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MODERN DIMENSIONAL ANALYSIS INVOLVED IN ENGINEERING PROBLEMS

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Abstract: *Complex engineering problems are solved recently more and more often with the help of methods which use the correlation between real parts (prototype) and an appropriately chosen model. The authors have made a brief review of these methods, emphasizing their limitations. Starting from this primary information, the authors present a much more efficient, unitary and particularly accessible method to all engineers and researchers, i.e. Modern Dimensional Analysis, developed by Prof. Szirtes. After presenting the method, the authors illustrate the applicability and flexibility of the method with the help of an example from the field of linear displacement calculation.*

Keywords: *prototype, model, Geometric Analogy, Similarity Theory, Classical and Modern Dimensional Analysis, static displacements*

INTRODUCTION

In order to solve some highly complex problems, either mechanical, thermal or of another nature, the engineer was obliged to correlate the theoretical results with the experimental ones based on the Theory of Models (TM). TM allows the reduction of both the number of experimental investigations and the graphics related to the reproduction of the studied phenomenon. In this case, the first time one define the prototype (the real part) and the model associated with it, which is usually made on a reduced scale, but can also be on a larger scale, if the analyzed phenomenon requires this [1; 54].

A first solution was offered by the Geometric Analogy (GA), where by defining homologous points-, lines-, angles-, surfaces- and volumes it

became possible to correlate the behavior of the prototype with that of the associated model.
For slightly more complex phenomena, the Theory of Similitude (TS) was used, where based on experimental investigations carried out on the model, the probable responses of the prototype could be deduced.
As shown in the works [1; 2; 10; 124], the similarity can be structural, where the focus is mainly on the geometric similarity of the two entities (prototype, respectively model), respectively functional, when it is assumed that similar processes are carried out in homologous times, thus ensuring the similarity of all physical quantities, which intervene in the description of the studied physical phenomenon. In turn, the functional similarity can be kinematic, respectively dynamic, when similar kinematic or dynamic phenomena will take place in homologous points and in homologous times, so that each variable (physical quantities) ω corresponds to a dimensionless ratio:

$$S_\omega = \frac{\omega_2}{\omega_1}[-]$$

constant in time constant in and in space, referred to as Scale Factors or Similarity Ratios, obtained on the model (ω_2), respectively on the prototype (ω_1).
Regarding the mathematical solution of the equations (usually complex in form) that describe the phenomenon studied in the theoretical way, it will be replaced by establishing some correlations between the dimensionless quantities based on some favorable groupings of the terms involved in the respective analytical equation; these dimensionless ratios are also called similarity criteria, and their correlations lead to the so-called criterion relations. Based on the experimental data, the criterion relations will lead, both to the simplification of the analysis of the phenomenon, and to the reduction of the experimental measurements necessary to establish some relevant and reliable correlations between the prototype and the model.
For each phenomenon, both the set of analytical equations and the set of criterion relations are uniquely defined.
For complex phenomena, the Theory of Similitude will be replaced by Dimensional Analysis, because the number of dimensionless variables, respectively of criterion equations increases a lot and the method becomes cumbersome.
Its classical variant, Classical Dimensional Analysis (CDA), also uses the correlation between prototype and model, by means of a set of dimensionless relations, set, which was established starting from Buckingham's π theorem [3; 4; 5; 6; 10; 12].
CDA does not eliminate experimental measurements, which it will require to be performed on the model, nor does it propose to explain the analyzed phenomenon from another point of view. The basic goal of CDA is to simplify and optimize the strategy of experimental investigations based on this set of dimensionless variables $\pi_j,\ j=1,\ldots,n$, and the two systems (prototype and

model), through their behavior, will fully respect the conditions imposed on all dimensionless groups.
In the case of CDA, the establishment of these dimensionless groups can be achieved by [1]:
• direct application of the theorem π;
• application of the method of partial differential equations on the basic relations of the phenomenon, followed by the normalization of the initially involved variables and their subsequent grouping in the desired dimensionless groups π_j;
• processing the complete and at the same time the simplest form of the equations related to the phenomenon, by transforming them into dimensionless forms, followed by the identification of these dimensionless groups.
It should be emphasized that all these above-mentioned theories are usually cumbersome, assuming among others:
• a substantial knowledge of higher mathematics;
• through knowledge in the field of the respective phenomenon;
• a laborious methodology for establishing the set of dimensionless groups;
• significant experience in the field, as well as intuition and ingenuity in establishing these dimensionless groups.
In a word, it is not a method accessible to the average engineer/researcher, but rather a theoretical one.
This is also the explanation for its widespread use/application by ordinary specialists. Their detailed presentation can also be found in the works [1; 2; 3; 4; 5; 6; 12].
Against these shortcomings, the methodology developed by Szirtes in his works [7; 8], brings significant simplifications to Dimensional Analysis, and this variant/version will be called Modern Dimensional Analysis (MDA).
MDA practically eliminates all the shortcomings of the previously presented methods, including CDA, being a method:
• easy, unitary and simple;
• which do not require deep knowledge in the field of the respective phenomenon, only to take into account all the variables, on which this phenomenon could depend to a certain extent;
• which ensures the establishment of the complete group (complete set) of dimensionless variables $\pi_j,\ j=1,...,n$;
• from the complete set thus established, based on simplifications of the phenomenon, simpler and easier to model variants can be found (primarily to simplify experimental investigations on the model);
• ensures the automatic elimination of those variables, whose influence is insignificant, whether the phenomenon does not depend on them, or whether their contribution is below a certain minimum threshold;
• allows a favorable selection of variables, so that the model becomes as flexible as possible, and experimental investigations are as simple, safe, reproducible and last but not least at a minimal cost price.
The methodology proposed by Szirtes, also illustrated in the next chapter, includes the following unique basic steps:

• review of all variables (together with their dimensions), which could have any influence on the studied phenomenon;
• choosing that set of variables, which can be chosen as size/magnitude a priori freely, both for the prototype and for the model, called independent variables; from the exponents of their dimensions, an invertible matrix is formed (so quadratic and with zero determinant), which will be matrix A;
• the rest of the variables, which will be the dependent variables, will be included in matrix B; they are chosen as size/magnitude a priori freely only for the prototype, and for the model they will result exclusively from the elements of the law of the model, which is to be deduced; the exception is that variable (or a small number of variables) related to the prototype, the determination of which is difficult, i.e. this is the purpose of modeling with MDA;
• the Dimensional Set (DS) consisting of matrices B, A, I and C will be formed, according to Figure 1

B	***A***
I_{nxn}	$C = -\left(A^{-1} \cdot B\right)^{T}$

Figure 1: The Dimensional Set

where: I_{nxn} is a unity matrix of order n, with n representing the number of dependent variables (the columns), identical to that of the number of dimensionless variables $\pi_j,\ j = 1,\dots,n$, which will lead to the formation of the set called the Model Law; the exponent (-1) refers to the inverse matrix, while (T) to the transposed matrix.
• The order of placing the variables within matrices A and B is optional and does not influence the Model Law to be deduced;
• Multiple variables with identical dimensions cannot be placed in matrix A, because the condition of its invertibility would be compromised;
• Instead, in matrix B there can be several variables with identical dimensions;
• With the help of matrix I_{nxn} lines $j = 1,2,\dots,n$ and C, all the dimensionless physical variables (quantities) sought $\pi_j\ j = 1,2,\dots,n$ are defined. Thus, the line j of the common matrix (I_{nxn} and C) will contain the exponents, which intervene in the definition of the dimensionless variable π_j and which will be equal to the product of a dependent variable (from matrix B, at the power of one), and the totality of the independent variables involved (from the matrix A, located at the powers contained in the row j of the matrix C.
• In order to deduce the Model Law, the expressions of all dimensionless variables π_j will equal unity, as will be shown next. It should be noted that, in each of these products, from the matrix I_{nxn} (a unit matrix), there is only one dependent variable, to the power of 1 (one), while from the matrix C: all the independent variables to the powers resulting from the calculation provided by the relationship calculation of the matrix C;
• In order to simplify the analysis, these obtained dimensionless variables π_j can be later merged, grouped for an easier analysis.

The next chapter will illustrate this unitary methodology. At the same time, a series of facilities will be highlighted regarding how the choice of independent variables can lead to obtaining the most flexible models.

1. STATIC DISPLACEMENT ANALYSIS

The illustration of the application of the MDA is provided by means of the calculation of the vertical displacement of a cantilever, made of a steel with the modulus of elasticity of longitudinal, rectangular section and length, required by a concentrated force (Fig. 2).

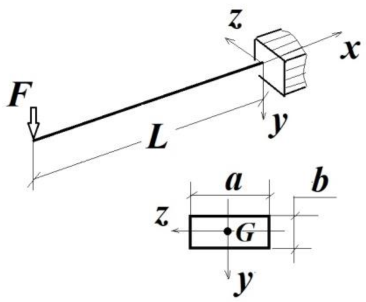

Figure 2: Analyzed clamped beam

For this prototype, it is required to find that model, which, based on the Dimensional Analysis, will allow the determination of the vertical displacenment "v" (in the direction of the force F) of the prototype with the help of the Law of the Model.

In the first instance, the following were chosen as variables: (v, E, L, F, a, b, g). The separation of lengths was applied, according to the three orthogonal directions (x, y, z), that is m_x, m_y, m_z, in order to increase the number of dimensions and thus, reduce the number of dimensionless variables π_j, which will form the Law of the Model.

After performing the calculations mentioned in the previous theory chapter, the Dimensional Set below (Variant *V1*) resulted, where the independent variables (L, F, a, b, g), included in Matrix A, and the dependent variables, included in Matrix B, respectively (v, E), were chosen.

Due to the fact that, in the matrix C, under the variable g, only values equal to zero are found, it turned out that it is a physically irrelevant variable and is eliminated from the matrix A, respectively g from all subsequent calculations.

		B				A		
V1		**v**	**E**	**L**	**F**	**a**	**b**	**g**
	mx	0	1	1	0	0	0	0
	my	1	-1	0	1	0	1	1
	mz	0	-1	0	0	1	0	0
	kg	0	1	0	1	0	0	0
	s	0	-2	0	-2	0	0	-2
	π1	1	0	0	0	0	-1	0
	π2	0	1	-1	-1	1	2	0

Based on the calculations, the component elements of the Model Law resulted $\pi_j = 1,\ j = 1,2$, where $\pi_j = 1,\ j = 1,2$ by equating each dimensionless variable with unity, i.e. and the substitution of the actual variables ω included in π_j their scales $S_\omega = \frac{\omega_2}{\omega_1}[-]$, the sought-after elements of the Model Law resulted, i.e.:

$$\pi_1 = \mathrm{v} \cdot b^{-1} = \mathrm{v}/b \Leftrightarrow S_\mathrm{v} = S_b$$

$$\pi_2 = \frac{E \cdot a \cdot b^2}{L \cdot F} \Leftrightarrow S_F \cdot S_L = S_E \cdot S_a \cdot (S_b)^2 \Rightarrow S_E = \frac{S_F \cdot S_L}{S_a \cdot (S_b)^2}$$

Having the model, and the independent variables chosen a priori $(L_1, L_2, F_1, F_2, a_1, a_2, b_1, b_2\, E_1)$, the size of the modulus of elasticity of the material of the model is determined, that is E_2, the model is made from this material. Based on the measurements on the model, the force F_2 will result in the displacement v_2, with the help of which, from the first element of the Model Law, one obtains in turn:

$$S_\mathrm{v} = S_b \Leftrightarrow \frac{\mathrm{v}_2}{\mathrm{v}_1} = \frac{b_2}{b_1} \Rightarrow \mathrm{v}_1 = \frac{\mathrm{v}_2 \cdot b_1}{b_2}$$

The calculation was resumed, without the variable g, obtaining the *V2* version, where by eliminating the variable g, it was also necessary to change the units of measure (in its place, kg, s, was introduced N, in order to be able to express all the remaining variables appropriately. Thus, the matrix A was formed by the elements related to the variables (L, F, a, b), and the matrix B remaining the same.

		B		A			
		v	**E**	**L**	**F**	**a**	**b**
V2	**mx**	0	1	1	0	0	0
	my	1	-2	0	0	0	1
	mz	0	-1	0	0	1	0
	N	0	1	0	1	0	0
	π1	1	0	0	0	0	-1
	π2	0	1	-1	-1	1	2

Following the calculations, although the number of variables involved was reduced, the same component expressions of the Model Law were obtained, i.e.:

$$\pi_1 = \mathrm{v} \cdot b^{-1} = \mathrm{v}/b \Leftrightarrow S_\mathrm{v} = S_b$$

$$\pi_2 = \frac{E \cdot a \cdot b^2}{L \cdot F} \Leftrightarrow S_F \cdot S_L = S_E \cdot S_a \cdot (S_b)^2$$

The calculations were resumed to find other versions, obtaining practically the same constitutive elements for the Model Law.

		B		A			
		v	**F**	**E**	**L**	**a**	**b**
	mx	0	0	1	1	0	0
V3	**my**	1	0	-2	0	0	1
	mz	0	0	-1	0	1	0
	N	0	1	1	0	0	0

	π1	1	0	0	0	0	-1
	π2	0	1	-1	1	-1	-2

$$\pi_1 = v \cdot b^{-1} = v / b \Leftrightarrow S_v = S_b$$

$$\pi_2 = \frac{F \cdot L}{E \cdot a \cdot b^2} \Leftrightarrow S_F \cdot S_L = S_E \cdot S_a \cdot (S_b)^2 \Rightarrow S_F = \frac{S_E \cdot S_a \cdot (S_b)^2}{S_L}$$

		v	**b**	**E**	**L**	**F**	**a**
V4	**mx**	0	0	1	1	0	0
	my	1	1	-2	0	0	0
	mz	0	0	-1	0	0	1
	N	0	0	1	0	1	0
	π1	1	0	0.5	-0.5	-0.5	0.5
	π2	0	1	0.5	-0.5	-0.5	0.5

$$\pi_1 = v \cdot \sqrt{\frac{E \cdot a}{L \cdot F}} \Leftrightarrow (S_v)^2 \cdot S_E \cdot S_a = S_L \cdot S_F \Leftrightarrow S_V = \sqrt{\frac{S_L \cdot S_F}{S_E \cdot S_a}}$$

$$\pi_2 = b \cdot \sqrt{\frac{E \cdot a}{L \cdot F}} \Leftrightarrow (S_b)^2 \cdot S_E \cdot S_a = S_L \cdot S_F \Rightarrow S_b = \sqrt{\frac{S_L \cdot S_F}{S_E \cdot S_a}}$$

		v	**L**	**E**	**b**	**F**	**a**
V5	**mx**	0	1	1	0	0	0
	my	1	0	-2	1	0	0
	mz	0	0	-1	0	0	1
	N	0	0	1	0	1	0
	π1	1	0	0	-1	0	0
	π2	0	1	-1	-2	1	-1

$$\pi_1 = v \cdot b^{-1} = v / b \Leftrightarrow S_v = S_b$$

$$\pi_2 = \frac{F \cdot L}{E \cdot b^2 \cdot a} \Leftrightarrow S_F \cdot S_L = S_E \cdot S_a \cdot (S_b)^2$$

		v	**a**	**E**	**L**	**F**	**b**
V6	**mx**	0	0	1	1	0	0
	my	1	0	-2	0	0	1
	mz	0	1	-1	0	0	0
	N	0	0	1	0	1	0
	π1	1	0	0	0	0	-1
	π2	0	1	1	-1	-1	2

$$\pi_1 = v \cdot b^{-1} = v / b \Leftrightarrow S_v = S_b$$

$$\pi_2 = a \cdot \frac{E \cdot b^2}{L \cdot F} \Leftrightarrow S_F \cdot S_L = S_E \cdot S_a \cdot (S_b)^2$$

In these Laws of the Model, by the express introduction of the dimensions a and b, the geometric similarity of the model with the prototype is imposed a priori, even if in variants V_4, V_5 and V_6 only one of these dimensions of the cross section is considered as an independent variable.

These component (constituent) elements of the Model Law are not influenced by the arrangement order of the variables neither in matrix A nor in matrix B.

It should be emphasized that the variables arranged in matrix A (which are actually the independent variables) can be freely chosen not only for the prototype, but also for the model, while the dependent ones (arranged in

matrix B), related to the model, will result exclusively through the application of the Model Law, being type/category II sizes.
For the prototype, all the variables (except for the dependency "v", which we are looking for), can be chosen a priori, being of category I.
Through measurements performed on the model, the category III quantity will result, which, through the Model Law, will provide the quantity sought for the prototype, which in the present case represents the vertical displacement "v" in the direction of the applied force from that point.

2. CONCLUSIONS

From what was presented previously, two important aspects can be noted, which are indisputable facilities of the MDA, namely:

• The inclusion of some variables, whose influence is insignificant (either they do not directly influence the respective physical phenomenon, or the order of magnitude of their contribution is too small) are automatically eliminated from the elements of the Model Law; in the present case, in the V_1 variant, the gravitational acceleration g was of this type;

• Through a favorable choice of the set of independent variables (those forming the matrix A) the condition of the geometric similarity of the model with the prototype can be easily waived (in the present case of imposing a rectangular section on the model as well), because if only the variable a as an independent variable, then the size of b will necessarily result from an element of the Model Law, which will more than likely move us away from the geometric similarity of the model to the prototype;

• If the merging of the variables is resorted to I_{zG}, for example instead of the variables a and b the axial moment of inertia will be put, then, as demonstrated by the authors in a previous work [9], in the matrix A they will make their place and other elements useful for the flexibility of the model, such as (F, I_{zG}, L, E), each offering new perspectives for the design of models that are more suitable for carrying out simple, safe and repeatable experimental investigations on the model.

		v	F	Iz	L	E
	mx	0	0	0	1	1
	my	1	0	3	0	-2
V7	**mz**	0	0	1	0	-1
	N	0	1	0	0	1
	π1	1	1	-1	1	-1

$$\pi_1 = \frac{\mathrm{v} \cdot F \cdot L}{E \cdot I_z} \Leftrightarrow S_{\mathrm{v}} \cdot S_F \cdot S_L = S_E \cdot S_{Iz}$$

from which, following the above-mentioned calculations, the sought calculation equation results:

$$\frac{v_2}{v_1}=\frac{S_E\cdot S_{Iz}}{S_F\cdot S_L}\Rightarrow v_1=v_2\cdot\frac{S_F\cdot S_L}{S_E\cdot S_{Iz}}.$$

In this case, if the influence is analyzed:

• the force F, it can be observed that the model will be able to be requested in a much wider range of values than those imposed by the previous variants, where the force was considered as a dependent variable;

• the axial moment of inertia I_{zG}, only their ratio to the prototype and to the model will be imposed, and the shape of the cross sections will not be important;

• length L, it will also be possible to design models with lengths as appropriate as possible for the purpose (experiments that can be carried out on the model);

• the modulus of elasticity E, then contrary to the expectations based on the strict observance of some geometric similarities, respectively of similarity between the prototype and the model, here it will be possible to use even models made/made of other materials compared to the prototype, which will also ensure a flexibility larger of the model.

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