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ON ENERGY IN PLASTIC JOINTS OF A TRUSS STRUCTURE

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Abstract: *For many truss structures as bus bodywork sections or tractor cabs, in the case of dynamic loads, plastic joints appear in some cross sections. The paper presents some considerations about the formation of these plastic joints as well as a method for calculating the elastic and plastic energy stored. So it is possible to determine the order of appearance of the plastic joints and calculate the ability of the structure to function.*
Keywords: *structure, plastic joint, energy, dynamic load*

1. INTRODUCTION

Using a method developed in the book [2] and the paper [3], Tofan and Ulea proposed in [4], [6] and [8] MathCAD representations of the bus bodywork section and in [5] a MathCAD model for the roll over test of a bus bodywork section.
In [7] Ulea and Itu used FEM in order to analyse a staticaly roll over test of a bus bodywork. In the case of real dynamic loads, plastic joints apear in some cross section. The creation of the finite element model was done using the MSC Patran software and the static analysis itself was done using the MSC Nastran solver. In figure 1 is presented the bus bodywork section and in figure 2 the roof nodes of the model.

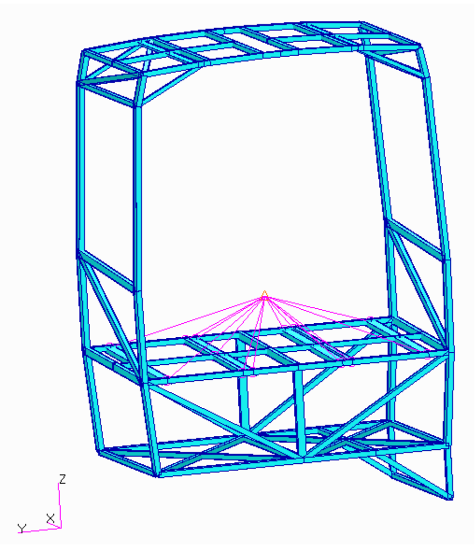

Figure 1: FEM model

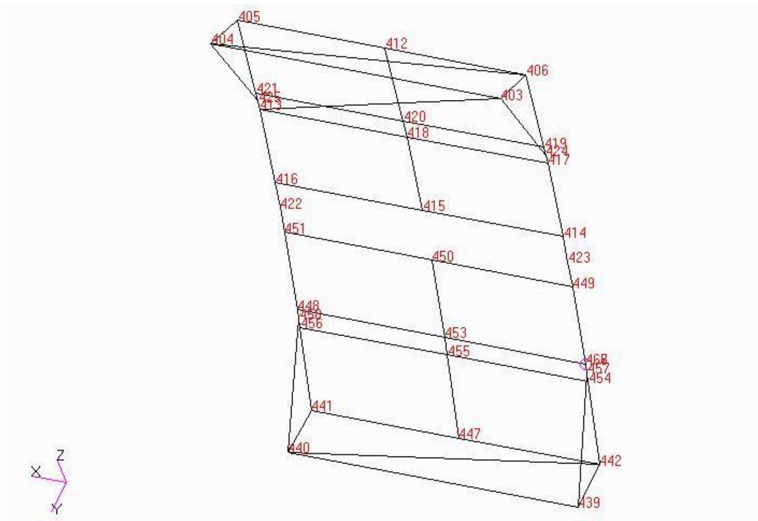

Figure 2: Roof nodes

In the table 1 are presented some results of the FEM calculations, arranged in descending order of the von Mises stress. Moments are calculated in relation to the global reference system.

Table1

Node	σ_{ech} [MPa]	Mix [Nmm]	Miy [Nmm]	Miz [Nmm]	N [N]	σ ax [MPa]
452	144,94	-6,55E+05	6,22E+03	3,33E+04	-1,33E+03	2,99
424	143,42	7,10E+05	-1,96E+04	1,91E+04	-1,64E+03	21,37
457	134,66	-7,26E+05	1,54E+04	3,62E+04	-8,84E+02	25,71
425	133,45	8,70E+05	2,55E+03	-3,00E+04	-1,98E+03	23,45

448	130,72	-8,09E+05	2,19E+03	3,57E+04	-1,57E+03	2,78

2. PLASTIC JOINTS

The analysis of Table 1 shows that in general the influence of bending moment on the y-axis, torque and axial force is much lower than that of the bending moment along the x-axis. Therefore, it can be considered as covering that the elastic bending moment in a node cross section is:

$$M_{ie} = \sigma_{ech}\, W_z \tag{1}$$

where σ_{ech} is the von Mises stress of Tables 1and W_z the section modulus corresponding to the type of rectangular tube used.
When converting a cross-section into a plastic joint, the plastic bending moment that occurs in the Prandtl elastoplastic material assumption is:

$$M_{ip} = 2\, \sigma_c\, S_z \tag{2}$$

where σ_c is the yield limit for the material, and S_z is the static moment of the half-section.
We calculate the ratio k between M_{ip} and M_{ie} with the relation:

$$k = \frac{M_{ip}}{M_{ie}} = \frac{2\sigma_c S_z}{\sigma_{ech} W_z} \tag{3}$$

When a node becomes a plastic joint, the neighboring cross sections are partially plastic as in Figure 3, where the hatched area represented the plastic zone. It can be shown in [1] that the boundary of the plastic zone is given by a parabola.

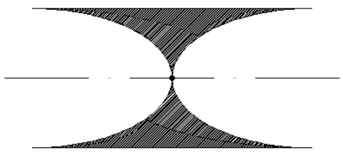

Figure 3: Plastic zone

The variation of the bending moment along the finite element related to the node is linear between M_A at one end and M_B at the other end. Note with α the angle of the slope of the moment.
Considering M_e the limit elastic bending moment, it can be written:

$$M_e = \sigma_c\, W_z \tag{4}$$

It is denoted by l the length of the element and by x_0 the length of the corresponding plastic area. If in B the bending moment becomes the plastic moment M_{ip} , the moment in A is kM_A.It can be written:

$$tg\alpha = \frac{M_{ip} - kM_A}{l} = \frac{M_{ip} - M_e}{x_0} \tag{5}$$

The relation of the plastic zone length is:

$$x_0 = \frac{M_{ip} - M_e}{M_{ip} - kM_A}\, l \tag{6}$$

For each node having a given cross section, a further load ΔF_0 should be added to produce a plastic joint:

$$\Delta F_0 = (k-1)F \tag{7}$$

where, in this case, F is the static load equal to15647 N.
That is why we have to look for the nodes with the lowest k coefficient of amplification.
The energy stored in the plastic joint section is in [Nmm]:

$$E_{p0} = M_{ip}.\varphi \tag{8}$$

where φ is the rotation of the section in [rad].
Taking into account the parabolic distribution of the plastic zone, the total energy stored in the plastic joint area is:

$$E_p = \beta .\, E_{p0} \tag{9}$$

where β is the coefficient of the plastic zone.
At a loading with a force ΔF_0 , the stored elastic energy is E_0.
After forming the plastic joint the remaining energy for loading is:

$$E = E_0 - E_p \tag{10}$$

Note with k_1 the relation:

$$k_1 = \sqrt{\frac{E}{E_0}} \tag{11}$$

The structure with the plastic joint must be loaded with an additional force:

$$\Delta F = k_1 . \Delta F_0 \tag{12}$$

The static moments and plastic bending moments for the used profile sections must be calculated.
The half cross section of a rectangular tube is shown in Figure 4.

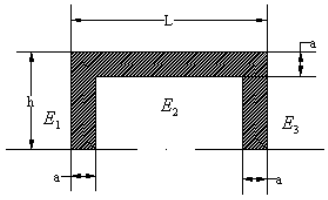

Figure 4: Half cross section

For rectangular tube 20x40x2, because of the asymmetry, the center of gravity is at y_g = 5.737 mm.
Using yield stress σ_c=280MPa we obtain:

S_z = 40. 2. 4,737 + 2. 2. 3,737^2. 0,5 + 2. 2. 14,263^2. 0,5 = 814 mm^3

M_{ip} = 814. 280 = 227920 Nmm

2. CALCULATION OF THE ANGLE φ

Suppose that the plastic joint is formed in node *n* . The adjacent elements to the node are *k* and *j*. The adjacent nodes are *m* and *p*.
The coordinates of the nodes in the undeformed state are in the plane yOz (y_{0m}, z_{0m}), (y_{0n}, z_{0n}) and (y_{0p}, z_{0p}).
Let be denoted with φ_0' and φ_0'', the angles, in radians, by the Oy axis of elements *k* and *j*, and with φ_0 the angle between elements *k* and *j*. Their relations are:

$$\varphi_0' = arctg\frac{z_{0m} - z_{0n}}{y_{0m} - y_{0n}} \tag{13}$$

$$\varphi_0'' = arctg\frac{z_{0n} - z_{0p}}{y_{0n} - y_{0p}} \tag{14}$$

$$\varphi_0 = \pi - \varphi_0' + \varphi_0'' \tag{15}$$

In the first state, nodal displacements in the yOz plane are (Δy_m , Δz_m), (Δy_n , Δz_n) and (Δy_p , Δz_p).
The coordinates of the nodes in the first state are obtained by collecting the old coordinates with the corresponding displacements:

$$\begin{aligned} y_{1m} &= y_{0m} + \Delta y_m \\ z_{1m} &= z_{0m} + \Delta z_m \\ y_{1n} &= y_{0n} + \Delta y_n \\ z_{1n} &= z_{0n} + \Delta z_n \\ y_{1p} &= y_{0p} + \Delta y_p \\ z_{1p} &= z_{0p} + \Delta z_p \end{aligned} \tag{16}$$

In the same way we calculate the angles φ_1' , φ_1'' and φ_1.
The angle with which the node *n* section is rotated is:

$$\varphi = \varphi_0 - \varphi_1 \tag{17}$$

It is insert in equation (8).

3. CALCULATION OF ENERGY STORED IN THE PLASTIC JOINT AREA

3.1 Bending

Since the use of relations (8) and (9) is difficult to apply, an E_p energy calculation methodology is presented.
For each point of the plastic area, before reaching the yield strength σ_c, Hooke's law is valid.
The specific deformation energy stored at each point (volume element) of the plastic area is:

$$E_s = \frac{{\sigma_c}^2}{2E} dV \tag{18}$$

where E is the Young's modulus and dV the elementary volume.
The energy stored in the plastic joint area is:

$$E_p = \int_v \frac{\sigma_c^2}{2E} dV = \frac{\sigma_c^2}{2E} V_p \tag{19}$$

where V_p is the volume of the plastic joint area.
Figure 3 shows that the plastic zone extends to the left and right of the plastic joint cross section. In Figure 4, B is the width, h half of the height and a the thickness of the rectangular tube.
For the left side of the plastic zone the following notations are made:

x_1 - the length of the plastic area

c_1 - the abscissa of the intersection of the parabola with the sole of the profile.

$$c_1 = \sqrt{\frac{a}{h}} x_1 \tag{20}$$

The volume of the left plastic area is:

$$V_{p1} = 2(x_1 - c_1)aB + \frac{4}{3}a(h-a)(x_1 - c_1) + \frac{2}{3}aBc_1 \tag{21}$$

With a similar formula, it is calculated the volume of the right plastic area V_{p2}.
The volume of the plastic joint area is:

$$V_p = V_{p1} + V_{p2} \tag{22}$$

This is replaced in the equation (19).

3.2 Torsion

Considering the thin-walled profile tube, the shear stress τ is given by Bredt's relation:

$$\tau = \frac{M_t}{2\Omega t} \tag{23}$$

in which t is the thickness of the wall, Ω is the area bounded by the median line of the profile and the torque tors

$$M_t = M_x , \tag{24}$$

If the section passes completely in the plastic range, the torque is calculated with the relation:

$$M_{tp} = 2\Omega t \tau_c \tag{25}$$

where the torque yield limit stress is:

$$\tau_c = 0{,}5\sigma_c \tag{26}$$

The lenght of the plastic area is:

$$x = \frac{kM_{xB} - M_{tp}}{k(M_{xB} - M_{xA})} l \tag{27}$$

The volume of the plastic area is:

$$V_p = xA \tag{28}$$

where A is the profile area.
The specific deformation energy stored at each point (volume element) of the plastic area is:

$$E_s = \frac{{\tau_c}^2}{2G} dV \tag{29}$$

where G is the shear modulus and dV the elementary volume.
The energy stored in the plastic joint zone is:

$$E_p = \int_v \frac{\tau_c^2}{2G} dV = \frac{\tau_c^2}{2G} V_p \tag{30}$$

4. CONCLUSIONS

The paper present a method to calculate the energy stored in plastic joints. First step is a static FEM analysis in order to arange the structure nodes after von Mises stresses. It is possible to define a coeficient k in order to calculate the necessary additional force to obtain a new plastic joint. In this way apears new plastic joints till the whole dynamic energy will be used.

REFERENCES

[1] Kaliszky, S. – Plasticity. Theory and Engineering Applications – Elsevier, Amsterdam, 1989

[2] Tofan, M.C., Goia, A.I., Ţierean, M., Ulea, M. - Deformatele structurilor – Editura Lux Libris, Braşov, 1995

[3] Tofan, M.C., Ulea, M., Goia, I., Burcă, I. – On Structure Deformations by Integrated Force Method – în Proceedings of the 2nd International Conference "Computational Mechanics and Virtual Engineering" COMEC 2007, October 11-13th, Braşov, ISBN 978-973-598-117-4, pag. 513-519

[4] Tofan, M.C., Ulea, M., Vlase, S.- MathCAD Application for Structures Fundamental Contours Base Generation - în Proceedings of the 2nd International Conference "Computational Mechanics and Virtual Engineering" COMEC 2007, October 11-13th, Braşov, ISBN 978-973-598-117-4, pag. 521-526

[5] Ulea, M., Tofan, M.C. - MathCAD Model for Roll Over Test of a Bus Bodywork Section- în Proceedings of the 2nd International Conference ″Advanced Composite Materials Engineering ″ COMAT 2008, 9 – 11 October 2008, Brasov, Romania, vol. 1B, ISSN 1844-9336, pag. 593-596

[6] Tofan, M., Ulea, M. - Contour Base Generation for Bus Bodywork Section- in Proceedings of the 3rd International Conference ″Advanced Composite Materials Engineering and International Conference Research & Innovation in Engineering ″ COMAT 2010, 27 – 29 October 2010, Brasov, Romania, vol. 3, ISSN 1844-9336, pag. 258-263

[7] Ulea, M., Itu, C. - FEM Analysis for Staticaly Roll Over Test of a Bus Bodywork Section - in Proceedings of the 3rd International Conference ″Advanced Composite Materials Engineering and International Conference Research & Innovation in Engineering ″ COMAT 2010, 27 – 29 October 2010, Brasov, Romania, vol. 3, ISSN 1844-9336, pag. 264 -267

[8] Ulea, M., Tofan, M.C.- Bus Bodywork Section MathCAD Representations - in the 5th International Conference ″Computational Mechanics and Virtual Engineering ″ COMEC 2013, 24-25 October 2013, Braşov, Romania, pag. 64-68