# PHOTOELASTIC AND NUMERICAL STRESS ANALYSIS OF A PIN ON A PLAN CONTACT SUBJECTED TO A NORMAL AND A TANGENTIAL LOAD 

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#### Abstract

Theoretical studies of contact stresses can be in some cases very complex. Several methods, experimental as well as numerical, are used to analyze these types of problems. In this paper two methods are used: the photoelasticity method and the finite element method. Stresses are determined in the neighborhood of the contact zone for a plan subjected to a normal load and a tangential load via a pin of cylindrical cross section. The purpose here is to study the effect of applying simultaneously a normal and a tangential load. Photoelastic fringes are used to obtain stress values, particularly in the neighborhood of the contact zone. Comparisons are made between experimental and simulated isochromatic fringes and isoclinic fringes. Relatively good agreements are observed.


Keywords: Photoelasticity, birefringence, isoclinic, isochromatic, contact.

## 1. INTRODUCTION

Stress initiation is mainly controlled by shear stress mechanisms, particularly for metallic parts. It is therefore very important to determine the type and the amplitude of the imposed mechanical stresses. Photoelastic fringes obtained experimentally with plan polarized light can help designers determine the stress field developed in mechanical parts particularly in the neighborhood of the contact zones. Several studies have been conducted [1-6], experimentally as well as numerically. In this work a contact problem between a pin and a plan is solved experimentally by the unwrapping of the isochromatic and the isoclinic fringe patterns. Two different ways of loading are studied, the first one is done with a normal load alone and the second one with a normal and a tangential load applied simultaneously on the model via a pin of cylindrical cross section. A finite element solution is used to simulate fringes and stresses developed in the stressed model particularly in the neighborhood of the contact zone. In our case, it is not necessary to separate the principal stresses; a comparison between experimental and simulated fringes is used to compare between experimental and simulated stresses in order to validate the finite element solution. A comparison is also made between experimental and
simulated values of the principal stresses difference along the vertical axis of symmetry for validation of the finite elements solution.

## 2. EXPERIMENTAL STRESS ANALYSIS

Let us recall, briefly, the principle of the photoelastic method based on the birefringent phenomenon (figure 1). The light intensity (eqn.1), obtained on the analyzer after traveling through the model and the different optical elements, is given by the following relation [7]:

$$
\begin{equation*}
I=a^{2} \sin ^{2} 2 \alpha \sin ^{2} \varphi / 2 \tag{1}
\end{equation*}
$$

The term represents the isoclinic fringes and the term represents the isochromatic fringes; these two terms give respectively the principal stresses directions and the values of their difference which can be obtained by using the following relation:

$$
\begin{equation*}
\sigma_{1}-\sigma_{2}=N f / e \tag{2}
\end{equation*}
$$

Where N is the fringe order obtained experimentally from the isochromatic fringe pattern, e is the model thickness and $f$ is the fringe value which depends on the light wave length used and the optical constant of the model material. Different tests can be used to determine this value; here a tensile test is used. The fringe orders recorded during loading of the specimen allowed us to determine the fringe constant which will be used later to analyze the stress field in the models ( $f=10.7 \mathrm{~N} / \mathrm{mm}$ ).


Figure 1: Light propagation in a polariscope
The model in the shape of a parallelepiped ( $50 \times 50 \times 4 \mathrm{~mm}$ ) is cut in a birefringent material. The model is then mounted on a loading frame (figure 2). A load is applied on the model via an aluminum pin of cylindrical cross section ( $\mathrm{R}=10 \mathrm{~mm}$ ). The applied equivalent load is set to $\mathrm{Fn}=485 \mathrm{~N}$. Plane polarized light is used to obtain the isochromatic and the isoclinic fringes. To obtain only the isochromatics, two quarter wave plates are added to the setting in the light path; one after the polarizer and the second one just before the analyzer. By doing so the light changes from plan polarized to circularly polarized light; therefore eliminating the isoclinics that might hide the isochromatic fringes.

Two tests are conducted, the first one with a normal load alone ( $\mathrm{F}_{\mathrm{n}}=485 \mathrm{~N}$ ) and the second one with the same normal load and a tangential load ( $F_{t}=40 \mathrm{~N}$ ) applied simultaneously via a pin of cylindrical cross section (Figure 2, right).

The applied load should be directly applied to the model. This is achieved by the use of a pulley and a bearing that should prevent friction during loading.


Figure 2: Model mounted on the loading frame (left), details of the model loading (right)

### 2.1. The normal load applied alone

Figure 3 shows the isochromatic fringe pattern obtained for the first test with a polychromatic light on a dark field circular polariscope; too quarter wave plates are used in order to eliminate the isoclinic fringes. The isochromatic fringes allowed us to obtain the values of the principal stresses difference in the model along the vertical axis of symmetry particularly in the neighborhood of the contact zone (figure 3). The value of the principal stresses difference increases from a value of 7.15 MPa starting from point at the origin to this maximum value of 73.34 MPa and then decreases as we move away from the contact zone to a lower value of about 2.92 MPa .



Figure 3: Isochromatic fringes and stresses variations along the vertical axis

### 2.2. The normal load and the tangential load applied simultaneously

Figure 4 shows isochromatic fringes obtained with polychromatic light on a white field background polariscope for the case of a normal and a tangential load applied simultaneously. These isochromatics show clearly the influence of
the tangential load; the fringes are not symmetrical anymore about the vertical axis. The values of the fringe orders taken from this isochromatic fringe pattern allowed us, by using eqn.2, to obtain the variation of the principal stresses difference along the vertical axis of symmetry (figure 4, right). Contrary to the first model, we can see that, for the numerical values, the difference of the principal stresses decreases from a value approximately equal to 82 MPa from the origin located at the upper surface of the model, to a minimum value of 49 MPa and rises again to a value of 56 MPa . Stresses decrease then, as we move away from the contact zone, to lower values. We can see that the numerical solution agrees relatively well with the experimental results with a slight difference which may be explained by experimental errors in the appreciation of the tangential load.


Figure 4: Isochromatic fringes (left) - Principal stresses difference (right)

## 3. NUMERICAL ANALYSIS

The following relation (eqn. 3) which can be obtained readily from Mohr's circle for stresses is used to calculate the principal stresses difference at any point of a stressed model.

$$
\begin{equation*}
\left.\sigma_{1}-\sigma_{2}=\left(\left(\sigma_{x}-\sigma_{y}\right)^{2}+4 \tau_{x y}{ }^{2}\right)\right)^{0.5} \tag{3}
\end{equation*}
$$

Using eqn. 1 into eqn. 3, the different values of the retardation angle $\varphi$ can be calculated at any point on the model using the following relation (eqn. 4):

$$
\begin{equation*}
\left.\varphi=2 \pi N=2 \pi \frac{e}{f}\left(\left(\sigma_{x}-\sigma_{y}\right)^{2}+4 \tau_{x y}^{2}\right)\right)^{0.5} \tag{4}
\end{equation*}
$$

The different values are used to determine the isochromatics over the whole model. The different values of the isoclinic parameter can be calculated with the following relation (eqn. 5) which can be obtained readily from Mohr's circle for stresses:

$$
\begin{equation*}
\alpha=\tan ^{-1}\left(2 \tau_{x y} /\left(\sigma_{x}-\sigma_{y}\right)\right) \tag{5}
\end{equation*}
$$

The different values give then easily the isoclinic fringe pattern. To achieve a better simulation of the applied load, an imposed displacement is applied to the model at the contact surface between the pin and the plan. The equivalent applied load is calculated then as the sum of the elementary vertical load components at the nodes located at the lower surface of the model which is in contact with the loading frame. The meshing is refined in the neighborhood of the contact zone (figure 5) to achieve a better simulation.


Figure 5: Finite element meshing
The program calculates the different values of the terms $\sin ^{2} \varphi / 2$ and $\sin ^{2} 2 a$ which represent respectively the isochromatics and the isoclinic fringe patterns. The simulated fringe patterns obtained can then be compared to the experimental ones. For both studied cases, the simulated isochromatic fringe patterns are similar to the isochromatic fringe patterns obtained experimentally.


Figure 6 : Isochromatic fringes: (A) experimental, (B) simulated
As for the isoclinic fringe pattern it is calculated for two polarizer and analyzer angles ( $0^{\circ}$ and $45^{\circ}$ ) for the first case studied; the isochromatics are purposely removed from the simulated fringe pattern (figure 7). The isoclinics obtained numerically are similar to the experimental ones. In our case it is enough to compare the isoclinic fringe patterns since the main purpose is to validate the finite element solution. If one needs to obtain the principal stresses trajectories, the isoclinic fringe patterns should be obtained for several angles to allow a higher precision in the construction of the principal stresses trajectories.


Figure 7: Comparison between simulated and experimental isoclinic fringes

## 4. Conclusion

With the help of these two tests, we have shown that two dimensional photoelasticity can be successfully used to validate the numerical approach (mesh, boundary conditions). For the case of a normal load alone, the isochromatic fringes are symmetrical about the vertical axis of symmetry. Stresses obtained experimentally are similar to the numerical ones. For the second case, for which a tangential load and a normal load are applied simultaneously, the influence of the tangential load on the developed stresses field is clearly observed; the isochromatic fringes are not symmetrical anymore about the vertical axis. The values of the principal stresses difference obtained along the vertical axis of symmetry showed clearly the stress variations and the maximum stress values. The next step of this study is to develop reliable finite element solutions for three dimensional contact problems in the presence of tangential loads.

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