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MATERIALE, ELEMENTE ŞI STRUCTURI COMPOZITE PENTRU CONSTRUCŢII

PREZENT ŞI PERSPECTIVE

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Contribution at designing of flexed pre-stressed elements

Contribuţii privind dimensionarea elementelor precomprimate încovoiate

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ABSTRACT

At present there are numerous studies regarding the elaboration of some optimal elements and structures of steel concrete, resulting in an important saving of material.

The researches made in this domain for the pre-stressed concrete are not so numerous. The situation may be explained, partially, by the dificulties generated by the great number and diversity of the parameters thet must be taken into consideration in defining the section of a pre-stressed element.

1. INTRODUCTION

At present there numerous studies regarding the elaboration of some optimal elements and structure of steel concrete, resulting in an important saving of material.

The researches made in this respect for the pre-sterssed concrete are not so numerous. The situation may be explained, partially, by the dificulties generated by the great number and diversity of the parameters that must be taken into consideration in defining the section of a pre-sterssed element.

The object of this paper is the plain girder, simply supported, with a section in the form of T,I or TT, having an rectilinear active reinforcement.

The aim is the determination of the optimal profile of the elements in the conditions of a minimum cost price.

The theoretical study trys to establish the most economical solutions for a range of constructional system met in practice, as follows: roof elements in T and TT (stright or curved), main girder (longitudinal or transversal to the structure), the girders for supporting rolling cranes etc.

2. THE OPTIMIZING OF THE DEPTH OF THE SECTION

We propose to determine the depth "hopt" for which the element of pre-stressed concrete presents a minimum cost price. Choosing this criterion of optimizing there will be taken in to account the components of the cost price, that refer to the consumption of concrete, reinforcement and concrete form.

The expenses refering to the manual labour and equipment are included in the total cost of the materials.

According to the usance of drawing up of the economic documentation the value of the concrete form is included in the cost of concrete.

For a certain section of a girder in T-form (Fig.1), having a rectilinear active reinforcement, situated at the interior part, the cost price per unit length of girder can be expressed as follow:

$$y_i = \left[b \cdot (h_0 + a) + (b_p - b) \cdot h_p\right] \cdot C_b + \gamma_a \cdot A_p \cdot C_{ap} + \frac{P_a}{l}, \quad (1)$$

where: b – the width of the girder, [m], h_0 – the useful depth, [m], a – the distance from the gravity center of the reinforcement, A_p, to the inferior fibre, [m], b_p – the width of the flange, [m], b- the width of the rib, reinforcement, [m^2], C_b – the unit price of the concrete, [lei/kg], P_a – the whole price of the non pre-stressed reinforcement, [lei/kg].

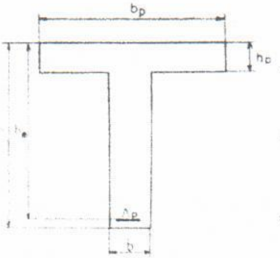

Fig. 1

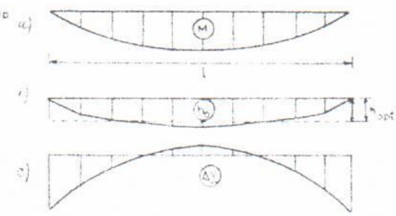

Fig. 2

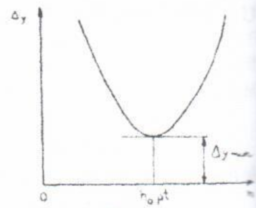

Fig. 3

The calculus is made, practically, by dividing the element in a number of *n* intervalls, all of them having the same length ΔX. Between two neighbouring portiong, *l* it is considered that the value of the bending moment is constant.

A certain economical depth of the section (h_e) that results from the analisys of the minimum of the cost price function corresponding to this value. Owing to the moment diagram which varies in function of the external loading scheme (Fig.2a), the economical girder will have a similar profile (Fig.2b).

Usually, the girdes are made with a constant depth or with a gentle slope, which results from the functional conditions.

Consequently, the function of the cost price varies with the quantity ΔY_i comparing with the most advatageous theoretical form. In this case, the depth "hopt", for which the total increasing of the cost price function (ΔY) will be minimum for the whole element, (Fig.3), correspond to the optimal form. The variation (ΔY_i) of the function (1) is obtain replacing h_0 by hopt h_e and A_p by ΔA_{pi}, this being the difference of reinforcement in the section *l* which is observed when we take into account the "hopt" instead of h_e.

If the interval ΔX is small enough, there may be considered that the variation of the cost price function is linear on the respective interval and it can be evaluated, aproximately, by the rule of the trapeze.

3. THE DETERMINATION OF THE ECONOMICAL DEPTH

The function of the cost price (1) has the following form:

$$Y_1 = f_1(h_0, A_p), \quad (2)$$

$$A_p = f_2(x), \quad (3)$$

$$x = f_x(h_0). \quad (4)$$

For a given section we can write the wellknown equations of mechanical equilibrium. The equation of the projection of the forces gives us the possibility of determination of the reinforcement quantity (A_p) in function of the height compressed zone (X) and the equation of the equilibrium of moments gives us the possibility to establish the value of X in function of h_0.

For the section in T-form, we must take into account supplementary the problem estimation of the position of the neutral axis.

3.1. The neutral Axis Passes the Plate ($X< h_p$)

Supposing that the coefficient of the work conditions, m_p,, is known, we can obtain from the equation of projection:

$$A_p = b[1+0,8(\beta_s - 1)]x_p \frac{R_i}{\sigma_l}, \tag{5}$$

Where: $\beta_s = b_p / b$ and $\sigma_l = m_p R_p$, R_p being the value of the calculus resistance of the pre-stressed reinforcement. If we not:

$$\eta = 1 + 0,8(\beta_s - 1) \text{ and } B = \left[b(h_0 + a) + (b_p - b)h_p\right]C_b + \frac{P_a}{l}$$

the relation (5) becomes:

$$A_p = \eta b x_p \frac{R_i}{\sigma_l} \tag{5'}$$

and the function of the price cost take the form:

$$Y_l^{(1)} = B + \eta b x_p \gamma_a \frac{R_i}{\sigma_l} \tag{6}$$

From the equation of the equilibrium of moment we can determine the height of the compressed zone:

$$x_p^{(1)} = h_0\left[1 - \sqrt{1 - \frac{M}{0,4\eta b h_0^2 R_i}}\right] \tag{7}$$

The domain of variation of the depth (h_0) is the interval $h_0 \in \left(h_{0\lim}^{(1)}, +\infty\right)$, where $h_{0\lim}^{(1)}$ has the following value:

$$h_{0\lim}^{(1)} = \sqrt{\frac{M}{0,4\eta b R_i}} \tag{8}$$

The economical height can be obtained from the condition of minimizing of the cost price function, and it has the following expression:

$$h_{0e}^{(1)} = \sqrt{\frac{M}{0,4\eta b R_i (1-\theta_1^2)}}, \tag{9}$$

where:

$$\theta_1 = \frac{R_i / \sigma_l}{\alpha_i + R_i / \sigma_l} \text{ and } \alpha_1 = \frac{C_b}{\eta \gamma_a C_{ap}}$$

Using the relation (7) we can estimate the precise value of the height of the compressed zone. If there results that $X_p<h_p$, the used relations are correct and we can determine further the quantity of reinforcement (A_p).

With the relation:

$$m_p = 1 - k\frac{A_p R_p}{\eta b h_0 R_i} \tag{10}$$

we can verify if the calculated coefficient m_p is the same with the one initially imposed. If these values are different the calculus must be done once again, taking into consideration the real value of m_p. This operation is repeated up to the difference between two successive values is small enough.

If in the compressed zone of the concrete is a known quantity of non-stressed reinforcement ($A'a$) which is introduced for constructive reasons, the previous relation have the following form:

a) The function of the cost price becomes:

$$y_i^{(2)} = B + \gamma_a \left[\eta b x_p \frac{R_i}{\sigma_l} C_{ap} + A_a^{'} \left(\frac{R_a}{\sigma_l} C_{ap} + C_{aa} \right) \right], \tag{11}$$

where: R_a – the calculated resistance of the non pre-stressed reinforcement, C_{aa} – the unit price of the non pre-stressed reinforcement, [lei/kg].

b) The height of the compressed zone can be determined with the relation:

$$x_p^{(2)} = h_0 \left[1 - \sqrt{1 - \frac{M - 0{,}8A_a^{'} R_a (h_0 - a^{'})}{0{,}4\eta b h_0^2 R_i}} \right] \tag{12}$$

c) The useful limit height is given by the expression:

$$h_{0\lim}^{(2)} = -\frac{A_a^{'} R_a}{\eta b R_i} + \sqrt{\frac{A_a^{'2} R_a^2}{\eta^2 b^2 R_i^2} + \frac{M + 0{,}8A_a^{'} R_a a^{'}}{0{,}4\eta b R_i}} \tag{13}$$

d) The economical height can be calculated with the expression:

$$h_{0e}^{(2)} = -\frac{A_a^{'} R_a}{\eta b R_i} + \sqrt{\frac{A_a^{'2} R_a^2}{\eta^2 b^2 R_i^2} + \frac{\eta b R_i M + 0{,}4A_a^{'} R_a (2\eta b R_i a^{'} + A_a^{'} R_a \theta_1^2)}{0{,}4\eta^2 b^2 R_i^2 (1 - \theta_1^2)}}, \tag{14}$$

a' being the distance betwwen the center of gravity of the non pre- stressed reinforcement and the superior fibre of the section.

3.2. The Neutral Axis Passes through the Rib ($X>H_P$)

From the equation of projection we have:

$$A_p = b(x + \beta h_p) \frac{R_i}{\sigma_l} \tag{15}$$

The function of the cost price in this case will be:

$$y_i^{(3)} = B + \gamma_a b C_{ap} \frac{R_i}{\sigma_l} (x + \beta h_p) \tag{16}$$

The equation of the equilibrium of moments gives us the possibility to determine the height of the compressed zone:

$$x^{(3)} = h_0 \left(1 - \sqrt{1 - \frac{M}{0{,}4 b h_0^2 R_i} + \frac{2\beta h_p}{h_0} - \frac{\beta h_p^2}{h_0^2}} \right) \tag{17}$$

The useful limit height has the following expression:

$$h_{0\lim}^{(3)} = -\beta h_p + \sqrt{\beta^2 h_p^2 + \frac{M + 0{,}4\beta b h_p^2 R_i}{0{,}4 b R_i}} \tag{18}$$

The domain of variation of the depth (h_0) is $h_0 \in \left(h_{0\lim}^{(3)}, +\infty \right)$.

The economical height, established on the basis of the same principle of minimizing of the cost price, can be determined with the relation:

$$h_{0e}^{(3)} = -\beta h_p + \sqrt{\beta^2 h_p^2 + \frac{M + 0{,}4\beta b h_p^2 R_i (1 + \beta\theta_2^2)}{0{,}4 b R_i (1 - \theta_2^2)}}, \tag{19}$$

where: $\theta_2 = (R_i / \sigma_l)/(\alpha_2 + R_i / \sigma_l)$ and $\alpha_2 = C_b / \gamma_a C_{ap}$.

Further, there can be calculated the height of the compressed zone and the corresponding quantity of the pre-stressed reinforcement.

Then we must verify and correct, according to the method previously described, the coefficient m_p, which is in this case the following:

$$m_p = \frac{1-k\alpha_p}{2} + \sqrt{\left(\frac{1-k\alpha_p}{2}\right)^2 + 0,8k\frac{b_p-b}{b}\cdot\frac{h_p}{h_0}}, \qquad (20)$$

where K and α_p have the significance indicated in the Romanian standard STAS 10 107-75.

For practical reasons there is not possible to vary the quantity of the reinforcement along the girder in accordance with the diagram of the bending moment.

In order to obtain a rational dimensioning, it is indicated to use follwing process:

We determine the economical height and the corresponding area of reinforcement in the most stressed section. Keeping a constant quantity of reinforcement along the whole gieder we calculate the useful height in different points using the relation:

$$h_0 = \frac{M + 0,4bR_i(x^2 + \beta h_p^2)}{0,8bR_i(x + \beta h_p)}, \qquad (21)$$

where $x = \frac{A_p}{b}\cdot\frac{\sigma_l}{R_i} - \beta h_p$.

In the case when in the zone of compressed concrete there is a non pre-stressed reinforcement (A'_a) introduced for constructive reasons, the previous relations are modified as follows:

a) The function of the cost price has the expression:

$$y_i^{(4)} = B + \gamma_a\left[b(x+\beta h_p)\cdot\frac{R_i}{\sigma_l}C_{ap} + A'_a\left(\frac{R_a}{\sigma_l}C_{ap} + C_{aa}\right)\right] \qquad (22)$$

b) The height of the compressed zone is determined with:

$$x^{(4)} = h_0\left(1 - \sqrt{1 - \frac{M}{0,4bh_0^2R_i} + \frac{2\beta h_p}{h_0} + \frac{2A'_aR_a}{bh_0Ri} - \frac{\beta h_p^2}{h_0^2} - \frac{2A'_aR_aa'}{bh_0^2R_i}}\right) \qquad (23)$$

c) The useful limit height is calculated with: (24)

$$h_{0\lim}^{(4)} = -\frac{\beta bh_pR_i + A'_aR_a}{bR_i} + \sqrt{\left(\frac{\beta bh_pR_i + A'_aR_a}{b^2R_i^2}\right)^2 + \frac{M + 0,4\beta bh_p^2R_i + 0,8A'_aR_aa'}{0,4bR_i}} \qquad (24)$$

d) The economical height has the expression: (25)

$$h_{0e}^{(4)} = -\left(\beta h_p + \frac{A'_aR_a}{bR_i}\right) + \left[\left(\frac{A'_aR_a + \beta h_pbR_i}{b^2R_i^2}\right)^2 + \frac{bR_iM + 0,4\beta b^2h_p^2R_i^2(1+\beta\theta_2^2)}{0,4b^2R_i^2(1-\theta_2^2)} + \right.$$
$$\left. + \frac{0,4A'_aR_a(2b\eta R_aa' + A'_aR_a\theta_2^2) + 1,6\beta h_pR_iA'_aR_a\theta_2^2}{0,4b^2R_i^2(1-\theta_2^2)}\right]^{1/2} \qquad (25)$$

4. CONCLUSIONS

By using the proposed method there can be determined the optimal and economical profiles of the pre-compressed girders, simply supported, with a rectilinear active reinforcement, in different schemes of loading.

The study can be extended to different forms of section.

By using the proposed solutions there is eliminated the calculus by tests, which is very used in present.

The fact that the optimal solution is in function of a certain economical relation which dictates the policy of prices, it can not be a limit for extending of the method.

The great volume of calculations that must be remade fter the continuous modifications of the prices can made by means of the data processing computers.

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