ESO ALGORITHM IN OPTIMIZATION CONTINUOUS BEAM WITH MULTIPLE SPANS

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Abstract

In last thirty years numerical analysis for structure of beams is one of main method in design structures of civil constructions like bridges. In this paper authors present a program which can compute internal force (bending moment shear force), displacements and rotation in every point of continuous beam and make an optimization with fully stressed method (ESO).

Keywords: Clayperon theorem, evolutionary structural optimization

1.Introduction

Emile Clapeyron derived the equation of three-moment first time in 1857 using the differential equations of beam bending. In fig.1 consider a continuous beam over several supports that carrying arbitrary loads (in this case take only distributed loads).

Using the moment-area theorem, we will analyze two adjoining spans of this beam to find the relationship between the internal moments of bending at each support and the loads applied to the beam. Applying the principle of superposition to this two-span segment, we can separate the moments caused by applied loads from the internal moments at the supports. The two-span segment can be represented by simply-supported spans carrying the internal moments M_S, M_C, and M_D, (fig.2) .In (fig.2) we can observe that positive moments create positive curvature in the beam and the internal moments M_S M_C, M_D are drawn in the positive directions. The areas under the moment diagrams due to the applied loads in the simply-supported spans are A_s and A_d; d_s represent the distance from the left support to centroid of area A_s, and d_d represent distance from the right support to the centroid of area A_d as shown in (fig.2). The moment diagrams due to unknown M_S, M_C, and M_D, are triangular, as shown in (fig.2).

From fig.2 we can observe that the elastic curve is a continuous beam, thus the rotation of the beam at the center support, is continuous across support. With other words angle of rotation to the left center is the same as angle of rotation to the right of center. This continuity condition may be expressed as:

$$\varphi_s = -\varphi_d \,, \qquad (1)$$

where

φ_s is the left angle of rotation at the center support ;

φ_d is the right angle of rotation at the center support.

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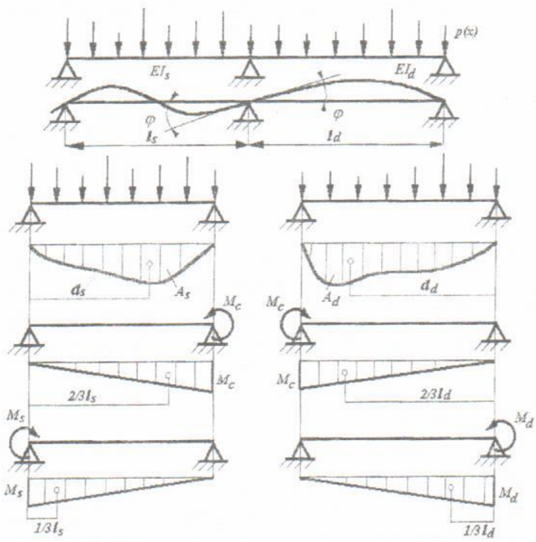


Fig.2 Diagram of bending moment for each span

Using the second moment-area theorem, and assuming, that the flexural rigidity (E is constant within each span, we can find the terms φ_s and φ_d in terms of unknown Moments, M_S, M_C and M_D and the known applied loads:

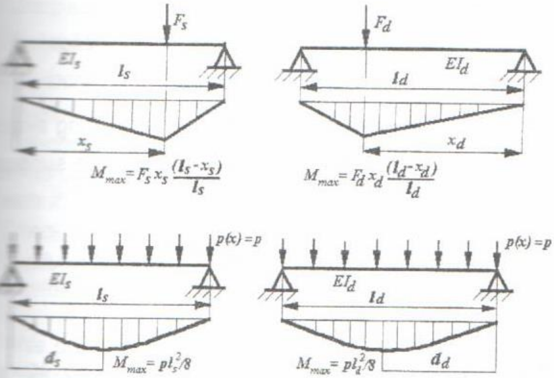
$$\varphi_s = \frac{1}{EI_s l_s}\left(d_s A_s + \frac{2}{3} l_s \frac{1}{2} M_c l_s + \frac{1}{3} l_s \frac{1}{2} M_s l_s\right); \varphi_d = \frac{1}{EI_d l_d}\left(d_d A_d + \frac{2}{3} l_d \frac{1}{2} M_c l_d + \frac{1}{3} l_d \frac{1}{2} M_s l_s\right) \quad (2)$$

Substituting relation (2) into equation (1) and re-arranging terms leads to the three-moment equation.

$$\frac{l_s M_s}{EI_s} + 2\left(\frac{l_s}{EI_s} + \frac{l_d}{EI_d}\right) M_c + \frac{l_d M_d}{EI_d} = -\frac{6 d_s A_s}{EI_s l_s} - \frac{6 d_d A_d}{EI_d l_d}; \quad (3)$$

If flexural rigidity for each span is equal ($EI_s = EI_s$), the three-moment equation became independent of EI. For application the three-moment equation numerically, the lengths, moment of inertia, and applied loads must specified for each span. Two commonly applied loads are, point loads and uniformly distributed loads. In figure 3 is shown expression of the right hand side in relation (3) for point loads acting on each span, and distributed loads on the left and right spans.

If the continuous beam have $n+1$ supports with n span, to find internal forces (bending moment and shear force), the three–moment equation is applied to $n-1$ adjacent pairs of spans. In matrix formulation for a continuous beam with n supports and $n-1$ span the three-moment equation became:



$$-\frac{6d_sA_s}{EI_sl_s}-\frac{6d_dA_d}{EI_dl_d}=-\frac{F_sx_s}{EI_s}(l_s-x_s)-\frac{F_dx_d}{EI_d}(l_d-x_d);\ (4.a)$$

$$-\frac{6d_sA_s}{EI_sl_s}-\frac{6d_dA_d}{EI_dl_d}==-\frac{p_sl_s^3}{4EI_s}-\frac{p_dl_d^3}{4EI_d};(4.b)$$

Fig.3 Diagram of bending moment for concentrated load and distributed loads

$$\begin{bmatrix} \frac{l_1}{EI_1} & 2\left(\frac{l_1}{EI_1}+\frac{l_2}{EI_2}\right) & \frac{l_2}{EI_2} & \cdots & 0 & 0 \\ 0 & \frac{l_2}{EI_2} & 2\left(\frac{l_2}{EI_2}+\frac{l_3}{EI_3}\right) & \frac{l_3}{EI_3} & \cdots & 0 \\ . & . & . & . & . & . \\ 0 & \cdots & \frac{l_j}{EI_j} & 2\left(\frac{l_j}{EI_j}+\frac{l_{j+1}}{EI_{j+1}}\right) & \frac{l_{j+1}}{EI_{j+1}}\ \cdots & 0 \\ . & . & . & . & . & . \\ 0 & 0 & \cdots & \frac{l_{n-1}}{EI_{n-1}} & 2\left(\frac{l_{n-1}}{EI_{n-1}}+\frac{l_n}{EI_n}\right) & \frac{l_n}{EI_n} \end{bmatrix} \begin{Bmatrix} M_0 \\ M_1 \\ . \\ M_j \\ . \\ M_n \end{Bmatrix} =$$

$$\begin{Bmatrix} -\frac{p_1l_1^3}{4EI_1}-\frac{p_2l_2^3}{4EI_2} & -\frac{p_1l_1^3}{4EI_1}-\frac{p_2l_2^3}{4EI_2} & . & -\frac{p_jl_j^3}{4EI_j}-\frac{p_{j+1}l_{j+1}^3}{4EI_{j+1}} & . & -\frac{p_{n-1}l_{n-1}^3}{4EI_{n-1}}-\frac{p_nl_n^3}{4EI_n} \end{Bmatrix}^T \quad (5)$$

Applying the end-moment conditions $M_0=0$ $M_n=0$ in relation (5) we obtain the final matrix form for three-moment equations. Three-moment equation for a continuous beam over *n* supports with *n-1* span with distributed loads obtained can be written in matrix form $[C]\{M\}=\{D\}$. Where, *[C]* is matrix of flexibility and is symmetric; *{M}* vector of bending moment: {L} is vector of distributed load. If a numbering convention is adopted in which support *j* lies between span *j* and span *j+1*, the three non-zero elements in row j of matrix *[C]* are given by:

$$c_{j,j-1}=\frac{l_j}{EI_j};c_{j,j}=2\left(\frac{l_j}{EI_j}+\frac{l_{j+1}}{EI_{j+1}}\right);c_{j,j+1}=\frac{l_{j+1}}{EI_{j+1}}\ . \quad (6)$$

Row *j* of vector *{D}* for the case of uniformly distributed loads (4.a) and the case of point load (4.b):

$$D_j = -\frac{p_j l_j^3}{4EI_j} - \frac{p_{j+1} l_{j+1}^3}{4EI_{j+1}}; (7.a)\ D_j = -\frac{F_j x_j (l_j - x_j)}{EI_j} - \frac{F_{j+1} x_{j+1} (l_j - x_{j+1})}{EI_{j+1}} .(7.b)$$

The internal bending moments at the supports are computed by solving system (5) $\{M\} = [C]^{-1}\{D\}$. Once the internal moments are found, the reactions at the supports can be computed from the static equilibrium (fig.4):

$$R_j = R_{j1} + R_{j2} = \frac{1}{2} p_j l_j + \frac{1}{2} p_{j+1} l_{j+1} - \frac{M_j}{l_j} - \frac{M_j}{l_{j+1}} - \frac{M_j}{l_{j+1}} - \frac{M_{j+1}}{l_{j+1}}; \quad (8)$$

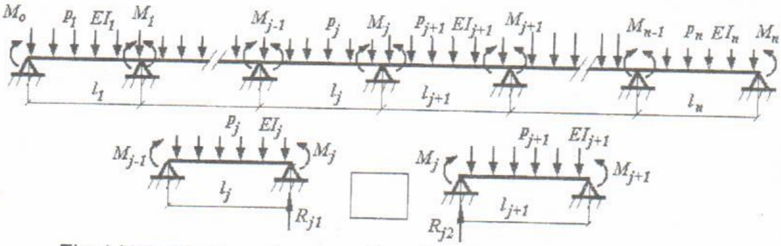


Fig.4 Internal shear force and bending moment at end of span

After we having computed the reactions and internal moments, we can find the shear force and moment diagrams from equilibrium equations. For example, consider span *j* between support *j* and support *j+1* the internal shear force at support *j* in span *j* are calculated with relation (9.a) and the internal force at support *j + 1* in span *j* is calculated with relation (9.b).

$$T_{j,j} = \frac{M_j - M_{j+1}}{l_j} - \frac{p_j l_j}{2}; (9.a)\ T_{j,j} = \frac{M_j - M_{j+1}}{l_j} + \frac{p_j l_j}{2}; (9.b)$$

After we calculate bending moment with Clayperon theorem for ends of each span calculate maximum bending moment along span and for these moments we could calculate stresses. In our analysis section is rectangular with height h and depth b. Maximum stress for each span is:

$$\sigma = \frac{6M_{i\max}}{bh^2}.$$

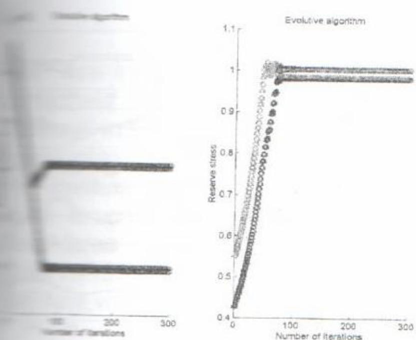
2.ESO algorithm for structure with rigid jointed

- *1 An initial structure is defined including loads and support conditions;*
- *2 Calculate maximum bending moment at end for each element of structure with Calyperon theorem;*
- *3 Calculate stress for maximum bending moment* $\sigma = \frac{6M_{i\max}}{bh^2}$;

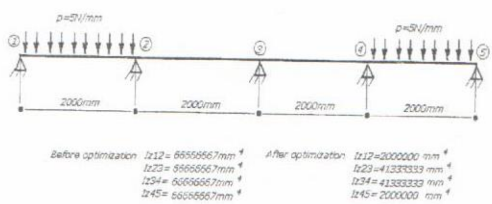
- Compare stress with target value σ_t;
- If absolute stress ($\sigma > \sigma_t$) is above target, increase height of section (h=h+inc) by a small increment; if absolute stress is below target($\sigma < \sigma_t$), decrease height of section (h=h-inc) by a small increment;
- If geometric property (height of section) diminished to zero $h \cong 0$ remove element from structure or if property reached some prescribed lower or upper bound then freeze it;
- Check to see if previously frozen member sizes need unfrozen;
- If volume change per iteration is within a small convergence tolerance or a prescribed iteration limit has been reached , then stop and print results, in not go to 2.

We present now two examples about how to use evolutionary structural optimization in stressed structures.

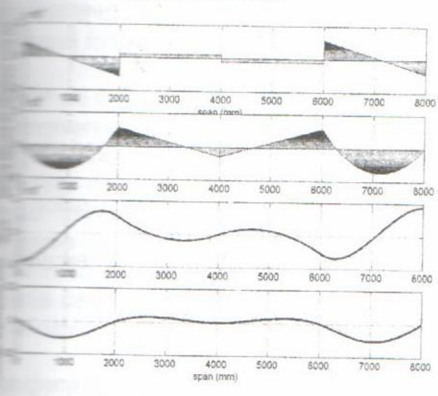
Example 1

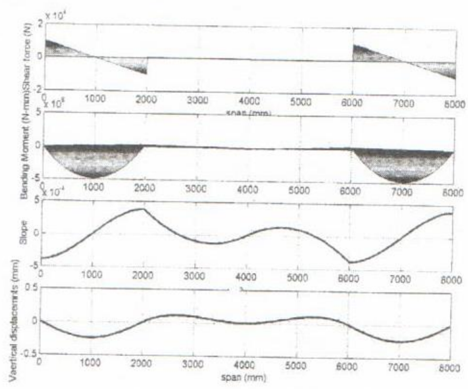


Process of iteration



Load and boundary conditions for structure

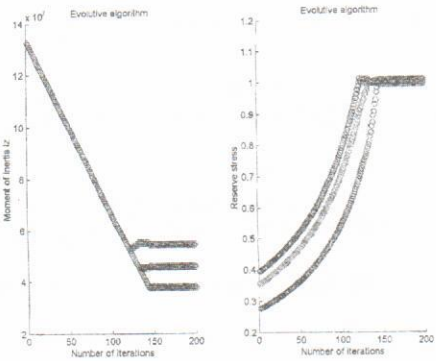




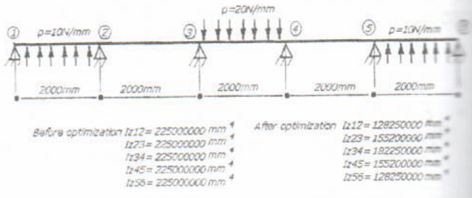
Before optimization After optimization(300 iterations-stress target=20Mpa)

Fig.5 Diagram of internal shear force and bending moment , angle of rotation vertical displacements

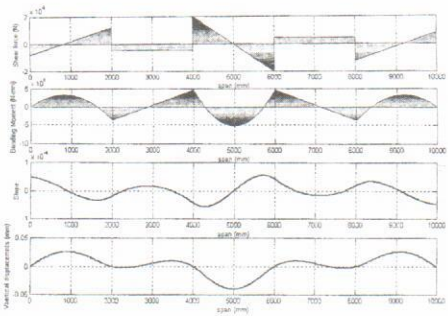
Example 2

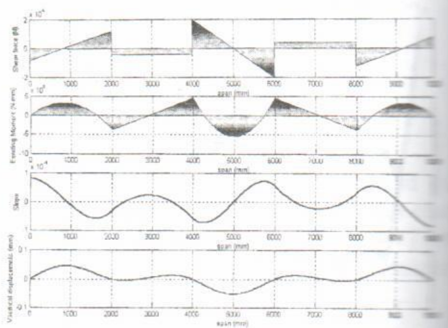


Process of iteration



Load and boundary conditions for structure





Before optimization After optimization(200 iterations-stress target=20MPa

Fig.6 Diagram of internal shear force and bending moment , angle of rotation vertical displacements

3.Conclusions

- Optimization with evolutionary structural optimization represents a good choice for complex structures with a high grad of redundancy.
- Method of fully stressed structure can be used for topologically optimization when initial structure is a layout.
- Evolutionary structural optimization is an iterative method that has an excellent convergence.

4.References

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