Transilvania University of Brasov
FACULTY OF MECHANICAL ENGINEERING

COMAT 2020 & eMECH 2020

Brasov, ROMANIA, 29-31 October 2020

THE DAMAGE PSEUDOSEVERITY

Marius-Vasile Pop[1], Gilbert-Rainer Gillich[1], Zeno-Iosif Praisach[1], Dorel Ardeljan[1]
[1] Babes-Bolyai University of Cluj-Napoca, ROMANIA, marius.pop@stud.ubbcluj.ro, gilbert.gillich@ubbcluj.ro, zeno.praisach@ubbcluj.ro, dorel.ardeljan@student.uem.ro

Abstract: *This paper presents the method that highlights the damage pseudo-severity that exists in the damaged beam for a specific position of the damage by using the relative frequency shift obtained from natural frequencies values and compares it with the square of the second derivative of the vibration mode shape from the analytical calculation. The natural frequencies are obtained from the numerical method for a healthy and damaged beam. For three damage depth, the damage is considered at the clamped end and at a specific position. Finally, the results confirm that for each vibration mode, regardless of the depth of the damage, the severity is the same.*
Keywords: *cantilever beam, natural frequency, relative frequency shift, damage severity*

1. INTRODUCTION

Methods to recognize the occurrence of damage by involving the analysis of vibration signals measured on the monitored structure require observing the modal parameter changes in the earliest state [1]. The principle of vibration-based damage detection is the following: when reducing the rigidity of one slice of the structure due to damage, the natural frequencies will decrease. This happened in different ways, depending on the damage type, location, and depth [2]-[4]. Thus, damages in form of transverse cracks affect the rigidity of the beam but also its dynamic behavior, by inducing changes in the mode shapes, modal curvatures, and natural frequencies [5]. In practice, measuring the natural frequencies is the simplest way to observe and quantify the alteration of the modal parameter because this measurement is made with simple and robust equipment [6]. However, it is necessary to use advanced algorithms to obtain accurate results when estimating the natural frequencies of beam-like structures [7]-[9]. Once the changes in the natural frequencies occurred due to damage captured, numerous methods to find the damage location and depth can be used, including statistical methods [10]-[12] and artificial intelligence [13]-[17].
In this paper we analyzed the frequency changes by using relative frequency shift which are occurring due to the appearance of a damage by comparing them with the strain energy loss express only by square of the second normalized derivative of the vibration mode shape. The paper introduces a new vision about the dynamic behavior of beams, aiming to show the correlation between relative frequencies shift changes and the mode shapes curvatures.

2. NUMERICAL APPROACH

Let's start from definition of relative frequency shift (RFS), which is given in [18] as

$$RFS_i = \frac{f_{Ui} - f_{Di}}{f_{Ui}} \tag{1}$$

where f_U is the natural frequency of the healthy beam, f_D is the natural frequency of the beam with a transverse crack, and $i=1, 2 \ldots$ represents the vibration mode.

Table 1: Geometry and mechanical parameters

L [m]	B [m]	H [m]	E [N/m^2]	ρ [kg/m^3]	ν
1.000	0.050	0.005	2x10^{11}	7850	0.3

For a cantilever beam with the geometry presented in fig. 1, and having the mechanical parameters (according to the ANSYS library for Structural Steel) presented in table 1, we consider in this study the first six vibration modes (i=1…6).

We calculated the natural frequencies of the healthy beam and that of the damaged beam by using finite elements method. The considered damage is along the whole width of the beam and is placed first at the clamped end (x=0), and the second time at x=0.2 m from the left beam end. For the depth of the damage a we consider three values, resulting: a/H=0.08, a/H=0.25 and a/H=0.5.
The results of the natural frequencies for healthy beam, damaged beam and relative frequency shift for the first six vibration mode and three degree of damage depth located at x=0 and x=0.2 m are presented in tables 2 and 3.

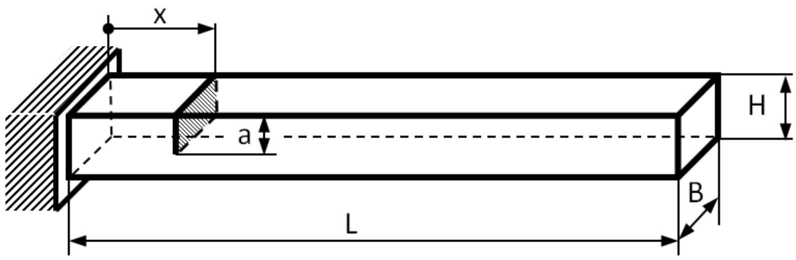

Figure 1: Clamped beam with damage

Table 2: Natural frequencies for healthy beam and damaged beam with damage at x=0

i	1	2	3	4	5	6
f_U	4.0899	25.6266	71.7545	140.6275	232.5200	347.4518
a/H	0.08					
$f_{D\_0.08}$	4.0833	25.5867	71.6466	140.4229	232.1934	346.9804
$RFS_{0\_0.08}$	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
a/H	0.25					
$f_{D\_0.25}$	4.0493	25.3844	71.1070	139.4158	230.6148	344.7456
$RFS_{0\_0.25}$	0.0099	0.0094	0.0090	0.0086	0.0082	0.0080
a/H	0.5					
$f_{D\_0.5}$	3.8939	24.5276	68.9644	135.6711	225.0692	337.2993
$RFS_{0\_0.5}$	0.0479	0.0429	0.0389	0.0352	0.0320	0.0292

Table 3: Natural frequencies for damaged beam with damage at x=0.2 m

i	1	2	3	4	5	6
a/H	0.08					
$f_{D\_0.08}$	4.0864	25.6264	71.7377	140.5431	232.4006	347.4064
$RFS_{0.2\_0.08}$	0.0008	0.0000	0.0002	0.0006	0.0005	0.0001
a/H	0.25					
$f_{D\_0.25}$	4.0685	25.6254	71.6537	140.1268	231.8321	347.1986
$RFS_{0.2\_0.25}$	0.0052	0.0000	0.0014	0.0036	0.0030	0.0007
a/H	0.5					
$f_{D\_0.5}$	3.9848	25.6213	71.3129	138.5827	229.8062	346.5043
$RFS_{0.2\_0.5}$	0.0257	0.0002	0.0062	0.0145	0.0117	0.0027

3. ANALYTICAL APPROACH

The analytical approach takes into account the dependence of the natural frequency for the damage beam and the natural frequency of the healthy beam, severity and square of the normalized second derivative of the vibration mode shape
By considering the relationship which define the changes of the natural frequencies for a damaged beam, presented in relation (2):

$$f_{Di}(x,a) = f_{Ui}\left(1-\gamma(a)\cdot\left(\bar{\phi}_i''(x)\right)^2\right) \tag{2}$$

where $\gamma(a)$ is the damage severity, and

$$\bar{\phi}_i''(x) = 0.5\left[\cos(\alpha_i x)+\cosh(\alpha_i x)-\frac{\cos(\alpha_i L)+\cosh(\alpha_i L)}{\sin(\alpha_i L)+\sinh(\alpha_i L)}\left(\sin(\alpha_i x)+\sinh(\alpha_i x)\right)\right] \tag{3}$$

is the second normalized derivative of the vibration mode shape for the cantilever beam and α_i is the dimensionless wave number.
For an established value of the damage depth, the damage severity can be expressed as follows:

$$\gamma(a) = \frac{f_{Ui} - f_{Di}(x,a)}{f_{Ui}} \cdot \frac{1}{\left(\bar{\phi}_i''(x)\right)^2} = RFS_i(x,a) \cdot \frac{1}{\left(\bar{\phi}_i''(x)\right)^2} \quad (4)$$

Figure 2, with black line, shows the square of the second normalized derivative of the vibration mode shape for the first six vibration mode of the cantilever beam.

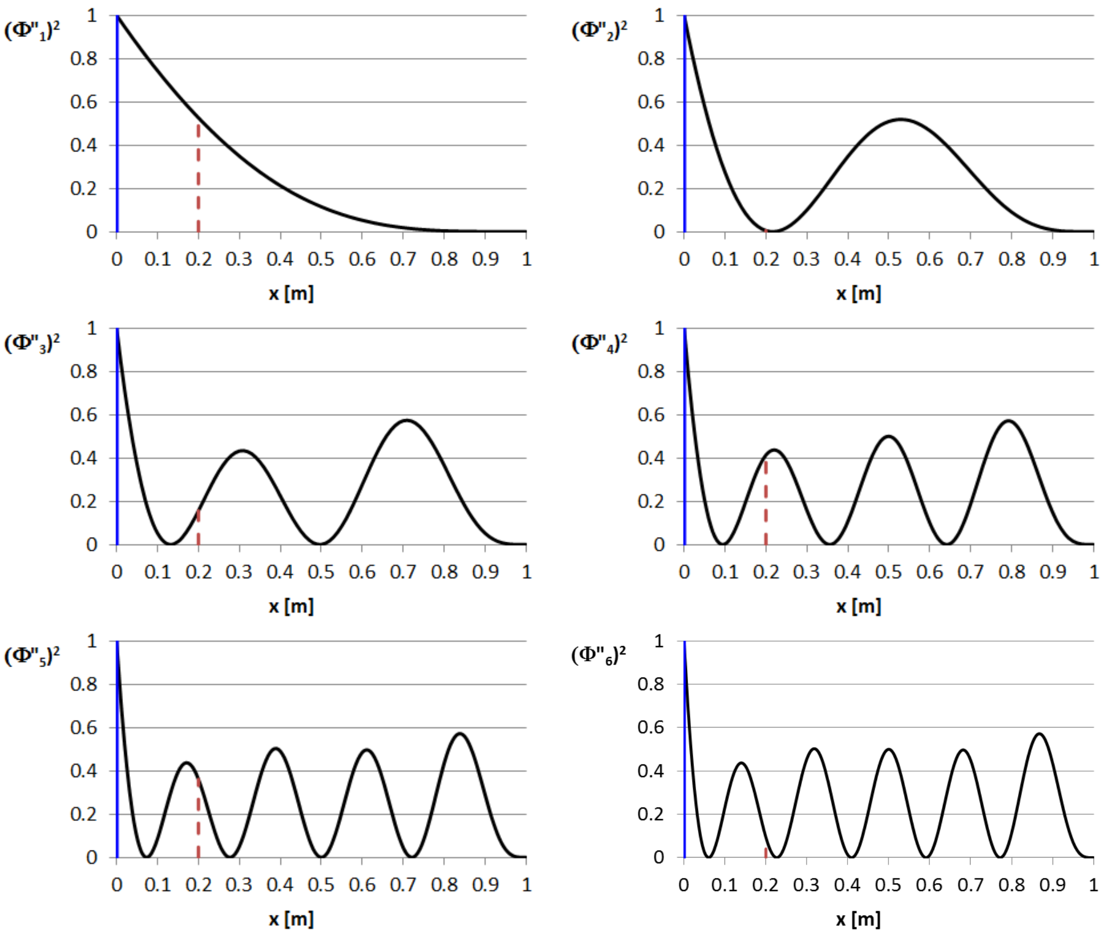

Figure 2: Square of the normalized second derivative of the vibration mode shape

By analyzing the figure 2 it can be observed that when x=0 we obtain $(\bar{\phi}_i''(0))^2$=1, but for x=0.2 m the square of the second normalized derivative of the vibration mode shape has smaller values (dashed red lines in figure 2). So, we have:

$$\gamma(a) = RFS_i(0,a) \cdot \frac{1}{\left(\bar{\phi}_i''(0)\right)^2} = RFS_i(0,a) \quad (5)$$

And in consequence we can state that the *RFS* found at the fixed end is the damage severity. For x=0.2 m, we can write the relation:

$$\gamma(a) = RFS_i(0.2,a) \cdot \frac{1}{\left(\bar{\phi}_i''(0.2)\right)^2} \quad (6)$$

But, from relation (4) we can deduce that:

$$RFS_i(0,a) \cdot \frac{1}{\left(\bar{\phi}_i''(0)\right)^2} = RFS_i(0.2,a) \cdot \frac{1}{\left(\bar{\phi}_i''(0.2)\right)^2} \quad (7)$$

Relation (7) can get the form:

$$RFS_i(0.2,a) = RFS_i(0,a)\cdot\frac{\left(\bar{\phi}_i''(0.2)\right)^2}{\left(\bar{\phi}_i''(0)\right)^2} \tag{8}$$

Considering that $\left(\bar{\phi}_i''(0)\right)^2 = 1$ and substituting $RFS_i(0,a)$ with $\gamma(a)$ in accordance with relation (5), we obtain

$$RFS_i(0.2,a) = \gamma(a)\left(\bar{\phi}_i''(0.2)\right)^2 = \gamma^*(a) \tag{9}$$

which we nominated the pseudo-severity of the beam. It is defined as the ratio between relative frequency shift for damaged beam when the damage is located at a given distance and when the damage is located at the clamped end for each vibration mode.
To validate the results obtained with relation (9), we calculate the square of the second normalized derivative of the vibration mode shapes for x=0.2 m and from numerical simulation results dividing the pseudo-severity to the severity. These results are presented in table 3.

Table 3: Comparison of values obtained for the square of the normalized modal curvatures from analytical calculation and numerical analysis

i	1	2	3	4	5	6
Analytically	0.5263	0.0049	0.1559	0.4135	0.3605	0.0932
FEM with a/H=0.08	0.5258	0.0047	0.1559	0.4125	0.3656	0.0964
FEM with a/H=0.25	0.5261	0.0049	0.1557	0.4132	0.3611	0.0936
FEM with a/H=0.50	0.5261	0.0048	0.1583	0.4126	0.3642	0.0933

Analyzing the values from table 3, it can be concluded that for each mode of vibration, the same values of the square of the second normalized derivative of the vibration mode shapes are found, irrespective to the damage depth, if the correct pseudo-severity was used. This means that the pseudo-severity can be used with confidence to characterize a crack with a given depth and location.

4. CONCLUSION

Relative frequency shift that occurs due to a transverse crack. It was shown that the severity of the crack takes the same value for all weak-axis bending vibration modes. It is found from the way the capacity of the beam to store energy is diminished due to the crack. The severity is always calculated for the crack located at the beam slice that suffer the biggest curvature/is subjected to the biggest bending moment. It is sufficient to calculate the severity for a given crack because the boundary conditions do not influence this parameter.
The pseudo-severity, which is actually the topic of this paper, depends on the position of the crack on the beam. Moreover, because the curvature at a given location on the beam is different for the different vibration modes, the pseudo-severity depends on the vibration mode number as well. The mathematical relation deduced herein to calculate the pseudo-severity is validated by simulations performed with a software dedicated for finite element analysis.

REFERENCES

[1] Bratu P., Vibraţiile sistemelor elastice, Editura Tehnica, Bucuresti, 2000.
[2] Gillich G.R., Praisach Z.I., Iancu V., Furdui H., Negru I., Natural Frequency Changes due to Severe Corrosion in Metallic Structures, Strojniški vestnik - Journal of Mechanical Engineering 61(12), 2015, pp.721-730.
[3] Vlase S., Dynamical response of a multibody system with flexible elements with a general three-dimensional motion, Romanian Journal of Physics, 57(3-4), 2012, pp. 676-693.
[4] Manoach E., Samborski S., Mitura A., Warminski J., Vibration based damage detection in composite beams under temperature variations using Poincaré maps, International Journal of Mechanical Sciences, 62(1), 2012, pp.120-132.
[5] Tufisi C., Gillich G.R., Hamat C.O., Gillich N., Praisach Z.I., Numerical Study of the Stiffness Degradation Caused by Branched Cracks and its Influence on the natural frequency drop, Romanian Journal of Acoustics and Vibration, 15(1), 2018, pp.53-57.

[6] Gillich G.R., Tufoi M., Korka Z.I., Stanciu E., Petrica A., The Relations between Deflection, Stored Energy and Natural Frequencies, with Application in Damage Detection, Romanian Journal of Acoustics and Vibration, 13(2), 2016, pp. 87-93.
[7] Mituletu I.C., Gillich G.R., Maia N.M.M., A method for an accurate estimation of natural frequencies using swept-sine acoustic excitation, Mechanical Systems and Signal Processing, 116, 2019, pp. 693-709.
[8] Ntakpe J.L., Gillich G.R., Mituletu I.C., Praisach Z.I., Gillich N., An Accurate Frequency Estimation Algorithm with Application in Modal Analysis, Romanian Journal of Acoustics and Vibration, 13(2), 2016, pp. 98-103.
[9] Onchis-Moaca D., Gillich G.R., Frunza R., Gradually improving the readability of the time-frequency spectra for natural frequency identification in cantilever beams, 2012 Proceedings of the 20th European Signal Processing Conference (EUSIPCO), 2012, pp. 809-813.
[10] Minda P.F., Praisach Z.I., Gillich N., Minda A.A., Gillich G.R., On the efficiency of different dissimilarity estimators used in damage detection, Romanian Journal of Acoustics and Vibration, 10fon the (1), 2013, pp. 15-18.
[11] Gillich G.R., Maia N., Mituletu I.C., Tufoi M., Iancu V., Korka Z., A new approach for severity estimation of transversal cracks in multi-layered beams, Latin American Journal of Solids and Structures, 13(8), 2016, pp. 1526-1544.
[12] Nitescu C., Gillich G.R., Abdel Wahab M., Manescu T., Korka Z.I., Damage severity estimation from the global stiffness decrease, 12th International Conference on Damage Assessment of Structures, published inȘ IOP Conf. Series: Journal of Physics: Conf. Series 842, 2017, art. 012034.
[13] Khatir S., Tiachacht S., Thanh C.L., Bui T.Q., Wahab M.A., Damage assessment in composite laminates using ANN-PSO-IGA and Cornwell indicator, Composite Structures, 230, 2019, art. 111509.
[14] Tran-Ngoc H., Khatir S., Le-Xuan T., De Roeck G., Bui-Tien T., Wahab M.A., A novel machine-learning based on the global search techniques using vectorized data for damage detection in structures, International Journal of Engineering Science, 157, 2020, 103376c.
[15] Zenzen R., Khatir S., Belaidi I., Le Thanh C., Wahab M.A., A modified transmissibility indicator and Artificial Neural Network for damage identification and quantification in laminated composite structures, Composite Structures, 2020, 112497.
[16] Khatir S., Wahab M.A., A computational approach for crack identification in plate structures using XFEM, XIGA, PSO and Jaya algorithm, Theoretical and Applied Fracture Mechanics, 103, 102240I.
[17] Manoach E., Warminski J., Kloda L., Teter A., Vibration Based Methods For Damage Detection In Structures, MATEC Web of Conferences 83, 2016, 05007.
[18] Negru I., Praisach Z.I., Gillich G.R., Vasile O., About the Neutral Axis Distortion due to Cracks and its Influence upon the Beams Natural Frequencies, Romanian Journal of Acoustics and Vibration, 12(1), 2015, pp.35-38.