Transilvania University of Brasov
FACULTY OF MECHANICAL ENGINEERING

ICMS 2019 & COMEC 2019
Brasov, ROMANIA, 21-22 November 2019

AN ALGORITHM TO FIND THE TWO SPECTRAL LINES ON THE MAIN LOBE OF A DFT

Gilbert-Rainer Gillich[1], Dorian Nedelcu[1], Andrea Amalia Minda[1], David Lupu[1]
[1] Universitatea "Eftimie Murgu" din Resita, ROMANIA,
gr.gillich@uem.ro, d.nedelcu@uem.ro, a.minda@uem.ro, david.lupu@messer.ro

Abstract: *Using interpolation methods to find the true harmonic component of a signal on an inter-line position in the spectrum is a usual practice in frequency estimation. The disadvantage of this method is that the points on which the interpolation is made have to be on the main lobe. Most researchers consider the maximizer and its biggest neighbor to satisfy this requirement, but in many cases this presumption is false. We propose an algorithm that permits finding the two points on the main lobe of the spectrum a certain number of samples. The evolution of the maximizer indicates which neighbor belongs to the main lobe. The algorithm is tested successfully for signals generated with known frequency and amplitudes. By estimating accurately the natural frequencies, the cracks can be detected in early stage by vibration-based methods.*
Keywords: *frequency evaluation, Discrete Fourier Transform, interpolation, algorithm, spectral line, crack detection*

1. INTRODUCTION

Damage assessment involving vibration-based methods require accurate frequency estimation. It is known that modal parameters suffer small changes if a crack arises in the structure [1-4]. If the modal parameters, frequencies in this case, are estimated with high accuracy, the occurrence of a crack can be observed in the early stage and technical respective administrative measures can be taken in due time. To improve the frequency readability, several signal post-processing methods were developed. Two types of approaches are known for fine resolution frequency estimation: the direct approach and the iterative approach [5,6].The simplest methods consider are direct methods, which base on interpolation made for two or three samples of the spectrum [7-14]. The frequency estimation made in this way implies negligible computational cost. In newer research it was shown that the three points belong to two spectral lobes, two being located on the main lobe whereas the third is always on a neighbor lobe [15]. Thus, the results obtained by interpolation are not that accurate as requested for damage detection. On the other hand, the methods based on two points consider the maximizer and its biggest neighbor. The position of the neighbor is not always on the main lobe, as shown in [15], so significant errors can result. Also, this method leads to significant errors when the inter-line frequency is close to zero [16]. An improvement of these methods is made in [17]. The iterative approach is more computation-cost extensive but improves a little bit the precision of the estimation process, see for instance [18,19].
In prior research [20-23], we developed iterative methods based on the sinc and pseudo-sinc functions. In this paper we introduce an algorithm that permits identifying the two spectral lines that belong to the main lobe. This algorithm can be used both for direct and iterative methods. Since the frequencies can be estimated with high accuracy, the algorithm significantly increase the applicability of vibration-based damage detection.

2. THE SPECTRAL LEAKAGE

The estimation of the frequency components of a signal supposes to convert the signal from the time domain into the frequency domain. This is made by assuring amplitudes to different lines in the spectrum which are equidistantly located. The distance between two spectral lines is nominated as frequency resolution and denoted Δf. If the position of a harmonic component fit the position of a spectral line, the energy of that harmonic is allocated just on to this line and in consequence the frequency is correctly estimated. Else, the energy is spread on several spectral lines, in order to ensure an output signal as close as possible to the original signal. In the latter case, the estimation precision depends on the frequency resolution Δf which is in inverse proportion to the signal time length t. The maximal error that is expected if the frequency estimation is performed with Discrete Fourier Transform (DFT) or other similar calculators is the half of the frequency resolution Δf. It is important to mention that the bigger the time length, the smaller the possible error.

The position of the amplitudes of the DFT samples is given by the *sinc* function, that is:

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \tag{1}$$

A suggestive image on DFT samples, marked with black bullets for the maximizer and its neighbors and red smaller bullets for the other samples, is given in Figure 1. The *sinc* function is represented with a green line. One can observe that the samples fit this curve. The phenomenon is known as leakage.

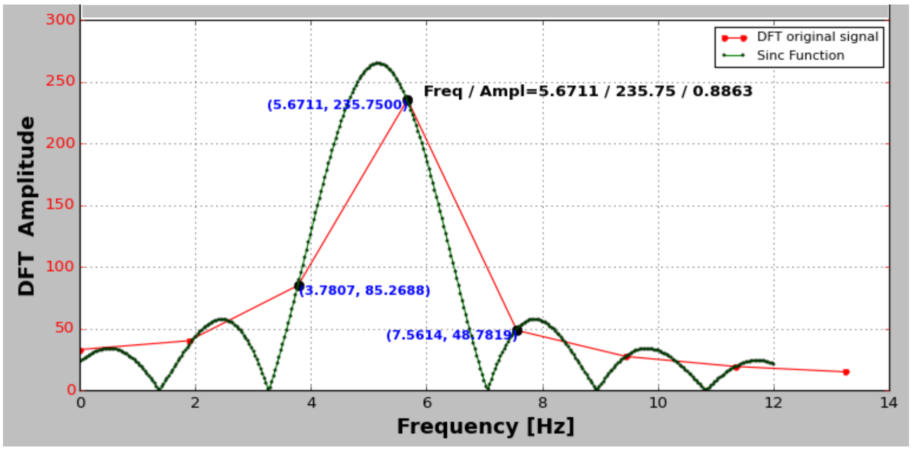
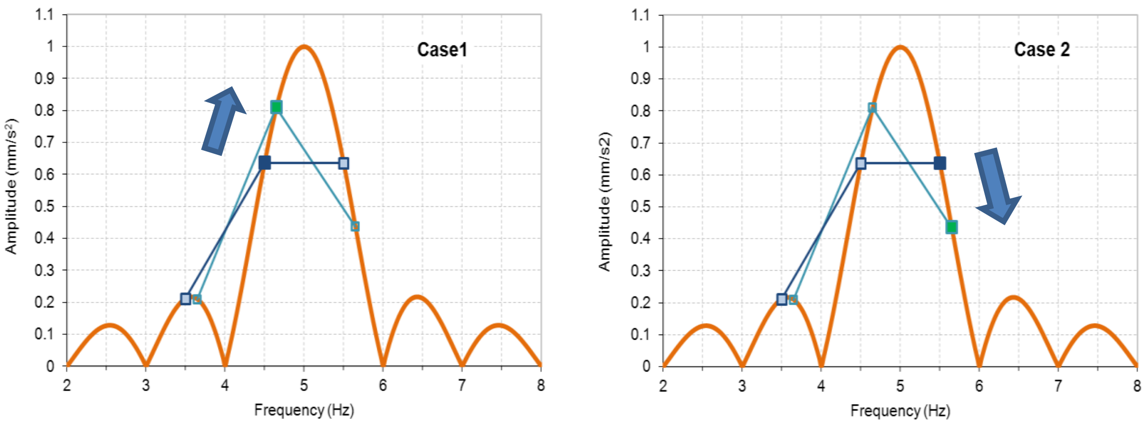


Figure 1: The DFT samples and the sinc function for the true frequency

Because no DFT sample is located at the frequency that corresponds to the true frequency f_{true}, post-processing is necessary to find f_{true}. This is made generally by searching and adjustment value δ that is added or subtracted from the measured frequency f_{m}. To this aim, it is necessary to know with precision which samples belong to te main lobe. Next section presents an algorithm to attain this information.

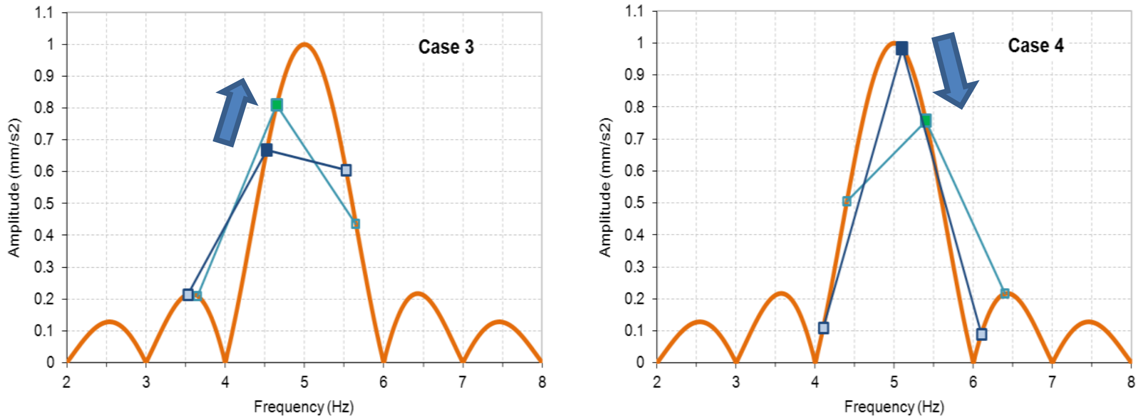
3. THE PROPOSED ALGORITHM

We start with analyzing the possible positions of the maximizer in regard to the other sample on the main lobe, and check how it is replaced when several samples in the signal are removed. To not alter the analyzed signal, these samples are artificially added to the original signal by zero-padding. Obviously, by removing several samples from the signal, the frequency resolution increases and the maximizer is pushed to the right in the spectrum. Figures 2, 3 and 4 show the possible evolution of the maximizer.
The first cases refer to the rare situation when the two samples on the main lobe have the same value. This situation is treated separately for the maximizer in the left position and right position.



a) b)

Figure 2: Two identical maximizer are found in the spectrum: a) the left one is considered in the analysis; b) the right one is considered in the analysis



a) b)

Figure 3: One maximizer is found in the spectrum: a) at the left side of the main lobe; b) at the right side of the main lobe

A third case considers the maximizer on the left side of the main lobe. A signal crop produces an increase of the maximizer, associated with a decrease of the other sample located on the main lobe. This case is shown in Figure 3.a. When the maximizer is positioned at the right side of the main lobe, a signal crop will produce the maximizer decrease, associated with an increase of the amplitude of the other sample belonging to the main lobe. This case is illustrated in Figure 3.b.

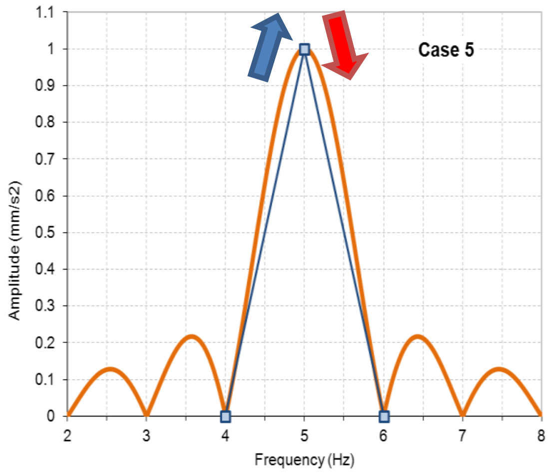


Figure 4: Uncertainty in the evolution of the maximizer (a false decrease can be obtained)

In the last possible case, the maximizer is located at the top of the main lobe. Thus, a crop of the signal can produce an increase followed by a decrease. This is associated with the shift of the maximizer to another spectral line that implies a precise assessment of the line on which the maximizer is located.

The parameters used the algorithm are:

- the frequency of the left neighbor of the maximizer for the zero-padded signal: x_1
- the frequency of the maximizer for the zero-padded signal: x_2
- the frequency of the right neighbor of the maximizer for the zero-padded signal: x_3
- the frequency of the left neighbor of the maximizer for the original signal: x_4
- the frequency of the maximizer for the original signal: x_5
- the frequency of the right neighbor of the maximizer for the original signal: x_6
- the amplitude of the left neighbor of the maximizer for the zero-padded signal: y_1
- the amplitude of the maximizer for the zero-padded signal: y_2
- the amplitude of the right neighbor of the maximizer for the zero-padded signal: y_3
- the amplitude of the left neighbor of the maximizer for the original signal: y_4
- the amplitude of the maximizer for the original signal: y_5
- the amplitude of the right neighbor of the maximizer for the original signal: y_6
- the number of samples to be added to the original signal by zero-padding: N_{ZP}
- the number of the original signal: N

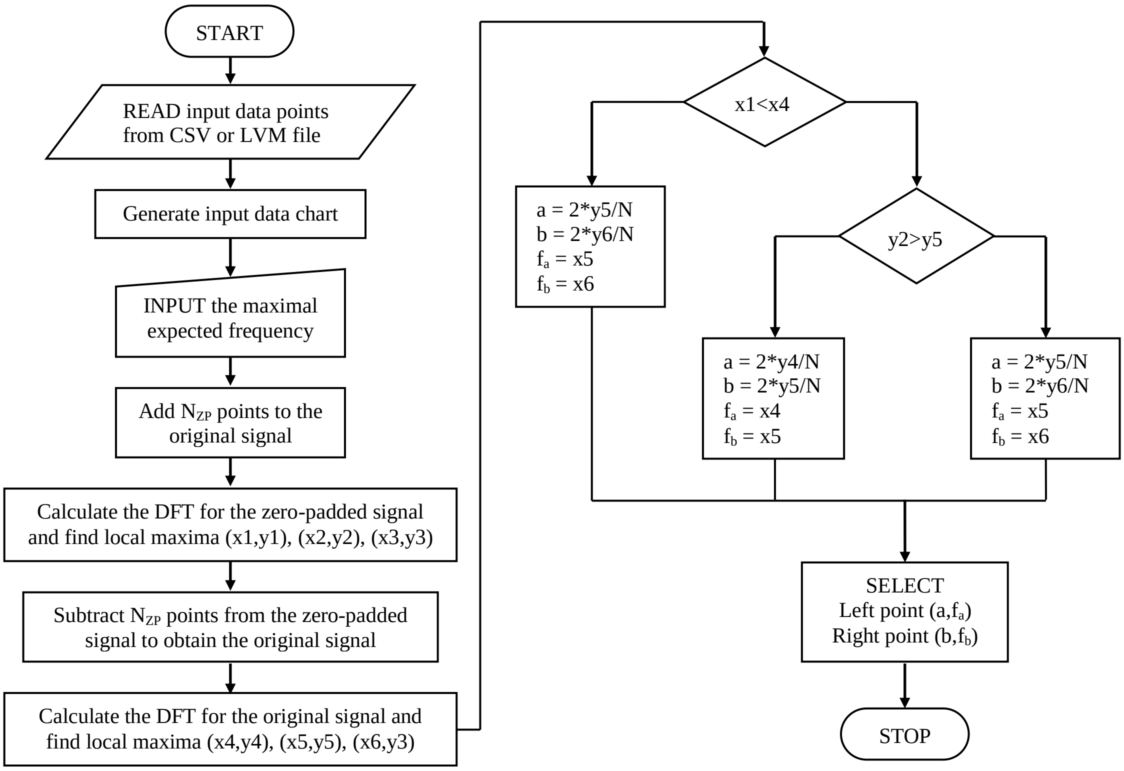


Figure 5: The flowchart of the algorithm

Figure 5 shows the flowchart of the algorithm. The algorithm was implemented by the authors in a software written in Python language, namely DFT SINC, that permits generating or importing a signal and estimating the frequencies of the signal components with high accuracy by two methods: interpolation, developed in [24] and *sinc*-based, developed in [25]. The software interface is shown in Figure 6.

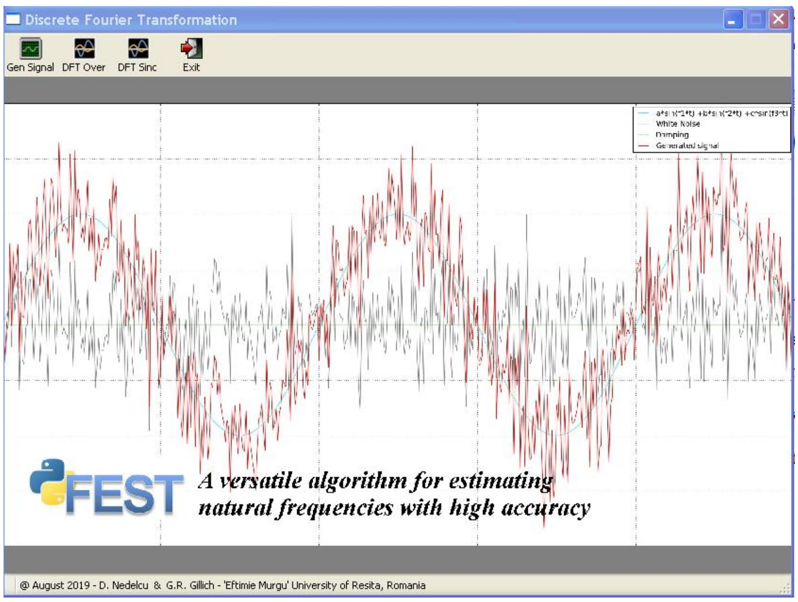


Figure 6: The DFT SINC interface

Tests made with DFT SINC have shown that the position of the DFT samples is always correctly recognized and the frequency estimation is properly performed. An example for case 5, which is the most difficult to be solved, is presented in Figure 7. In fact, it represents a zoom on the main lobe highlighting the position of the second DFT sample. The maximizer is on the right side of the lobe, see Figure 8, thus we identified correctly that the second sample is on the left lobe side.

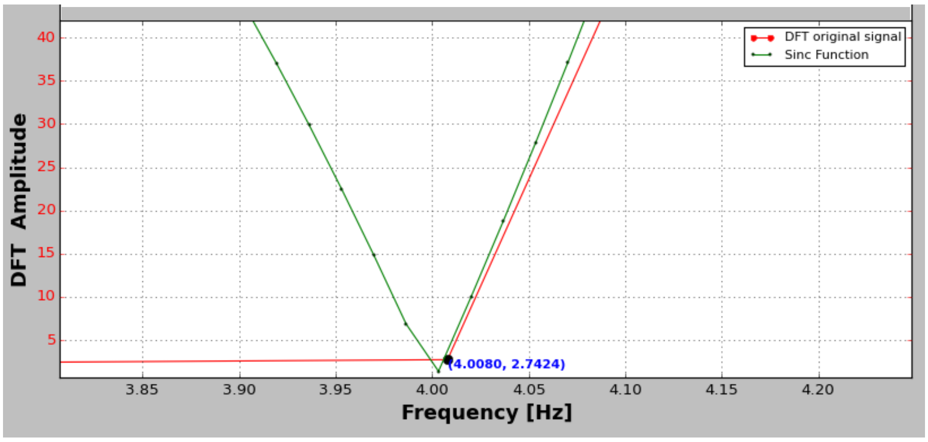


Figure 7: Zoom on the main lobe to highlight the position of the DFT sample with smaller amplitude

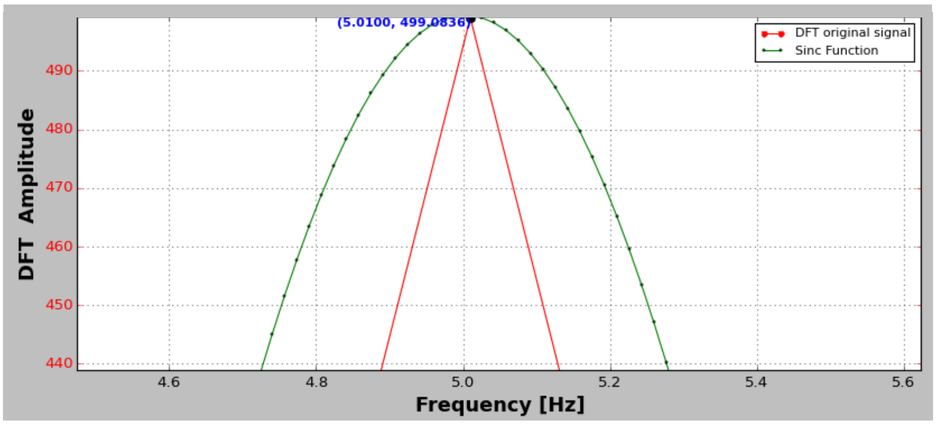


Figure 8: Zoom on the top of the main lobe to highlight the position of the maximizer at the right lobe side

The insignificant error of positioning the sample on the curve plotted based on the *sinc* function also shows that the frequency estimation is done with high accuracy. This is also observable in figure 8, where the frequency 5.01 Hz is indicated instead of % Hz, which is the frequency of the generated signal. Which such an accuracy in frequency estimation, the detection of cracks in early state is made possible [26].

3. CONCLUSION

The paper introduces an algorithm to find the DFT samples that belong to the main lobe of the spectrum. It can be used to find the point of interpolation for numerous frequency estimation methods that us the interpolation to find the inter-line position of the true frequency. The algorithm was tested for numerous signals with known frequency and proved to be reliable.

REFERENCES

[1] Ostachowicz W.M., Krawczuk C., Analysis of the effect of cracks on the natural frequencies of a cantilever beam, Journal of Sound and Vibration, 150(2), 1991, pp. 191–201.
[2] Vlase S., Marin M., Öchsner A., Considerations of the transverse vibration of a mechanical system with two identical bars, Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications, 233(7), 2019, pp. 1318-1323.
[3] Tufoi M., Hatiegan C., Vasile O., Gillich G.R., Dynamic Analysis of Thin Plates with Defects by Experimental and FEM Methods, Romanian Journal of Acoustics and Vibration,10(2), 2013, pp. 83-88.
[4] Bratu P., Elastic systems vibrations,Technical Publishing House, Bucharest, 2000.
[5] Liao J.-R., Chen C.-M., Phase correction of discrete Fourier transform coefficients to reduce frequency estimation bias of single tone complex sinusoid, Signal Processing, 94, 2014, pp. 108-117.
[6] Yao S., Wu Q., Fang S., An Improved Fine-Resolution Method for Frequency Estimation of Real-Valued Single-Tone Using Three DFT Samples, IEEE Access, 7, 2019, pp. 117063-117074.

[7] Rife D.C., Vincent G.A., Use of the discrete Fourier transform in the measurement of frequencies and levels of tones," Bell Systems Technology Journal, 49(2), 1970, pp. 197-228.
[8] Rife D.C., Boorstyn R.R., Single tone parameter estimation from discrete-time observations, IEEE Trans. Information Theory, vol. IT-20, no. 5, 1976, pp. 591-598.
[9] Grandke T., Interpolation Algorithms for Discrete Fourier Transforms of Weighted Signals, IEEE Transactions on Instrumentation and Measurement, Vol. 32, 1983, pp. 350-355.
[10] Quinn B.G., Estimating Frequency by Interpolation Using Fourier Coefficients, IEEE Transactions on Signal Processing, Vol. 42, 1994, pp. 1264-1268.
[11] Jain V.K., Collins W.L., Davis D.C., High-Accuracy Analog Measurements via Interpolated FFT, IEEE Transactions on Instrumentation and Measurement, Vol. 28, 1979, pp. 113-122.
[12] Ding K., Zheng C., Yang Z., Frequency Estimation Accuracy Analysis and Improvement of Energy Barycenter Correction Method for Discrete Spectrum, Journal of Mechanical Engineering, 46(5), 2010, pp. 43-48.
[13] Voglewede P., Parabola approximation for peak determination, Global DSP Magazine, 3(5), 2004, pp. 13-17.
[14] Jacobsen E., Kootsookos P., Fast, accurate frequency estimators, IEEE Signal Processing Magazine, 24(3), 2007, pp. 123-125.
[15] Gillich G.R., Mituletu I.C., Signal Post-processing for Accurate Evaluation of the Natural Frequencies. In: R. Yan, X. Chen, S. Mukhopadhyay (eds), Structural Health Monitoring. Smart Sensors, Measurement and Instrumentation, Vol. 26, Springer Cham, 2017, pp. 13-37.
[16] Xiang J.Z., Qing S., Wei C., A novel single tone frequency estimation by interpolation using DFT samples with zero-padding, in: Proc. IEEE ICSP, Chengdu, China, Mar. 2017, pp. 277-281.
[17] Candan C., A method for fine resolution frequency estimation from three DFT samples, IEEE Signal Processing Letters, 18(6), 2011, pp. 351-354.
[18] Reisenfeld S., Aboutanios E., A new algorithm for the estimation of the frequency of a complex exponential in additive Gaussiannoise, IEEE Communications Letters, 7(11), 2003, pp. 549-551.
[19] Aboutanios E., Mulgrew B., Iterative frequency estimation by interpolation on Fourier coefficients, IEEE Transactions on Signal Processing, 53(4), 2005, pp. 1237-1242.
[20] Ntakpe J.L., Gillich G.R., Mituletu I.C., Praisach Z.I., Gillich N., An Accurate Frequency Estimation Algorithm with Application in Modal Analysis, Romanian Journal of Acoustics and Vibration, 13(2), 2016, pp. 98-103.
[21] Gillich G.R., Mituletu I.C., Negru I., Tufoi M., Iancu V., Muntean F., A Method to Enhance Frequency Readability for Early Damage Detection, Journal of Vibration Engineering & Technologies, 3(5), 2015, pp. 637-652.
[22] Minda P.F., Praisach Z.I., Gillich N., Minda A.A., Gillich G.R., On the Efficiency of Different Dissimilarity Estimators Used in Damage Detection, Romanian Journal of Acoustics and Vibration, 10(1), 2013, pp. 15-18.
[23] Gillich G.R., Mituletu I.C., Praisach Z.I., Negru I., Tufoi M., Method to Enhance the Frequency Readability for Detecting Incipient Structural Damage, Iranian Journal of Science and Technology, Transactions of Mechanical Engineering, 41 (3), 2017, pp. 233–242.
[24] Mituletu I.C., Gillich G.R., Maia N.M.M., A method for an accurate estimation of natural frequencies using swept-sine acoustic excitation, Mechanical Systems and Signal Processing, 116, 2019, pp. 693-709.
[25] Minda A.A., Gillich G.R., Sinc Function based Interpolation Method to Accurate Evaluate the Natural Frequencies, Analele Universitatii Eftimie Murgu Resita, 24(1), 2017, pp. 211-218.
[26] Gillich G.R., Maia N., Mituletu I.C., Tufoi M., Iancu V., Korka Z., A new approach for severity estimation of transversal cracks in multi-layered beams, Latin American Journal of Solids and Structures, 13(8), 2016, pp. 1532-1544.