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OPTIMUM DYNAMIC ABSORBER FOR INCREASING DYNAMIC STIFFNESS

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Abstract: *Spindle system is one of the most important part of a machine tool, its dynamic characteristics directly affect the cutting ability of the whole machine. Thus, the machining performance can be raised remarkably by improving the dynamic stiffness of the spindle system. Traditionally, the vibration of mechanical systems are damped by attaching an absorber to the original system. The engineering literature contains a large number of papers on many aspects of this subject. This paper, which focuses on the aspect of the problem, discusses the design of optimun vibration absorber for linear damped system. The results are presented for many combinations of system parameters as well as for many input frequencies.*
Keywords: *dynamic stiffness, optimun vibration absorber*

1. INTRODUCTION

The performance of machine tools and mechanical systems in general can be substantially improved by increasing dynamic rigidity.
For a time-invariant linear dynamic system whose behavior is described by the equation

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F\} \tag{1}$$

dynamic stiffness $K(j\omega)$ is defined, by analogy with static stiffness, as the ratio between the Fourier transform of the input quantity and the output quantity

$$K(j\omega)=\frac{F(j\omega)}{X(j\omega)}=\frac{1}{W(j\omega)} \qquad C(j\omega)=W(j\omega)=A(\omega)\exp\Phi(\omega) \qquad \boldsymbol{C}(j\omega)=\boldsymbol{H}^{-1}(j\omega). \tag{2}$$

where is the frequency transfer function, it is dynamic failure (compliance).
where $W(j\omega)$ is the frequency transfer function, $C(j\omega)$ it is dynamic failure (compliance).
The amplitude of compliance, assuming that SE has only one mode of motion of its own, is $A(\omega)$

$$A(\omega)=\frac{1}{\sqrt{(k^2-m\omega^2)^2+c^2\omega^2}}. \tag{3}$$

To reduce the amplitude, $A(\omega)$, one can intervene in the sense of increasing the parameters *k* and *c* and in the one of decreasing the equivalent mass *m*. In this paper we choose the way to increase the dynamic rigidity by increasing the dampings in the elastic structure. This is done by attaching an auxiliary mass absorber that will be optimally tuned. Details on the types of absorbers used in the construction of machines and the location can be found in the paper [1]. Two constructive solutions that recommended for frames, housings, etc. are shown in fig. 1, wherein 1 is the vibratory structure, 2 is the auxiliary mass and 3 is the rubber element.

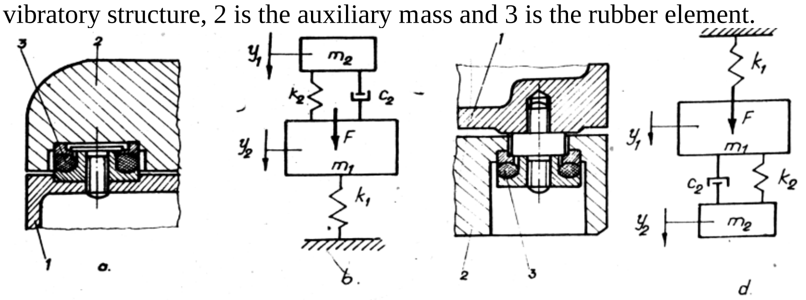


Figure 1 [1] Two constructive solutions

In the dynamic models of Fig. 1-b, d, related to the constructions from fig.1-a, c, we will also consider the damping c_1 from the elastic structure with the equivalent mass m_1. With the help of this model, important qualitative information can be obtained on the optimal design of vibration absorbers for linear systems.

2. THE OPTIMAL DYNAMIC ABSORBER

The problem of designing an optimal vibration absorber for damped linear systems has been studied by a number of authors. Most authors study the problem, in case of harmonic excitations, either by analytical methods [2], [5], or by approximating the analytical solution [4], and by numerical optimization method [6], [3], [7].
In this paper, the problem of determining the optimal parameters of the absorber, in order to reduce the amplitude of the main system response, will be addressed using the principles of optimization. We will formulate the problem as a nonlinear optimization problem. Solving it is done with a quasi-Newtonian relaxation method, using the search algorithm of BFGS - Broyden, Fletcher, Goldfarb, Shanno, for which I wrote a program in the language MATLAB.
The dynamic equations, written in matrix form, for the whole system (primary and secondary) are

$$M\ddot{y} + C\dot{y} + Ky = f \tag{4}$$

where

$$\boldsymbol{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} \boldsymbol{c_1} + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}, \boldsymbol{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}, \ \boldsymbol{f} = \begin{Bmatrix} F\sin\omega t \\ 0 \end{Bmatrix}.$$

If we enter the notations

$$\begin{aligned}
&\omega_i = \sqrt{k_1/m_1}\,;\ \varsigma_i = c_i \big/ 2\sqrt{m_i k_i}\,;\ i = 1,2 \\
&\Omega = \omega/\omega_1, \Omega_r = \omega_2/\omega_1, \sigma = m_2/m_1 \\
&\mathrm{A} = \left[\Omega^4 - \Omega^2 - \Omega^2\Omega_r^2(1+\sigma) - 4\varsigma_1\varsigma_2\Omega^2\Omega_r^2 + \Omega_r^2\right]^2 \\
&\quad + \Omega^2\Omega_r^2\left[\varsigma_1\Omega^2 + \varsigma_2\Omega^2(1+\sigma) - \varsigma_1{\Omega_r}^2\right]^2
\end{aligned} \tag{5}$$

we will obtain the dimensionless amplitudes of the main and secondary system

$$\frac{Y_1}{F/k_1} = \frac{1}{\mathrm{A}}\sqrt{\left[\left(\Omega_r^2 - \Omega^2\right)^2 + 4(\varsigma_2\Omega\Omega_r)^2\right]};\frac{Y_2}{F/k_1} = \frac{1}{\mathrm{A}}\sqrt{\Omega_r^4 + 4\varsigma_2^2\Omega^2\Omega_r^2}\,. \tag{6}$$

3. FORMULATION OF THE OPTIMIZATION PROBLEM

We formulate the problem (7) of optimal shock absorber design as a non-linear programming problem: to minimize the objective function $f(x)$ with restrictions of equality and inequality

$$\begin{aligned}
&\min_{x \in \Omega \subset R^n} f(x) \\
&g_i(x) = 0 \qquad i = 1,..m_e \\
&g_i(x) \le 0 \qquad i = m_e + 1,..m \\
&x_l \le x \le x_u
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
&\min\left(\max\left[X_1(F/k_1)\right]\right) \\
&\varsigma_1, \sigma\ date \\
&\boldsymbol{x} = \{\varsigma_{\boldsymbol{2}}, \Omega_r\}; \begin{cases} 0 \le \Omega_r \le \Omega_r^s \\ 0 \le \varsigma_2 \le \varsigma_2^s \end{cases}
\end{aligned} \tag{7}$$

Its values for σ and ς_1 which the numerical calculations were made, as well as the optimization results are presented in tables 1 and 2.
The variations according to the ratio of masses, σ , of the values $\max[Y_1(F/k_1)]$, of the optimum of the frequency ratio Ω_r and of the optimum of the damping factor ς_2 are shown in Fig. 1, 2, 3, 4.

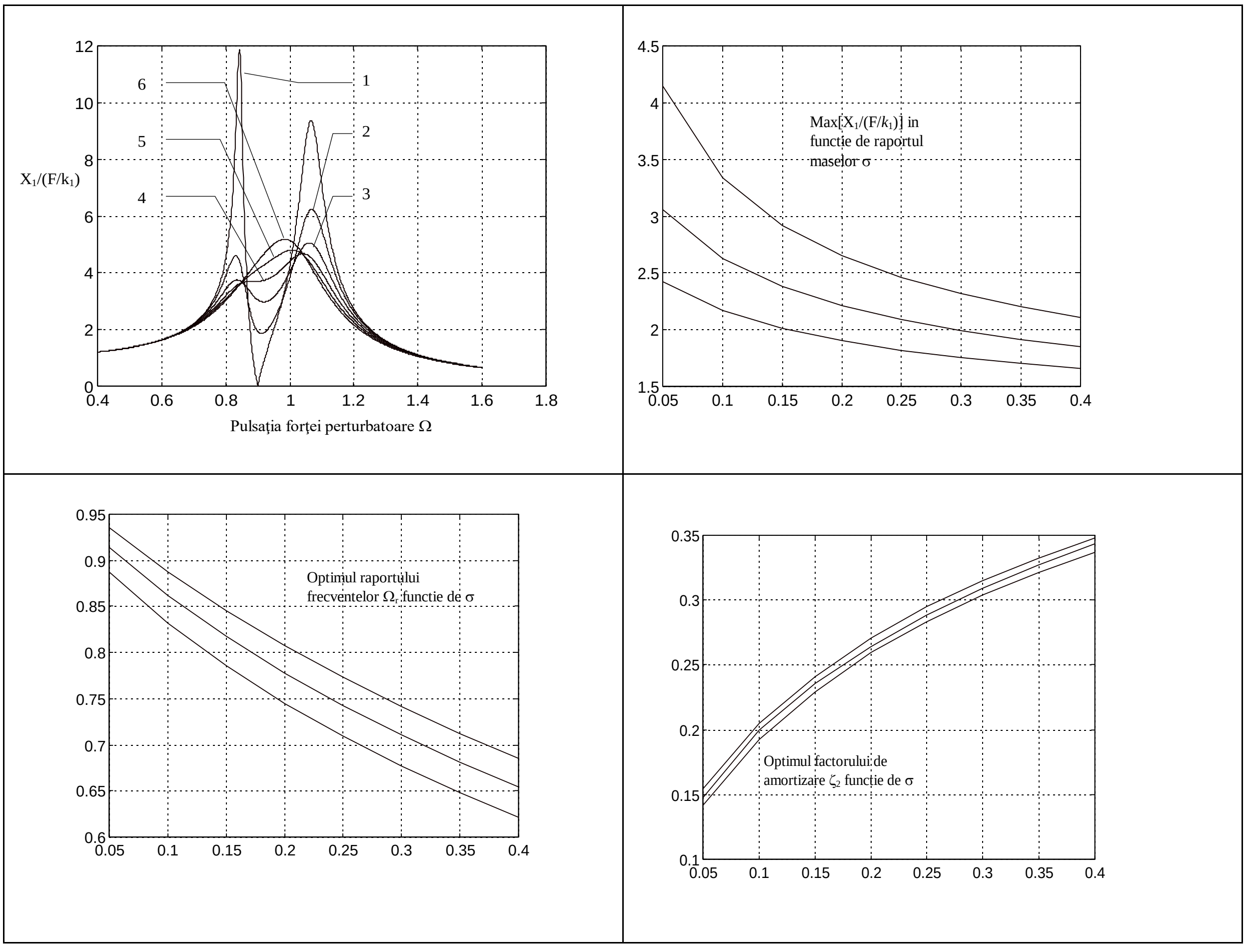
Table 1: Optimization results

σ	Ω_r		
	ζ_1=0.05	ζ_1= 0.1	ζ_1= 0.15
5.0000e-002	9.3533e-001	9.1384e-001	8.8767e-001
1.0000e-001	8.8751e-001	8.6189e-001	8.3210e-001
1.5000e-001	8.4533e-001	8.1738e-001	7.8569e-001
2.0000e-001	8.0722e-001	7.7803e-001	7.4518e-001
2.5000e-001	7.7277e-001	7.4269e-001	7.0930e-001
3.0000e-001	7.4128e-001	7.1070e-001	6.7707e-001
3.5000e-001	7.1237e-001	6.8151e-001	6.4791e-001

4.0000e-001	6.8570e-001	6.5471e-001	6.2134e-001

Table 2: Optimization results

ζ_2			max $[Y_1/(F/k_1)]$		
ζ_1=0.05	ζ_1=0.1	ζ_1=0.15	ζ_1=0.05	ζ_1= 0.1	ζ_1= 0.15
1.4160e-001	1.4786e-001	1.5418e-001	4.1462e+000	3.0566e+000	2.4256e+000
1.9262e-001	1.9971e-001	2.0507e-001	3.3368e+000	2.6225e+000	2.1667e+000
2.2897e-001	2.3571e-001	2.4070e-001	2.9193e+000	2.3765e+000	2.0112e+000
2.5952e-001	2.6442e-001	2.7079e-001	2.6513e+000	2.2101e+000	1.9021e+000
2.8341e-001	2.8871e-001	2.9468e-001	2.4605e+000	2.0870e+000	1.8194e+000
3.0383e-001	3.0909e-001	3.1513e-001	2.3156e+000	1.9911e+000	1.7537e+000
3.2143e-001	3.2701e-001	3.3263e-001	2.2008e+000	1.9135e+000	1.6997e+000
3.3696e-001	3.4319e-001	3.4774e-001	2.1072e+000	1.8491e+000	1.6543e+000



4. CONCLUSION

We notice that for given ς_1 we can reduce the amplitudes for both the main and the secondary system by increasing the mass ratio. Another conclusion to be drawn from the analysis of the results is that for a given mass ratio, by increasing the damping factor ς_1, the optimum damping factor ς_2 increases while the optimal frequency ratio and the maximum optimal amplitude decrease.

If the main system is poorly damped, for example $\varsigma_1 = 0.05$, we can reduce the amplitude of the main system by 75% using an auxiliary mass equivalent to 40% of the mass of the main system ($\sigma = 0.4$).

The results presented in the paper, for the optimal values of the objective function are better than those presented in the papers [3], [4]. This is due to the optimization algorithm used.

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