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CONTACT OF ELASTIC SOLIDS BY MATHCAD METHOD

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Abstract: When two non-conforming solids are through into contact they touch initially at a single point along a line. In this paper its make a application of Hertz theory used the Math Cad program for the contact of two cylinders.
Key-Words: - Contact, Hertz, non-conforming, Math Cad, stress, deformations.

1. Introduction

A theory of contact is required to predict the shape of this area of contact and how it grows in size with increasing load, the magnitude and distribution of surface traction normal and possibility tangential, transmitted action the interface.
Finally it should enable the components of deformation, and stress in both bodies to be calculated in the vicinity of the contact, analytical and numerical by Math Cad method.

2. Geometry contact of elastic solids. Non-conforming surfaces in contact.

We shall now consider the deformation as a normal load P is applied. Two solids of general shape are shown in cross-section after deformation in Fig.1.Before deformation the separation between two corresponding surface points $S_1(x, y, z_1)$ and $S_2(x, y, z_2)$ is given by equation (1):

$$h = Ax^2 + By^2 = \frac{1}{2R^{'}}x^2 + \frac{1}{2R^{''}}y^2 \tag{1}$$

Where: A and B are positive constants and $R^{'}$ and $R^{''}$ are defined as principal relative radii of curvature, and $h=z_1-z_2$.
From the symmetry of this expression about *0* the contact region must extend an equal distance on either side of O. During the compression distant points in the two bodies T_1 and T_2 move towards O, parallel to the Z-axis, by displacements δ_1 and δ_2 respectively. If the solids did not deform their profiles would overlap as shown by the dotted lines in Fig.1. Due the contact pressure the surface of each body is displaced parallel in Oz by an amount $\bar{u}_{z1}$ and $\bar{u}_{z2}$ (measured positive into each body) relative to the distant points T_1 and T_2. If after deformation, the points S_1 and S_2 are coincident within the contact surface then

$$\bar{u}_{z1} + \bar{u}_{z2} + h = \delta_1 + \delta_2 \tag{2}$$

or

$$\bar{u}_{z1} + \bar{u}_{z2} = \delta - Ax^2 - By^2 \tag{3}$$

Where: x and y are the common coordinates of δ_1 and δ_2 projected onto the x-y plane. If S_1 and S_2 lie outside the contact area so that they do not touch is follows that

$$\bar{u}_{z1} + \bar{u}_{z2} \le \delta - Ax^2 - By^2 \tag{4}$$

To solve the problem, it is necessary to find the distribution of pressure transmitted between the two bodies at their surface of contact, such that the resulting elastic displacements normal to the surface satisfy equation (3) within the contact area and equation (4) outside it

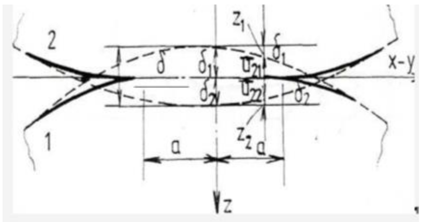


Fig.1 -*Two solids of general shape in cross- section after deformation*

Before proceeding to examine the problem in elasticity, however, it is instructive to see how the deformations and stresses grow as the load in applied on the basis of elementary dimensional reasoning. For simplicity we shall restrict the discussion to solids of revolution in which the contact area is a circle of radius and two-dimensional bodies in which the contact area is an infinite strip of width 2a.

We note that in Fig.1, $\delta_1 = \bar{u}_{z1}(0)$ and $\delta_2 = \bar{u}_{z2}(0)$, so that equation (2) can be written in non-dimensional form

$$\left\{\frac{\bar{u}_{z1}(0)}{a} - \frac{\bar{u}_{z1}(x)}{a}\right\} + \left\{\frac{\bar{u}_{z2}(0)}{a} - \frac{\bar{u}_{z2}(x)}{a}\right\} = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{x^2}{a} \quad (5)$$

Putting $x=a$ and writing $\bar{u}_z(0) - \bar{u}_z(a) = d$, the deformation within the contact zone (5) becomes

$$\frac{d_1}{a} + \frac{d_2}{a} = \frac{a}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad (6)$$

Provided that the deformation is small, $d \le a$, the state of strain in each solid is characterized by ratio *d/a*. Now the magnitude of the strain will be proportional to the contact pressure divided by the elastic modulus, therefore, if p_m is the average contact pressure acting mutually on each solid (6) becomes

$$p_m / E_1 + p_m / E_2 = a(1/R_1 + 1/R_2)$$

i.e.

$$p_m = \frac{a(1/R_1 + 1/R_2)}{1/E_1 + 1/E_2} \quad (7)$$

Thus, for a given geometry and materials, the contact pressure and the associated stress increase in direct proportion to the linear dimension of the contact area. To relate the growth of the contact to the load, two and three dimensional contacts be examined separately.

1) In the contact of cylinders, the load per unit axial length $P=2ap_m$, whence (7)

$$a= [P\,(1/E_1+1/E_2)/(1/R_1+1/R_2)]^{1/2} \quad (8)$$

and

$$p_m= [P(1/R_1+1/R_2)/(1/E_1+1E_2)]^{1/2} \quad (9)$$

from which we see that the contact width and contact pressure increase as the square root of the applied load.

2) In the contact of spheres, or other solids of revolution, the compressive load $P=pa^2p_m$. Hence from (7)

$$a= [(P(1/E_1+1/E_2)/(1/R_1+1/R_2)]^{1/2} \quad (10)$$

and

$$p_m=[P(1/R_1+1/R_2)^2/(1/E_1+1/E_2)]^{1/3} \quad (11)$$

In this case, the radius of the contact circle and the contact pressure increase as the cube root of the load.

In the case of three-dimensional contact the compression of each solid δ_1 and δ_2 are proportional to the load indentations d_1 and d_2, because the approach of distant points

$$\delta=\delta_1+\delta_2=d_1+d_2= [P^2\,(1/E_1+1/E_2)^2(1/R_1+1/R_2)]^{1/3} \quad (12)$$

The approach of two bodies due to elastic compression in the contact region is thus proportional to $(\text{load})^{2/3}$.
In the case of two dimension contact displacement δ_1 and δ_2 are not proportional to d_1 and d_2 but depend upon the arbitrarily chosen datum for elastic displacements.

3. Hertzian contact computer assisted process design using Math Cad method

For the understanding the hertzian models, it was study first the constituent equations for the vertical displacement u_z.
The hypothesis I is associate to establishment the path in a median elastic plane dependent by the curves of the conjugated surface and the elastics contacts of the two surface cylinders the account of contact verifying the consigns equations (fig.2).

$$(z_0+u_0)+(z_1+u_1)=h,\quad h = h_0+h_1 \tag{13}$$

The external point of the contact, verifiable the non-equation

$$(z_0+u_0)+(z_1+u_1)<h \tag{14}$$

Take by P_z (x, y), the distribution of the contact pressure we have:

$$u_i{}'\frac{1}{\pi E_i^*}\iint\frac{P_z(\xi,\eta)}{r}d\xi d\eta\,,$$

$$E^*{}_i=\frac{E}{1-\nu^2}\quad I=0...n \tag{15}$$

Where: E_i, ν_I - is the Young and Poisson coefficients of this two materials.

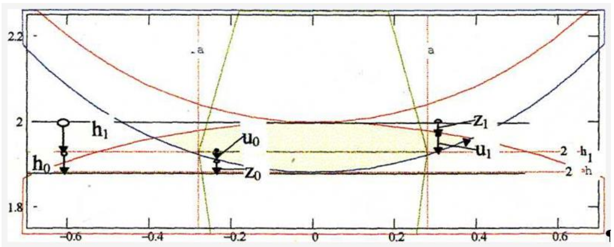
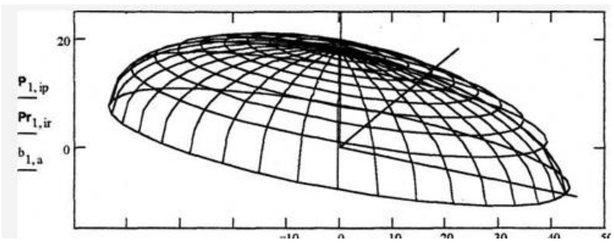


Fig.2 -*Hertzian contact, definition of contact domain*



-50 -40 -30 -20 -10 0 10 20 30 40 50

Fig.3 –*Ellipsoidal distribution of the pressure in Hertz contact*

For construct an imagine of the sliding in the hertz plain I am stimulated one of two sphere by the plane structural complex by beam elements, for 7 radial level and twenty one angular (266 elements, 21*7=147 nodes).
To fix to structure embed for the contour 0… 20, 41, 62, 146 radial sliding for the 21, 42…126 nodes.
It rested that the slides for the contact plane and at the same time is making be determinates the pressure of contact distribution (fig.4).
The impose reshuffle of force is corresponding of flatten in the profile plane (fig.5).

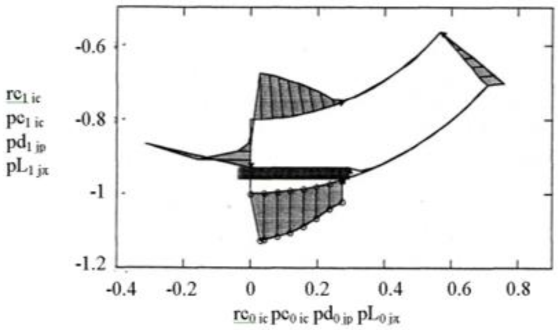


Fig.4- *Pressure of contact distribution*

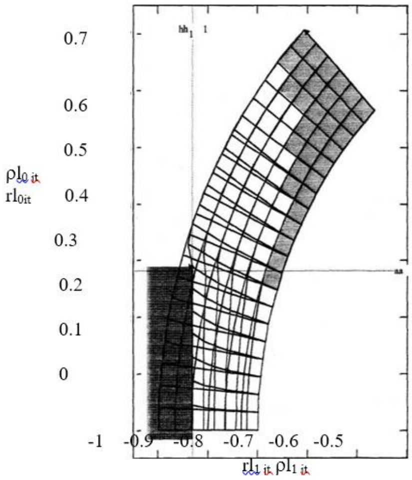


Fig.5 – *Flatten in the profile plane*

4. Conclusion

The numerical methods are one of the best methods to determinations the tensions in the roles and rolling ways (for example) It is very important the projects of the profile of roles for determinations of the state of tensions.

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