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CONTRIBUTIONS TO THE MATHEMATICAL MODELING OF CONVECTIVE DRYING PROCESSES OF FRUIT AND VEGETABLES

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Abstract: *The paper explores the theoretical way of carrying out the process of convective drying of vegetables and fruits. Starting from Newton's Convection Heat Transfer Law, the mathematical expressions of the drying rate, the amount of moisture evaporated, the amount of heat taken up by the material, the convection coefficient, the drying time, the amount of humidity trapped by the air flow in -a time interval, etc. Attention is drawn to the fact that the theoretical mathematical modeling also uses coefficients specific to each vegetable or fruit, which are determined only experimentally. The mathematical model presented in the paper can also be used in the design of convective drying systems for vegetables and fruits.*
Key Words: *vegetables and fruits, convective drying, mathematical modeling.*

1. INTRODUCTION

Heat transfer by convection is based on Newton's law, written in form:

$$\dot{Q} = \alpha A \Delta t \left[W \right] \quad , \qquad (1)$$

where : α is the heat transfer coefficient by convection, in W / m^2K; A - the area of the heat exchange area, in m^2; Δt - temperature difference, in K [1], [3].

Determining the heat transfer coefficient by convection is difficult because it depends on many factors, which can be grouped into three main categories: hydrodynamic factors; thermophysical factors and geometric factors. For this reason, for its determination, we turn to the theory of similarity, by means of which we obtain the following equations that describe a certain heat transfer case. The only similarity criteria that contain the convective heat transfer coefficient are: the Nusselt criterion used in the stationary regime and the biot criterion used in the transient regime.

In the vast majority of cases we obtain functions that contain products of the following factors:

$$Nu = f\left(Fo \Pr \mathrm{Re}\, Gr\, Ga\, Ar\, K ... \right) \qquad (2)$$

From this, the value of the convective heat transfer coefficient can be deduced.

The definition of convection heat transfer through Newton's law (1) makes all the factors influencing the convection process to be included in the convection coefficient: the type of motion, the flow regime, the physical properties of the fluid, the shape and orientation of the surface heat exchange. [2]

Convection is a process of energy, mass and impulse transfer. The energy is stored in the fluid particles and transported as a result of their movement.

The intensity of the heat transfer through convection depends to a great extent on the fluid mixing movement.

Depending on the cause of the movement, convection is classified into:

- free or natural convection, when the mixing motion is the result of the differences in density produced by the temperature gradients;
- forced convection, when the motion of the mixture is the result of external causes, such as pumps, fans etc.

The basic relationship of Newton's convection heat transfer allows the heat to be exchanged between a fluid and the surface of a wall:

$$\dot{Q} = \alpha \cdot A \cdot |t_f - t_p| = \alpha \cdot A \cdot \Delta t \qquad (3)$$

or:

$$q_s = \frac{\dot{Q}}{A} = \alpha \cdot |t_f - t_p| = \alpha \cdot \Delta t, \qquad (4)$$

where: Q is the heat flux transferred by convection, in W; q_s - the unitary surface flux, in W/m^2; α - heat exchange coefficient by convection, in $W/m^2\ {}^0C$; t_f, t_p - fluid temperature, respectively, of the wall surface at ${}^0C$; Δt - temperature difference between fluid and wall, in ${}^0C$ or K; A - area of heat exchange (contact) between wall and fluid, in m^2.

For to control the process of convection drying of vegetables and fruits in order to conserve them it is necessary to know the speed of killing, the amount of moisture to be removed, the dimensions of the drying wall etc.

The drying rate, m is defined as the moisture mass dm taken from the surface unit S, in the time unit $d\tau$ [4], [5], [6], [7]:

$$m = \frac{dm_{um}}{Sd\tau}\left[kg/m^2 s\right] \ . \qquad (5)$$

The relationship between the drying rate and the length of the process is the basic element for the construction of a drying plant. This relationship is complex and the use of an analytical method of establishing it is not possible. Practically, the drying rate is determined experimentally by drawing out the drying curves specific to the different vegetable or fruit species. Figure 1 shows the shape of the drying curve, which establishes the relationship between moisture and the duration of the drying process, and Figure 2 shows the variation in the drying rate for the two drying phases.

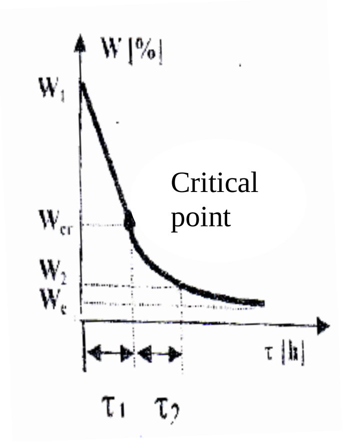

Fig.1. *Form of the drying curve* [8]

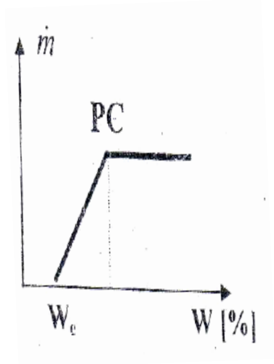

Fig.2. *Variation of drying speed* [8]

The factors on which drying speed depends are grouped into the following categories:

- *nature of the product*, given by its structure, chemical composition, the connection of the moisture with the material etc .;
- *the form of the product to be dried*;
- *initial and final humidity value*;
- *parameters of the drying agent* (temperature, speed, humidity);
- *the way in which the contact between the heat and the product is established*.

Although it is impossible to take into account all the parameters and factors on which the drying is dependent, it is still possible to determine the duration of the drying process. The drying curve (Figure 2) for vegetables and fruits shows two distinct areas separated by the critical point. The first portion represents a straight line and corresponds to the constant drying time, and the second is in the form of an exponential curve, with the drying rate decreasing continuously (the second drying period) [8].

During the constant drying rate, the partial pressure of the vapors at the surface of the vegetables and fruit is equal to the value of the pressure corresponding to that of the free surface of a liquid. The drying rate does not depend on the thickness of the material layer nor on the initial moisture value. This depends only on the temperature regime of the drying, the speed of the drying agent and its humidity. In the second drying period, the drying rate is determined by the rate of moisture diffusion inside the material and depends on the thickness of the

layer of material to be dried, as well as its humidity value. The influence of air velocity and its humidity is generally low.
In the case of constant moisture drying of the drying agent, the drying rate is constant during the first period and the amount of evaporated moisture is proportional to the amount of heat taken up by the material:

$$dm_{um} = \frac{dQ}{r}[kg] \quad , \tag{6}$$

where *r* is the latent heat of vaporization of water, in *kJ / kg*.
According to Newton's equation, the amount of heat taken up by the material over the time interval *dt* is:

$$dQ = \alpha S\left(t - t_{\sup}\right)d\tau\left[kJ\right], \tag{7}$$

where: α is the convective heat transfer coefficient, in W / m^2K; *S* - surface of the material to be dried, in m^2; *t* - temperature of the thermal agent at 0 ^0C; t_{sup} - temperature of the surface of the material to be dried at 0 ^0C.
By replacing in relation (6) the amount of heat given by the relation (7), we obtain:

$$dm_{um} = \frac{\alpha S}{r}\left(t - t_{\sup}\right)d\tau = K_t S\left(t - t_{\sup}\right)d\tau\,, \tag{8}$$

where $K_t = \alpha / r$ is the substance transfer coefficient, in kg / m^2sK.
The drying rate equation for the first period has the expression:

$$m_I = \frac{dm_{um}}{Sd\tau} = K_t\left(t - t_{\sup}\right). \tag{9}$$

If the surface moisture of the vegetables or fruit is high, the temperature of the evaporation surface t_{sup} is equal to the free surface temperature of the evaporating water and can be determined by the wet thermometer ($t_{sup} = t_u$).
So:

$$m_I = K_t\left(t - t_u\right), \tag{10}$$

where the temperature difference t-t_u can be measured with the psychrometer.
Typically, the determinant of the process, the drying rate, is expressed as the difference between the partial pressure of the water vapor at the saturation state and the state:

$$m_I = k_p\left(p_s - p_v\right), \tag{11}$$

where k_p is the mass transfer coefficient equal to the product of the value of the convective heat transfer coefficient and a coefficient of proportionality, in kg / m^2sPa.
Based on the similarity theory, the coefficient of convection can be determined by:

$$mk_p = b\alpha = bA\left(w\rho\right)^n\,, \tag{12}$$

where: *A* is a constant dependent on the nature of the drying agent; *w* - speed of the drying agent, in *m / s*; ρ - density of the drying agent, in kg / m^3; *n* - an exponent corresponding to the flow regime.
In the case of air, the mass transfer coefficient is:

$$k_p = 0{,}0745\left(w\rho\right)^{0{,}8}. \tag{13}$$

Using the expression of the mass transfer coefficient, given by the relation (13), the drying rate for the air is obtained:

$$m_I = 0{,}0745\left(w\rho\right)^{0{,}8}\left(p_s - p_v\right). \tag{14}$$

From the relationship (14) it follows that the drying rate in the first period depends only on the parameters of the drying agent, specified by the difference (p_s-p_v) and the value of the velocity w ρ .

The drying process in the second period shows a decreasing rate of drying rate and can be expressed approximately by a straight line connecting the critical and steady humidity. For this period, it is recognized that the drying rate is proportional to the difference between free and steady-state humidity:

$$m_{II} = \frac{dm_{um}}{Sd\tau} = K_u\left(W - W_e\right), \tag{15}$$

where: K_u is the coefficient of evaporation; W, W_e - the moisture content of the material at a given time, respectively in equilibrium.

By replacing this expression $dm_{um} = m_m dW$, one obtains:

$$\frac{m_m dW}{Sd\tau} = K_u\left(W - W_e\right), \tag{16}$$

or after integration:

$$\frac{m_m}{S}\ln\frac{W_{cr} - W_e}{W_2 - W_e} = K_u\tau_2 , \tag{17}$$

where: W_2 is the moisture content of the material at the end of the second drying period; τ_2 - duration of this period; m_m - mass of dry material.

From the relationship (17) we can determine the drying time in the first period, which is equal to the drying rate at the critical point:

$$\tau_2 = \frac{m_m}{K_u S}\ln\frac{W_{cr} - W_e}{W_2 - W_e}. \tag{18}$$

The relation (18) can also determine the drying time in the first period, which is equal to the drying rate at the critical point:

$$\frac{m_m dW}{Sd\tau} = K_u\left(W_{cr} - W_e\right). \tag{19}$$

After the integration of the relation (19) between the initial and the critical humidity, respectively between time 0 and τ1, we obtain:

$$\tau_1 = \frac{m_m}{K_u S}\ln\frac{W_1 - W_{cr}}{W_{cr} - W_e}. \tag{20}$$

The calculation method presented for the determination of the duration of the two phases leads to approximate results, and experimental data is needed for their correction. Typically, moisture content of the air varies continuously when passing through the drier, in the direction of the increase, while the temperature value decreases.

In fact, it can be considered that in this case the drying rate is proportional to the difference x_s-x, the difference in moisture content:

$$\frac{dm_{um}}{\tau dS} = K_x\left(x_s - x\right), \tag{21}$$

where: dS is the elemental area of the material to be dried, in m^2; K_x - mass transfer coefficient.

The amount of moisture involved in the airflow over a period of time is:

$$dm_{um} = m_a \tau dx. \tag{22}$$

By introducing the relation (22) into (21) we obtain:

$$m_a\frac{dx}{x_s - x} = K_x dS. \tag{23}$$

If the relation (23) integrates between the initial air humidity x_i and the final air x_f, the total area required for the first drying period is obtained:

$$S_1 = \frac{m_a}{K_x} \ln \frac{x_s - x_i}{x_s - x_f} \quad (24)$$

The relation (24) can only be applied to the first phase, in which the drying is carried out at a constant speed; the critical drying area decreases: $S' = nS$, where $n = W / W_{cr}$ is the ratio between free and critical humidity.
In this case the relation (23) becomes:

$$m_a \frac{dx}{x_s - x} = K_x \frac{W}{W_{cr}} dS \quad (25)$$

The relationship (25) allows to determine the total area required for the second drying period. The numerical value of the K_x coefficient shall be determined experimentally for the same conditions as the plant is assumed to work.

3. CONCLUSIONS

1. Convective drying of vegetables and fruit is frequently used for the preservation and storage of vegetables and fruit for long periods of time.
2. In the case of convective drying, technical equipment shall be used in the design of which all the physico-biological characteristics of the processed materials must be known in order to ensure the proper quality of the finished products and to minimize the energy consumptions required to bring the products to optimum humidity to keep them for longer periods.
3. Theoretical research by the mathematical modeling of the kinetics of the convective drying process of vegetables and fruits offers the possibility of reducing experimental research, but also making available to designers and manufacturers dedicated to the improvement of the technical equipment used for this purpose.

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