DESIGN APPROACH FOR THE VEHICLE'S STEERING LINKAGE FITTED ON RIGID AXLE

Ion PREDA, Dinu COVACIU, Gheorghe CIOLAN

Abstract: *The steering system of a vehicle consists of two subsystems: the steering linkage, which correlates the steering angles of all the steerable road-wheels, and the actuation subsystem, which connects the driver's steering-wheel with one mobile element of the steering linkage. In the case of the vehicles with rigid steering axles, the most used steering linkage is a simple mechanism, composed of four bars (one being the base) connected with four joints. However, the design of the steering linkage isn't an easy task, mainly because the wanted correlation function is difficult to specify and because the packaging restrictions are very important. The present paper shows a possible approach to define the optimization function and the dimensions of the correlation steering mechanism.*

Keywords: *steering system, steering linkage, Ackermann steering, trapezoid mechanism kinematics, rigid axle, optimization.*

1. INTRODUCTION

Accordingly with its functions, the steering system of a vehicle is composed of two subsystems. One is the steering linkage (the steering transmission), having the task to correlate the values of steering angles of all the steerable road-wheels. The other is the actuation subsystem, which connects the driver's steering-wheel with one mobile element of the steering linkage and, possibly, helps (assists) the driver in effort or decision.

The steering linkage is normally a three-dimensional mechanism with bars, able to correlate the pivoting angles of an axle's left and right wheels, but also the medium pivoting angles of the wheels appertaining to two or more steering axles, if is the case.

The oldest and simplest correlating mechanism (steering linkage) is a four-bar mechanism, also called steering trapezoid or trapezoidal steering mechanism. This can be fitted on a rigid axle and consists (as presented schematically in Figure 1) in the axle's beam (as ground or mechanism base – segment **AB**), two steering-arms (spindle-arms or cranks – segments **AD** and **BC**) and the tie-rod (track rod – segment **CD**). The steering-arms are attached to the axle's beam by two pivots (the king-pins **A** and **B**) and to the tie-rod by two ball-joints (**C** and **D**).

With respect to the steering trapezoid kinematics, the design implies the completion of more steps, as:

- the definition of the objective function to be fulfilled by the mechanism;
- the obtainment of the actual positioning function, i.e. the actual behavior of the mechanism;
- the optimization of mechanism.

2. THE BASIC OBJECTIVE FUNCTION OF THE STEERING LINKAGE

To ensure pure rolling of the wheels during cornering, the projections on the ground of all the wheels' spinning axes must be concurrent in a unique point, the turn center $\mathbf{O_A}$, as it is presented in Figure 2. That means the steering angles of the wheels are a function of turn radius R ($\mathbf{O_AG}$), wheel base w_b ($\mathbf{P_rP_f}$) and wheel track t ($\mathbf{P_{fi}P_{fo}}$), ensuring the inside wheel of an axle to be steered to a greater angle than the outside wheel [1], [7], [9], [15]. That allows the inside wheel to steer a tighter radius and avoids the tires scrubbing when the vehicle travels around a curve with small lateral tire forces (normally at low speed).

Known probably from about 3800 years ago, this geometrical condition of steering was introduced in 1758 in the construction of the horse drawn carriages by the English manufacturer Erasmus Darwin. Lather, in 1817, in Munich, the German carriage builder Georg Lankensperger applied also this condition, which was patented in 1818 by his agent in England, Rudolph Ackermann. That's why this basic design condition, respected at the manufacture of the first automobiles, is known for long time as conventional or "true Ackermann" steering.

The vehicle's steering scheme in Figure 2 shows in a graphic way that the steering angle of the inner wheel must be greater as the one of the outer wheel, i.e. $\delta_i > \delta_o$.

Starting from the expressions of the tangent functions of these angles:

$$\tan\delta_i = \frac{w_b}{R_r - \frac{t}{2}} \qquad \tan\delta_o = \frac{w_b}{R_r + \frac{t}{2}} \qquad (1)$$

and eliminating R_r (the distance from the turning center $\boldsymbol{O_A}$ to the middle point of the rear axle $\mathbf{P_r}$), it results the quantitative form of the Ackermann condition:

$$\frac{1}{\tan\delta_o} - \frac{1}{\tan\delta_i} = \frac{t}{w_b} \qquad (2)$$

From this equation, the basic design objective specification can be expressed mathematically as:

$$\delta_{iA}(\delta_o) = \operatorname{atan}\frac{w_b\tan\delta_o}{w_b - t\tan\delta_o} = \operatorname{atan}\frac{1}{\frac{1}{\tan\delta_o} - \frac{t}{w_b}} \qquad (3)$$

In the previous equations, the notations are: w_b – the vehicle wheelbase; t – the track width of the steering wheels (at the level of the ground); δ_i, δ_o – the steering angles of the inner and outer wheels.

Using equation 3 it is easy to demonstrate that, in the case of the true Ackermann steering, the difference of the steering angles $\Delta\delta = \delta_i - \delta_o$ (also called toe-out angle) will increase as the turning radius become smaller. That means, the concordance of the steering geometry with the Ackermann condition will cause the road wheels to toe-out when the steering wheel is turned. This is why the Ackerman condition is referred often as toe-out on turns.

3. THE ACTUAL FUNCTION OF THE STEERING LINKAGE

The kinematics of the four-bar trapezoidal mechanism was largely studied during the years [1], [7], [9], [15]. It is demonstrated that, even this type of steering linkage is a spatial mechanism (due to the kingpin inclination and caster inclination of the wheel pivots), its kinematics is very close to the one of a planar mechanism [1]. Therefore, in Figure 1 the contour **ABCD** schematize the skewed shape of the planar mechanism considered as steering linkage.

It observes that, due to the kingpin angle, the distance t_p between the pivots (kingpins) at the level of the mechanism plane is smaller as the track t of the front wheels (in the scheme, the scrub radii were considered small and, so, negligible).

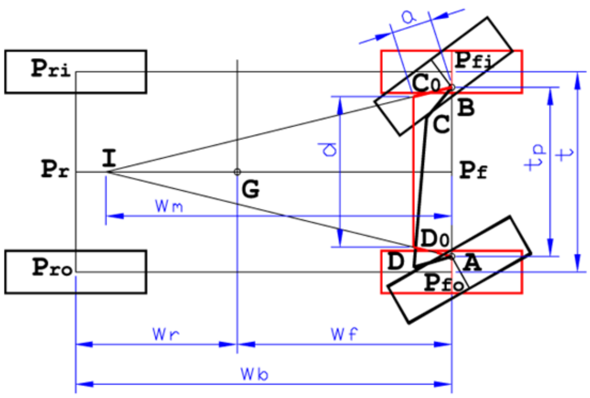

Fig. 1. Schematic of the trapezoidal steering linkage

Due to the necessity to ensure similar vehicle behavior during the left or right turn, the mechanism must be symmetrical when the vehicle travels in straight line. That means the corresponding contour $\mathbf{ABC_0D_0}$ has a symmetrical trapezoidal shape, implying not only an equal length a for the steering arms (the segments **AD** and **BC**), but also equal position angles φ_0 between the steering arm and the center plane of the wheel ($\varphi_0 = <\mathbf{C_0IP_f} = <\mathbf{D_0IP_f}$).

The current angles of the inner and outer steering wheels are respectively: $\delta_i = <\mathbf{C_0BC}$, $\delta_o = <\mathbf{D_0AD}$.

To obtain the actual function of the trapezoidal steering linkage, it was started here from the projections of the tie-rod (segment **CD**) on the longitudinal and transversal axes of the vehicle:

$$\begin{aligned} y &= t_p - a[\sin(\varphi_0 - \delta_o) + \sin(\varphi_0 + \delta_i)] \\ x &= a[\cos(\varphi_0 - \delta_o) - \cos(\varphi_0 + \delta_i)] \\ d^2 &= x^2 + y^2 \end{aligned} \tag{4}$$

Making the notations:

$$\begin{aligned} &S_o = \sin(\varphi_0 - \delta_o) \quad C_o = \cos(\varphi_0 - \delta_o) \\ &S_i = \sin(\varphi_0 + \delta_i) \quad C_i = \cos(\varphi_0 + \delta_i) \\ &E_1 = \frac{\left(1 + \frac{b^2 - d^2}{2a^2} - \frac{b}{a} S_o\right)}{C_o} \quad E_2 = \frac{\left(S_o - \frac{b}{a}\right)}{C_o} \end{aligned} \tag{5}$$

and making some calculations, it obtains:

$$\begin{aligned} &S_i = -\frac{E_1 E_2 + \sqrt{1 - {E_1}^2 + {E_2}^2}}{1 + {E_2}^2} \\ &\delta_{im}(\delta_o) = \operatorname{atan} S_i - \varphi_0 \end{aligned} \tag{6}$$

As can be seen, the actual inner steering angle δ_{im} is a complicated function given by the equations (6) and (5), but depends only by the argument δ_o.

4. CONSTRUCTIVE LIMITATIONS

Because the equation (6) has a different form as the equation (3), it is obvious that the trapezoidal steering mechanism is not able to generate correctly the true Ackermann condition. For that reason, sustained efforts were dedicated during the years by the engineers and mathematicians to obtain methods for the constructive optimization.

In this respect, two different approaches were used mainly in the steering linkage design. One method tries to find a more complex mechanism, able to fulfill the exact requirements of the Ackermann condition. For example, in the paper [5] it is demonstrated that some types of more complex mechanisms are able to generate exactly the required Ackermann function (corresponding to the equation 3). But this approach has fewer chances to be used in practice due to the complexity, reliability and packaging problems.

The other method consists in the optimization of the simpler trapezoidal four-bar mechanism, which means to minimize somehow the errors between the wanted correlation function and the actual way the mechanism steers the wheels of the same axle.

Due to the simplicity (and symmetry) of the four-bar steering linkage, the number of parameters to be optimized is reduced: the lengths d and a of the tie-rod and steering arm.

For the manufacturing and assembling process, the position angle φ_0 (of the steering arm to the wheel's center plane) presents a bigger importance, and may replace one length (generally d) in the optimization process.

Using the notations in Figure 1, the angle φ_0 can be calculated easily from the lengths of the mechanism bars:

$$\varphi_0 = \operatorname{asin}\left(\frac{t_p - d}{2a}\right) \tag{7}$$

In many scientific works it is stated that the steering arm's lengths a has a minor importance for the optimization process [1], [14] (for that, it is normally adopted from constructive reasons). But this length a, as the other dimensions t_p and d, present high importance for the packaging space of the steering system.

5. TURNING OF THE VEHICLE WITH ELASTIC TIRES

If the Ackermann condition was very suitable for the firsts automobile generations, having stiff wheels with solid tires and then with high pressure pneumatic tires, the modern wheeled vehicles can present large tire lateral deflections.

As is presented in Figure 2, the tires experience lateral slip angles α in order to generate lateral forces (and vice-versa), and the turn center moves from the point $\mathbf{O_A}$ (corresponding to the Ackermann condition) to another turn center **O**, placed on a circle having the center in the vehicle's center of gravity **G** and a radius equal with the turning radius R (it is remembered here that the turning radius is the distance from the turning center to the vehicle's center of gravity).

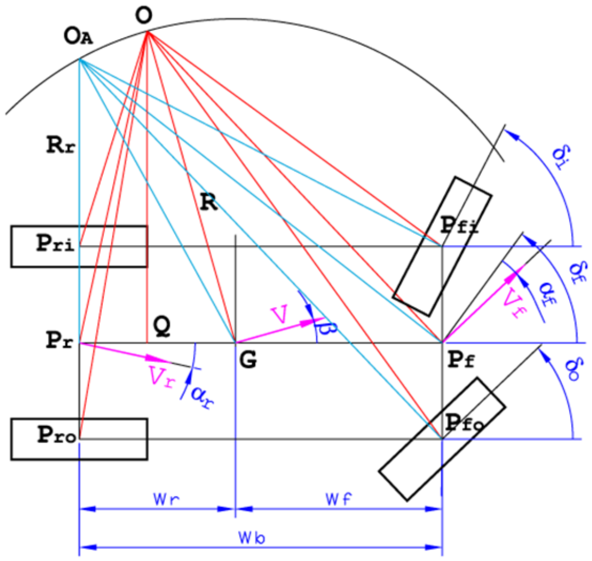

Fig. 2. Principle of conventional Ackermann steering and true steering

That means, the objective function of the steering linkage (the wanted steering angle correlation to be generated) depends not only on the vehicle's wheelbase w_b and turning radius R, but will change in movement with respect to the dynamic conditions: a combination of the wheels' steered angles and slip angles, responsible for the magnitude and orientation of the cornering forces.

In other words, if the vehicle will negotiate a tighter turn or will travel with a higher speed, the slip angles α_f and α_r of the front and rear tires will increase and the turning center (the point **O**) will move forward.

Studying the triangles $\mathbf{OQP_f}$ and $\mathbf{OQP_r}$ in Figure 2, it results the equations:

$$w_b = QP_f + QP_r \qquad OQ = R\cos\beta$$
$$\tan(\delta_f - \alpha_f) = \frac{QP_f}{OQ} \qquad \tan(\alpha_r) = \frac{QP_r}{OQ} \qquad (8)$$

and then, considering small side slip angle β for the vehicle, it obtains:

$$QP_f + QP_r = OQ\left(\tan(\delta_f - \alpha_f) + \tan(\alpha_r)\right)$$
$$OQ \approxeq R \qquad \delta_f \approxeq \frac{w_b}{R} + \alpha_f - \alpha_r \qquad (9)$$

Presented in the technical literature ([3], [6], [8], [9], [13], [15]), the last equation shows that, excepting the cases at which the tires' lateral forces are negligible (movement with small speed on horizontal grounds), the actual mean steering angle δ_f imposed by the driver differs on the Ackermann mean steering angle

$$\delta_{fA} = \frac{w_b}{R} \qquad (10)$$

That means an ideal steering linkage should have a variable correlating function. But, for the moment, that is not implemented (yet) into practice. In this situation it is necessary to optimize the steering trapezoidal mechanism to the most wanted driving conditions.

6. OBJECTIVE FUNCTION OF THE STEERING LINKAGE IN REAL DRIVING CONDITIONS

In a series of papers (as [12]), the authors of this work presented some possibilities to process experimental data, mainly obtained from GPS technology, in order to study the vehicle dynamics. These procedures are also suitable to experimentally indicate the main driving parameters useful for steering analysis.

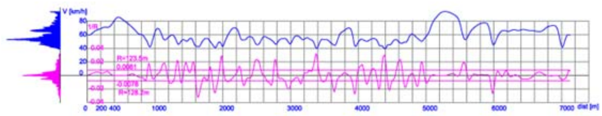

Fig. 3. Speed v and trajectory curvature ($1/R$) vs. traveled distance on a mountain road

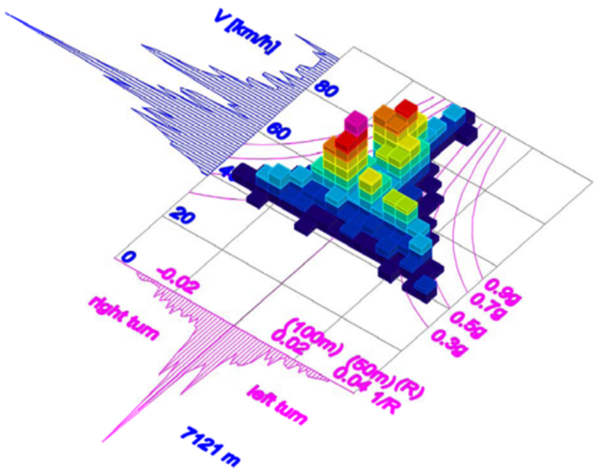

Fig. 4. Mono- and bi-parametric probability density functions for speed v and trajectory curvature ($1/R$) vs. distance – mountain road

In the Figure 3 are presented some evolutions of a car's speed and trajectory curvature ($1/R$, the inverse of the turning radius) as functions of the travelling distance. It was recorded the driving condition on 7.1 km on a mountain uncongested road. The speed was directly measured while the curvature was derived on the base of vehicle's successive positions [12].

Processing these records, it was obtained the mono-parametric and bi-parametric probability density functions for the speed and trajectory curvature, as is presented in Figure 4. The representation height and the colors indicates which pairs speed – curvature (v, $1/R$) or speed – turning radius (v, R) presents a bigger probability to be found on the road.

Starting from the series of values (v, $1/R$) it will be obtained the centripetal acceleration a_{cp} of the vehicle

$$a_{cp} = \frac{v^2}{R} \qquad (11)$$

With some measurements (or assumptions) about the position of the vehicle's center of gravity (coordinates w_f and w_r in Figures 1 and 2) and about the lateral stiffness of the front and rear tires (C_f and C_r), it is possible to obtain firstly the ratio (α_f/α_r) of the lateral slip angles of the tires and then the estimations of these values (α_f and α_r, Figure 2).

Now, with the equation 9 it can be determined the mean steering angle δ_f of the front wheels. Finally, using the dimensions and the points **Q**, **O**, $\mathbf{P_f}$, $\mathbf{P_{fi}}$ and $\mathbf{P_{fo}}$ presented in Figure 2, it can be estimated the ideal values of the steering angles δ_{fi} and δ_{fo} that correspond to the driving conditions imposed by the pair speed – radius (v, R).

7. OPTIMIZATION OF THE STEERING LINKAGE

Choosing the most probable or the most demanding driving situations, depending on the motor vehicle destination, an objective function can be founded. Generally, this is different if compared with Ackermann condition.

Supposing known the objective function, there are many methods proposed in the literature for the optimization of the steering linkage [1], [2], [4], [6], [10]. Some are based on empiric data and requirements ([1], [2], [6]), while the others ([4], [10]) are using new and performant algorithms, able to find global or local maximums for the optimization function, based on the error between the objective and actual correlation functions of the steering linkage.

The authors propose that the optimization function to be the sum of the weighted error squares of a cloud of points evenly disposed on the curvature (1/R) axis as was used in [11]. The weighting function can be founded experimentally, as it is presented with magenta color in the lower-left side of the Figure 4.

8. CONCLUSION

The vehicles with rigid steering axles are fitted generally with very simple steering linkage, which can be mounted on the axle beam. The robustness, reliability and cost make the four-bar trapezoidal mechanisms to be preferred for the correlation of the steering angles.

Despite its constructive simplicity, an optimized design of the steering linkage presents some challenges for the engineers.

In the present work, there were presented some of the packaging restrictions and it was indicated a possible way to define (starting from experimental data obtained with GPS systems) the objective (wanted) correlation function of the steering angles.

The suggested way for the dimensional optimization is based on method of the least squares applied for the weighted errors of the steering angles.

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