$, and $\gamma_5 = 0.289376[\mathrm{rad}]$; $|N_4|_{\min} = 20013.398[\mathrm{N}]$ for $\alpha_5 = 1.490286[\mathrm{rad}]$, $\beta_5 = 1.318827[\mathrm{rad}]$, and $\gamma_5 = 0.265055[\mathrm{rad}]$; $|N_5|_{\min} = 0.003685[\mathrm{N}]$ for $\alpha_5 = 1.267035[\mathrm{rad}]$, $\beta_5 = 1.255546[\mathrm{rad}]$, and $\gamma_5 = 0.445395[\mathrm{rad}]$; $\left(\sum_{i=1}^{5}|N_i|\right)_{\min} = 869505.071[\mathrm{N}]$ for $\alpha_5 = 1.193347[\mathrm{rad}]$, $\beta_5 = 1.280752[\mathrm{rad}]$ and $\gamma_5 = 0.485330[\mathrm{rad}]$.

For the second case one obtains: $|\Delta x|_{\min} = 0.000001[\mathrm{m}]$ for $\alpha_5 = 1.310287[\mathrm{rad}]$, $\beta_5 = 1.350817[\mathrm{rad}]$, and $\gamma_5 = 0.344343[\mathrm{rad}]$; $|\Delta y|_{\min} = 0.000539[\mathrm{m}]$ for $\alpha_5 = 1.314115[\mathrm{rad}]$, $\beta_5 = 0.965626[\mathrm{rad}]$, and $\gamma_5 = 0.672543[\mathrm{rad}]$; $|\Delta z|_{\min} = 0.00000569[\mathrm{m}]$ for $\alpha_5 = 1.279858[\mathrm{rad}]$, $\beta_5 = 1.219041[\mathrm{rad}]$, and $\gamma_5 = 0.464891[\mathrm{rad}]$; $\left(\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}\right)_{\min} = 0.002187[\mathrm{m}]$ for $\alpha_5 = 1.095497[\mathrm{rad}]$, $\beta_5 = 1.020457[\mathrm{rad}]$, and $\gamma_5 = 0.763008[\mathrm{rad}]$; $|N_1|_{\min} = 0.008567[\mathrm{N}]$ for $\alpha_5 = 1.100687[\mathrm{rad}]$, $\beta_5 = 1.387306[\mathrm{rad}]$, and $\gamma_5 = 0.510199[\mathrm{rad}]$; $|N_2|_{\min} = 1.137601[\mathrm{N}]$ for $\alpha_5 = 1.350476[\mathrm{rad}]$, $\beta_5 = 0.971034[\mathrm{rad}]$, and $\gamma_5 = 0.650114[\mathrm{rad}]$; $|N_3|_{\min} = 14419.090[\mathrm{N}]$ for $\alpha_5 = 1.095739[\mathrm{rad}]$, $\beta_5 = 1.020745[\mathrm{rad}]$, and $\gamma_5 = 0.767847[\mathrm{rad}]$; $|N_4|_{\min} = 74.96161[\mathrm{N}]$ for $\alpha_5 = 1.323615[\mathrm{rad}]$, $\beta_5 = 1.194129[\mathrm{rad}]$, and $\gamma_5 = 0.457567[\mathrm{rad}]$; $|N_5|_{\min} = 0.000041[\mathrm{N}]$ for $\alpha_5 = 1.288224[\mathrm{rad}]$, $\beta_5 = 1.006448[\mathrm{rad}]$, and $\gamma_5 = 0.647810[\mathrm{rad}]$; $\left(\sum_{i=1}^{5}|N_i|\right)_{\min} = 680762.2[\mathrm{N}]$ for $\alpha_5 = 1.296042[\mathrm{rad}]$, $\beta_5 = 0.971863[\mathrm{rad}]$ and $\gamma_5 = 0.675963[\mathrm{rad}]$.

4. CONCLUSIONS

In this paper we have studied the minimization of the displacements (along one direction or considering it by Euclidian norm) and of modulii of the tensions in one particular bar or as sum of modulii. The reader may observe that even in the particular cases the problem has an analytical solution, the only possibility to obtain a solution being by use of numerical calculation. The additional bar can have an end in a zone of space defined by a parallelepiped for which one knows $x_{\min}$, …, $z_{\max}$. Of course, depending on the problem this zone may be another particular one (not necessary a parallelepiped, but a sphere, an ellipsoid etc.) or may be a reunion of particular zones. The problem may simplify in the planar case when only two equations are obtained, but neither in this case may an analytical solution be obtained in the general situation. In fact, analytical solution looks to be possible only for very particular systems of bars.

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