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## THE ADVANTAGES OF MODERN DIMENSIONAL ANALYSIS IMPLEMENTING IN THE ADITIVE MANUFACTURING TECHNOLOGIES OPTIMIZATION

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**Abstract:** Nowadays the Additive Manufacturing (AM) represents a very promising manufacturing process, having several facilities and advantages in comparison with the classical manufacturing technologies. Also, one can underline that there are a huge number of unexplored directions, which assure for AM to becoming soon a very competitive manufacturing process, with undoubted low-cost-, reduced material consumption-, as well as optimal stiffness-, and competitiveness technology. In this sense, one of the less-explored ways represents exactly the involvement of the dimensional methods in gaining an optimal, high-competitive final product. Like this, instead of the real structural element, named prototype, the engineers will perform high-accuracy tests on the attached reduced-scale models, whose experimental results are extended to prototype by means of the deduced Model Law (ML). The authors, based on their previous theoretical as well as experimental investigations, offer a short overview of these advantages, based on Thomas Szirtes' approach of dimensional analysis, referred to below Modern Dimensional Analysis (MDA).

**Keywords:** Additive Manufacturing (AM), Fuse Deposition Modelling (FDM), Geometric Analogy (GA), Theory of Similarity (TS), Modern Dimensional Analysis (MDA).

### 1. BRIEF OVERVIEW OF AM

Nowadays AM represents a very promising way to obtaining high quality-, cheap-, and low time-consumption-, as well as low pollution/waste products (unique components or spare parts, too). Since 1986, by Rapid Prototyping, the first complex products were obtained [1-4]. Later, the technology was applied not only for plastics, but also for plastic-metal, respectively only metal parts [5]. The Fuse Deposition Modelling (FDM), representing the authors' main field of analysis/investigation, was one of the earlier technologies [6-7]. It was followed by Power Bed Fusion processes, widely analyzed and applied [8-11]. The Sheet Lamination processes, which followed them, gained a huge and efficient application [12-15]. Nowadays, the Directed Energy Deposition,

respectively the Automated Fiber Placement techniques are widely and successfully applied [16-18]. The common casting technologies, combined with *AM*, offer several facilities and advantages both in obtaining complex final products and in improving their initial manufacturing process from point of view of cost, time, accuracy, waste amount, as well as durability. A fruitful combination of the plastic and metal layers, offers, mainly in the cases of the casting molds and cores manufacturing, new research directions. In these latter cases, due to the combination of the mechanical and thermal loadings, their stress-strain states become very complex and so, the involvement of the dimensional methods can represent a very promising modality/approach. It means that instead of the testing of the real structural element, named *prototype*, one can perform high-accuracy tests on an attached, reduced-scale one, named *model* [19-26]. It is well-known fact that in most cases, performing tests on the prototype is difficult, while on the model, they can be performed much easier, precisely, repeatable and cheaply. By the adequately deduced *MLs*, constituted strictly from dimensionless variables, one can extend the obtained results on model to the prototype, forecasting its (latter's) behaviors. In the following, the authors briefly review the main dimensional methods, together with their advantages and limits.

## 2. THE MAIN DIMENSIONAL METHODS AND THE PRINCIPLE OF *MDA* WITH ITS ADVANTAGES

In order to obtain easier and cheaper information on structural elements, as well as on large structures, mathematicians and engineers introduced the dimensional methods.

For the relatively simply cases the Geometric Analogy (*GA*) satisfies the imposed requirements. Here the geometric similarity is compulsory; it supposes rigorous proportionality of lengths, as well as angular equality for the prototype and the attached model. One can define homologous points, lines, surfaces and volumes; consequently, the attached model has a very limited flexibility with respect to the prototype. The Theory of Similarity (*TS*) solves a little bit more complex phenomenon, allowing both structural and functional similarity. In this case, the analyzed phenomena occurs so that, at homologous times, in homologous points, each involved  $\eta$  significant variable are described by distinct (separate)

$$S_{\eta}[-] = \frac{\eta_2}{\eta_1} = const. \quad (1)$$

constant ratio of the values, corresponding to model ( $\eta_2$ ) and prototype ( $\eta_1$ ).

The  $S_{\eta}$  dimensionless ratios are the so-called *scale factors*, which are always constant in time and space for the given phenomena; their number coincides with the involved variables' number.

In principle, instead of some solutions of complicated equations, one can apply relatively simply correlation between a reduced numbers of

$$\pi_j, j = 1...n \quad (2)$$

dimensionless variables, which constitute the *ML*; at *TS*, they result by means of suitable grouping of the adequate terms of governing equations.

One has to mention, that the involvement of the above-mentioned *MLs* assure a significant diminishing of the measurement's volume.

For a large number of the dimensionless variables, the Classical Dimensional Analysis (*CDA*) was applied, based on the well-known Buckingham's  $\pi$  theorem.

At first sight, seems that *CDA* offers a relatively easy manner for analysis of complex phenomena. Upon closer analysis, we notice its main disadvantages, based between others on its difficulties in deducing the demanded/requested  $\pi_j, j=1\dots n$  dimensionless variables.

One has to mention that for obtaining these  $\pi_j, j=1\dots n$  dimensionless variables, *CDA* offers three main modalities, namely from:

the Buckingham's  $\pi$  theorem;

the partial differential equations applied to fundamental differential relations of the analyzed phenomenon, when the initial variables, by suitable grouping, offer these dimensionless quantities;

the complete, but in the same time the simplest equation(s) which describe the phenomena, which will be transformed into dimensionless forms, offering finally the desired  $\pi_j$  groups.

Consequently one can mention the main shortcomings of the *CDA*, namely:

the protocol in obtaining the desired set of  $\pi_j$  groups is rather chaotic, arbitrarily, and strongly depending on the ingenuity as well as of the involved specialist's experience;

for the involved specialist, there are required solid knowledge in the field of the analyzed phenomenon, as well as in higher mathematics, too;

only rarely (occasionally) can be obtained the complete *ML*, mainly due to the fact that there are only a limited number of the involved mathematical relations related to the phenomena;

for common engineers or specialists, involved in prototype-model correlation analysis, *CDA* does not represent an easy approach.

Compared to this, the methodology developed by Szirtes [27-28], hereafter named Modern Dimensional Analysis (*MDA*), offers an efficient solution practically for all above-analyzed shortcomings.

Consequently, the *MDA* represents a simply, unitary, as well as particularly accessible methodology, with the following main advantages:

the involved specialist, instead of thorough connoisseur in the phenomenon as well as in higher mathematics, only has to identify the set of the involved variables, of course together with their dimensions, which have (or can present) a certain extent influence on the analyzed phenomena;

it has an unitary, simply and user-friendly protocol, which assures at once to eliminate automatic all insignificant/irrelevant variables;

in all cases *MDA* assures obtaining the complete set of the  $\pi_j, j=1\dots n$  dimensionless variables, as well as the complete *ML*; this is practically impossible the all afore-mentioned methods, excepting some particular cases; this *ML* is very flexible, suitable for several particular cases, corresponding to simplified approaches of the phenomena;

by an a priori choosing/setting of the directly related variables to the conceived experimental investigations on model, hereafter named *independent variables*, *MDA* assures an additional flexibility, which represents a significant advantage,

non-existent in all the methods mentioned above; their a priori chose is possible/admitted both for the prototype and model;  
 this set (of the independent variables) assures defining the most suitable model, which will offer for the involved model the most simply, lower-cost testing conditions, safety, as well as repeatable experimental investigations;  
 The rest of the variables, hereafter named *dependent variables*, only for the prototype can be chosen a priori; their magnitudes for the model are strictly obtained by applying a given (suitable) element of the *ML*;  
 between the dependent variables there are also a small number of variables of the prototype, whose magnitude cannot be obtained more easily (with low cost or accessible experimental measurements) and whose determination is actually the purpose of this dimensional analysis; thus, these afore-mentioned prototype's variables are obtained by applying the *ML*;  
 furthermore, *MDA* removes the restriction of the geometric similarity of the model with the prototype, e.g. the shape of the cross-sections can be different at the model from the prototype; in this case, instead of selecting/choosing as independent variables the cross-sectional dimensions, one will substitute them by the  $I_z$  second order moment of inertia of cross-section;  
 if the material is considered as independent variable, choosing by mean of  $E$  Young modules, than one can accept different material for the model, respectively for the prototype;  
 in the case of choosing instead of them the  $E \cdot I_z$  flexural stiffness (rigidity), than neither the shape of the cross-section, nor the type of material must be identical in the prototype and model; the single request/condition remaining that their

$$S_{E \cdot I_z} = \frac{E_2 \cdot I_{z,2}}{E_1 \cdot I_{z,1}} \quad (3)$$

scale factor to remain the same (to be constant), with the afore-mentioned indexing (2- for model and 1-for prototype).

Of course, this evaluation can be continued, but only these, afore-mentioned facilities underline the suitability of *MDA* in *AM* process optimization.

### 3. CONCLUSIONS

Based on the authors' previous theoretical and experimental investigations, including *MLs'* validation for different structural elements manufactured by *AM* [29-31], one can conclude the followings:

The *MDA* offers several incontestable facilities, starting from choosing different materials up to adopting different shape of cross-sections for prototype and model;

It represents a simply, unitary, as well as particularly accessible methodology; the involved specialist, instead of thorough connoisseur in the phenomenon as well as in higher mathematics, only has to identify the set of the involved variables, of course together with their dimensions, which have (or can present) a certain extent influence on the analyzed phenomena;

it assures at once to eliminate automatic all insignificant/irrelevant variables;

in all cases *MDA* assures obtaining the complete set of the  $\pi_j, j=1...n$  dimensionless variables, as well as the complete *ML*; this is practically impossible the all afore-mentioned methods, excepting some particular cases;

this *ML* is very flexible, suitable for several particular cases, corresponding to simplified approaches of the phenomena;  
the deduced *ML* for a complex phenomenon it can serve to obtain particular cases, with more simply and cheapest models;  
in addition, the deduced *ML* for a given structural element, allows its application (extension) to complex structures, made of these structural elements, by taking into account the homologous points of the structure in relation to the basic structural element.

All of these advantages can be followed in the authors' previous mentioned works. Among the following goals of the authors is the analysis of complex structures, made of several materials (plastic combined with metals), obviously through *AM* technology, with immediate application to molds for casting unique pieces, respectively of complex shapes.

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