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OPTIMIZATION DESIGN OF FLEXIBLE ROTOR

Dumitru D. Nicoara
Transilvania University of Brasov, tnicoara@unitbv.ro

Abstract: *The paper deals with the optimization of the rotor–bearing systems in static and dynamic range. The goal of this study is to find out the position of the bearings, the diameters of the shaft (different diameters for several segments of the shaft) in order to minimize certain cost functionals as: static stiffness, amplitudes and "energy" of the dynamic response, critical rotating speeds and receptance, i.e. the diminishing of the vibrations. Some constraints are imposed: the maximum stress, the minimum diameter, distances between bearings, constant volume (weight) of the shaft, etc. Unbalanced response, i.e. synchronous harmonic excitation as well as asynchronous harmonic response is analyzed and therefore several conclusions resulted.*
Keywords: *rotor bearing systems, finit element, design, optimization*

1. INTRODUCTION

The dynamic performances of the rotor-bearing systems are strongly influenced by the design parameters such as: distances between the bearings, diameters of the different portions of the shaft, bearing preload, bearing spacing etc. In most papers this influence is studied by varying the parameters and analyzing of its effect on the system. In this paper we propose a method for optimal determination of these parameter using the optimization principles [3], [9], [13]. The optimization of the rotor–bearing systems is realized in static and dynamic range. The proposed methodology is equivalent with a passive control of rotor-bearing systems. In the papers [9], [10] we has introduced four types of objective functions based on the modal analysis of the rotor-bearing system. In this paper we define four types of optimization problems constructed with thes objective functions. By coupling of the finite element method to the methods of non-linear optimization with constraints thise problems have been solved. The goal of the optimization is the determination of the design parameters: the position of the bearings, the diameters of the shaft (different diameters for several segments of the shaft) so as the static stiffness and dynamic stiffness defined as the inverse of receptance matrix to be maximized (the goal being the diminishing the vibrations).

2. FINITE ELEMENT MODEL. ROTOR EQUATIONS

The model of the rotor-bearings systems comprises of a continuous elastic shaft, with several rigid disks, mounted on anisotropic elastic bearings. The finite element model, [2], [4], [10], [11] could use one of the following three beam finite element types:

a. Beam C^1 finite element type based on Euler-Bernoulli beam model;
b. Beam C^1 finite element type based on Timoshenko beam model;
c. Beam C^0 izoparametric finite element type based on Timoshenko beam model;

The beam finite element has two nodes. For the static analysis, a 2D problem, there are two degrees of freedom (DOF) per node, one displacement perpendicularly on the beam axis and the slope of the deformed beam. In the case of the dynamic analysis four degrees of freedom (DOF) per node are considered: two displacements and two slopes measured in two perpendicular planes containing the beam [10], [11]. Thimoshenko beam model is finally adopted as the beam might be short and therefore the effect of the shear force must be considered. The gyroscopic effect and damping in bearings may be taken into account.
The linearized bearing are commonly modeled as four spring coefficients and four damping coefficients, i.e.,

$$\begin{bmatrix} k^b_{yy} & k^b_{yz} \\ k^b_{zy} & k^b_{zz} \end{bmatrix} \begin{Bmatrix} q^b_y \\ q^b_z \end{Bmatrix} + \begin{bmatrix} c^b_{yy} & c^b_{yz} \\ c^b_{zy} & c^b_{zz} \end{bmatrix} \begin{Bmatrix} \dot{q}^b_y \\ \dot{q}^b_z \end{Bmatrix} = \begin{Bmatrix} f^b_y \\ f^b_z \end{Bmatrix}$$

$$\left[k^b_{ij}\right] = \begin{bmatrix} k^b_{ij} & 0 \\ 0 & 0 \end{bmatrix}, \ \left[c^b_{ij}\right] = \begin{bmatrix} c^b_{ij} & 0 \\ 0 & 0 \end{bmatrix} \quad i, j = y, z$$

where the superscript *b* denotes the bearing, c^b_{ij} , k^b_{ij} are the linearized directional bearing damping and stiffness due to *j* directional motion, respectively, and f^b_y , f^b_z are the bearing forces in the *y-x* and *z-x* planes, respectively.
The equation of an anisotropic rotor-bearing systems which consists of a flexible nonuniform shaft, rigid disk and anisotropic bearings may be written as
The equations may be written as

$$\boldsymbol{M}\,\ddot{\boldsymbol{q}} + \left(\boldsymbol{C} + \boldsymbol{\Omega}\boldsymbol{G}\right)\dot{\boldsymbol{q}} + \boldsymbol{K}\,\boldsymbol{q} = \boldsymbol{F} \tag{1}$$

$$\mathrm{M} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}_{N\times N}, \quad \mathrm{C} = \begin{bmatrix} c_{yy} & c_{yz} \\ c_{zy} & c_{zz} \end{bmatrix}_{N\times N},$$

$$\mathrm{G} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}_{N\times N}, \quad \mathrm{K} = \begin{bmatrix} k_{yy} & k_{yz} \\ k_{zy} & k_{zz} \end{bmatrix}_{N\times N},$$

$$\mathrm{F} = \begin{Bmatrix} f_y(t) \\ f_z(t) \end{Bmatrix}_{N\times 1}, \quad q(t) = \begin{Bmatrix} q_y(t) \\ q_z(t) \end{Bmatrix}_{N\times 1}$$

where **q** is the global displacement vector, whose upper half contains the nodal displacements in the y-x plane, while the lower half contains those in z-y plane, and where the positive definite matrix **M** is mass (inertia) matrix, the skew symmetric matrix **G** is gyroscopic matrix, and the nonsymmetric matrices **C** and **K** are called the damping and the stiffness matrices, respectively.

The authors elaborated several computer codes in MATLAB [8] programming language. Optimization toolbox is used to perform the computer programs, which have very good performances in order to study complex rotor with two or more bearings with several rigid disks. The finite element model of the rotor-bearing system was validated by means of experimental measurements which were performed in the dynamic range [3].

3. OPTIMIZATION MODELS

In the paper two optimization problem were defined. The first is a static problem for which the objectiv function is the static stiffness, that is the displacement *v* under the force *F* applied on the shaft, divided by force. The design parameters are: the distance s_i between the bearings and the diameters d_i of the different portions of the shaft. The problem is subject to some constraints: the maximum and minimum limits for the distances s_i and diameters d_i , the bending stress must be smaller than an available value σ_a and the volume *V* of the shaft must be constant. This may be written as :

$$\min_{s_1,s_2,d_3,d_1,d_2} \; v/F$$

$$\begin{cases} s_1^l \le s_1 \le s_1^u \\ s_2^l \le s_2 \le s_2^u \end{cases} \quad \begin{cases} d_1^l \le d_1 \le d_1^u \\ d_2^l \le d_2 \le d_2^u \\ d_3^l \le d_3 \le d_3^u \end{cases} \quad \begin{cases} \sigma \le \sigma_a \\ V = const. \end{cases} \tag{2}$$

The second problem is a dynamic one. In this case the objectiv functions may be chosen depending on the shaft excitation: synchronous or asynchronous excitation. All objective functions are the measure of dynamic stiffness [10].

Rotating unbalance (synchronous excitation): the objective function is (a) the receptance for a given rotating speed, or (b) the average receptance for an interval of rotating speeds, or (c) the lowest critical rotating speed (with or without the gyroscopic effect). The optimization problems obtained in these three cases are represented in Eq. (3).

$$\begin{array}{lll} (a) & (b) & (c) \\ \min\limits_{s_k,d_k} \dfrac{A_u}{A_F} & \min\limits_{s_k,d_k} \dfrac{1}{\Omega_1-\Omega_2}\displaystyle\int_{\Omega_1}^{\Omega_2}\dfrac{A_u}{A_F}\,d\Omega & \max\limits_{s_k,d_k} \omega^{cr} \\ s_k^i \le s_k \le s_k^s & s_k^i \le s_k \le s_k^s & s_k^i \le s_k \le s_k^s \\ d_k^i \le d_k \le d_k^s & d_k^i \le d_k \le d_k^s & d_k^i \le d_k \le d_k^s \\ \Omega = \Omega_0 & \Omega \in (\Omega_1, \Omega_2) & V = const. \\ V = const. & V = const. & \end{array} \tag{3}$$

In the above equations A_u is the amplitude of the displacement, A_F is the force amplitude, Ω is the rotor spin speed and ω is the whirl speed. The first two objective functions (a) and (b) lead to a pseudo-static problem, the response is found out by solving a linear system of equation, in complex if damping and/or gyroscopic effect are considered.

Asynchronous harmonic excitation is mainly due to defects in bearing rollers and/or rings [4]; in this case the objective function is the "energy" of the response and its corresponding optimization problem is

$$\min_{s} \; \frac{1}{T_{\max}}\int_0^{T_{\max}} \{u\}^T\{u\}\,dt \;,\; \{u\} = \begin{Bmatrix} v \\ w \end{Bmatrix} \tag{4}$$
$$s^i \le s \le s^s\,;$$
$$d^i \le d \le d^s$$

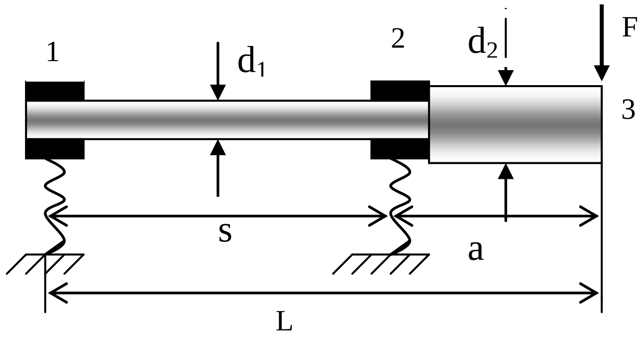
where T_{max} is the integration total time corresponding to n shaft rotations, $\{\boldsymbol{u}\}$ is displacement vector composed by the two displacements, v and w, measured perpendicularly on the shaft. In this case it is necessary to compute the dynamic response of the rotor–bearing system and the θ -Wilson step by step integration method was used.

Some constraints are imposed: the maximum stress, the minimum diameter, distances between bearings, constant volume (weight) of the shaft, etc. Unbalanced response (synchronous harmonic excitation) as well as asynchronous harmonic response are analyzed and therefore several conclusions resulted.

4. NUMERICAL EXAMPLES

Example 1.

The first example illustrates the static optimization of an two bearings shaft, Fig. 1. The optimization problem is of the type (3.a) and is defined by the Eq. (5). Table 1 shows the initial data. Figures 2 and 3 show the variation of the vertical displacement v under force F, with the distances s, for several cases: rigid bearings and elastic bearings (k=10^7 N/m), for Euler-Bernoulli beam model, or Timoshenko beam model. It can be noticed that the bearing stiffness strongly influences the optimum values of the design parameters. Timoshenko beam model is suitable as the distances between bearings may become small and the influence of the shear force may not be neglected.



$$\min_{s,d_1,d_2} v/F \qquad \begin{cases} s^i \le s \le s^s \\ d_1^i \le d_1 \le d_1^s \\ d_2^i \le d_2 \le d_2^s \\ \sigma_a \le \sigma_a^s \\ V = const. \end{cases} \tag{5}$$

Fig. 1: Shaft configuration

Table 1: Initial data of the rotor configuration

d_1 = 80 mm	F = 50 000 N
d_2 = 80 mm	σ_a = 100 Mpa,
a = 100 mm	$k = 10^7$ N/mm
d_{min} = 40 mm	$E = 2*10^5$ Mpa

The design parameters are s, d_1, and d_2. Timoshenko C^1 beam element was used. Table 2 and Fig.2-3 shows the results of the optimization of the above system, for different values of the shaft volume.

Table 2: Results of the optimization

Volume [mm^3]	s [mm]	d_1, d_2 [mm]	σ_1, σ_2 [Mpa]	v_{max}/F
7.5398e+005	50.5	80 / 80	100 / 100	3.39e-6
1.0053e+006	68.86	88.68 / 85.94	73.1 / 80.25	2.40e-6
1.5080e+006	85.28	102.58 / 101.12	47.2 / 49.25	1.58e-6

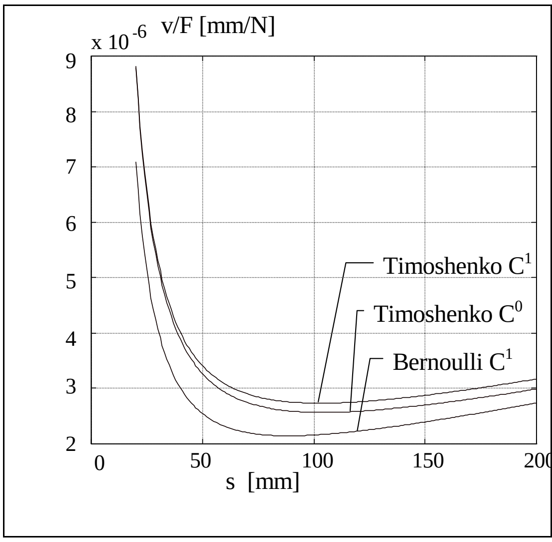


Figure 2: Static stiffness - elastic bearings

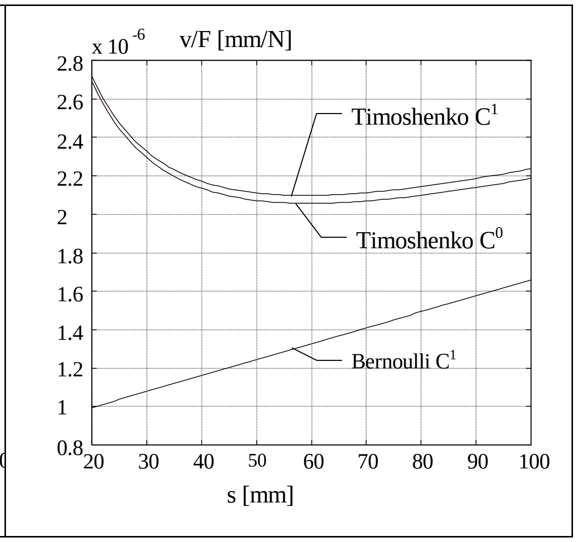


Figure 3: Static stiffness - rigid bearings

Example 2.

The second problem is a dynamic one. We consider the rotor configuration from Fig. 4. For the case of the linearized bearings model and Timoshenko beam model, we shall define and resolve three optimization problems:

(OP1) Find out the optimal value s_{opt} of the distance s between the bearings so that the dynamic stiffness (calculated in disk station) be maximum.

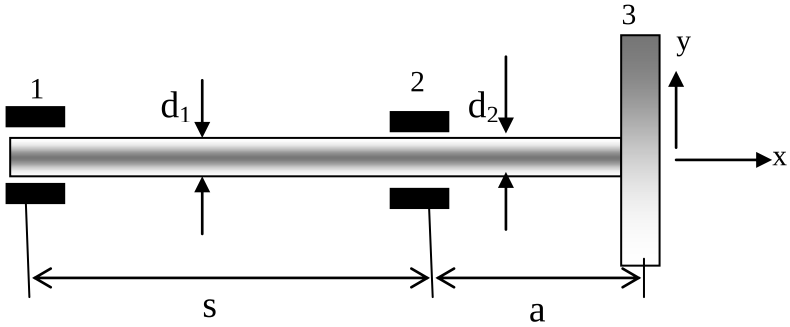


Fig. 4

(OP2) Given $s = s_{opt}$, already determined in the previous problem, find out the optimum diameters d_1 and d_2 so that the average receptance to be minimized. In this case we have: $\Omega \in (0,9000)$ [rpm], $L = a + s = const., V = const.$

(OP3) In this problem the design parameter are the distance between bearings and the diameters of the two shaft segments. The objective function is average receptance.

Table 3 shows the rotor data. Tables 4, 5 and 6 show the optimal value of the parameter s, for the first optimization problem, obtained with the three objective functions: receptance, average receptance and the lowest whirl speed. It can be noticed that the values of the resulting design parameters are very close for the objective functions used in this work.

Table 3 Rotor data Example 2

Shaft	Disk	Bearings
$E = 2.068e11$ N/m^2 $\rho = 7833$ Kg/m^3 $d_1 = d_2 = 0.08$ m a = 0.1 m	m = 75 Kg $J_T = 0.190$ Kg m^2 $J_P = 0.368$ Kg m^2 e = 0.01 m	Stations 1 and 2 $k_{yy} = 5e8$ N/m; $k_{yz} = k_{zy} = 0$ $k_{zz} = 3e8$ N/m $c_{yy} = c_{zz} = 1e4$ Ns/m $c_{zy} = c_{yz} = 0$

Table 4 Optimal value of the distance *s*

Objective function – receptance			
	Ω [rpm]	s_{opt} [m]	Receptance [m/N]
1	2000	0,3374	$1,26*10^{-8}$
2	4000	0,3404	$1,37*10^{-8}$
3	6000	0,3467	$1,64*10^{-8}$
4	8000	0,3591	$2,30*10^{-8}$
5	9000	0,3615	$3,32*10^{-8}$

Table 5 : Optimal value of the distance *s*

Objective function - average receptance, $d_1 = d_2 = 0,08$ [m]			
	Ω [rot/min]	s_{opt} [m]	Average receptance [m/N]
1	Ω∈(0, 9000)	0,3507	$1,65*10^{-8}$
2	Ω∈(0, 15000)	0,3	$3,57*10^{-8}$

Table 6 Optimal value of the distance *s* the lowest whirl speed

Objective function – the lowest whirl speed – no gyroscopic effect	
s_{opt} [m]	ω_1^{cr} [rpm]
0,3523	10.553
Objective function – the lowest whirl speed – whit gyroscopic effect	
0,3193	10.057

For the second problem, the optimization result had been calculated with the cost functional the average receptance for $s = s_{opt}$, determined in the first problem and under the following conditions: $L = s_{opt} + a = const.$, $\Omega \in (0, 9000)$ [rpm]. The results are shown in Table 7.

Table 7

Objective function - average receptance, $L = s_{opt}+a = const.$, $s_{opt} = 0,3507$			
Ω [rpm]	d_1[m]	d_2[m]	Average receptance [m/N]
$\Omega \in (0, 9000)$	0,0796	0,0811	$1,65*10^{-8}$

For the third problem the results obtained with the average receptance as objective function are shown in Table 8.

Table 8

Objective function - average receptance, $\Omega \in (0, 15.000)$ [rpm],			
s [m]	d_1[m]	d_2[m]	Average receptance [m/N]
0,2616	0,0889	0,0903	$1,45*10^{-8}$

We notice that dynamic stiffness has considerably increased with over 100% for an average increasing of the diameters with 11%. The ratio between the optimal distance between the bearings and the length a of the console is $s_{opt} / a = 2,6$.

3. CONCLUSION

In this paper we proposed several optimization model for rotor-bearing systems. These models permit the maximization of the static and dynamics stiffness of the systems, i.e. the diminishing of the vibrations. The solutions of the optimization problems are obtained by the coupling of the finite element method with the nonlinear optimization methods with constraints. Optimization computer codes has been realized in MATLAB programming language. The finite element model of the rotor-bearing system was validated by means of experimental measurements which were performed in the dynamic range. The method is very useful for the design engineers from the very beginning of the design, offering to the designer the optimal values of the parameters.

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