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THE DYNAMIC ANALYSIS OF JOINTED BAR STRUCTURES WITH THE SIMULINK PROGRAM

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Abstract: This article describes a simple and efficient method for the dynamic analysis of spatial jointed bars by means of the Matlab and Simulink program package. The masses and rigidity matrices are created and assembled in Matlab and the calculation of the dynamic response is done in Simulink. Because the Simulink package includes integrating procedures of the differential equations of motion, the response of the motion, velocity and acceleration in each node and for each degree of freedom may each be calculated by means of this package.

Key words: finite element method, the dynamic response of structures.

1. GENERAL ASPECTS

The equation of motion of a three-dimensional truss structure yields after the mass, stiffness and damping matrices of the bar elements have been assembled in the global mass, stiffness and damping matrices.

The equation of motion for a spatial truss structure has the following matricial expression:

$$[M]\{\ddot{D}\}+[C]\{\dot{D}\}+[K]\{D\}=\{F\}\,. \tag{1}$$

In order to determine the response in displacement, velocity and acceleration for a bar structure described by (1) equation, boundary conditions and initial conditions must be applied. The boundary conditions are being applied by imposing known displacements for the structure. Initial conditions assume knowing the velocity and displacements vector in each node and on each degree of freedom, at a t=0 moment of time.

$$\{\dot{D}\}=\{\dot{D}_0\};\{D\}=\{D_0\}\text{ at } t=0. \tag{2}$$

Usually, before the determination of the dynamic response of the bar structure, a modal analysis is being performed in order to determine natural angular frequencies and vectors. The dynamical characteristics of the bar structures are being yielded by solving the characteristic equation:

$$([K]-\omega_i^2[M])\{D\}=\{0\}\,,\quad i=1...n \tag{3}$$

where,

[K] -global stiffness matrix ;
[M] -concentrated mass matrix;
{D}-nodal displacement vector;
ω_i - natural angular frequencies.

The concentrated mass matrix for a bar element yields from equally dividing the mass in the two nodes. The concentrated mass matrix in the global reference system is:

$$[M]^{(e)} = \frac{\rho Al}{2}\begin{bmatrix} l^2 & ml & nl & 0 & 0 & 0 \\ ml & m^2 & mn & 0 & 0 & 0 \\ nl & nm & n^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & l^2 & lm & nl \\ 0 & 0 & 0 & ml & m^2 & mn \\ 0 & 0 & 0 & nl & nm & n^2 \end{bmatrix}. \tag{4}$$

In order to determine the distributed mass matrix, form functions are considered, which have been used in the modeling of the displacement field. The distributed mass matrix is being determined with the following expression:

$$[M]^{(e)} = \frac{\rho Al}{6}\begin{bmatrix} 2l^2 & 2ml & 2nl & l^2 & lm & nl \\ 2ml & 2m^2 & 2mn & ml & m^2 & mn \\ 2nl & 2nm & 2n^2 & nl & nm & n^2 \\ l^2 & lm & nl & 2l^2 & 2lm & 2nl \\ ml & m^2 & mn & 2ml & 2m^2 & 2mn \\ nl & nm & n^2 & 2nl & 2nm & 2n^2 \end{bmatrix}. \tag{5}$$

In order to calculate the dynamic response, representation in state space is being used, which contains the positions and the velocities corresponding to the structure's degrees of freedom.

In space state, the state vector is being defined, which contains the positions and the velocities corresponding to the structure's degrees of freedom, as following:

$$\begin{gathered} \{x_1\, x_2\, x_3 .. x_{2n}\} = \{y_1\, y_2\, y_3 .. y_n\, \dot{y}_1\, \dot{y}_2\, \dot{y}_3 .. \dot{y}_n\}; \\ \{\dot{y}\} = \{\dot{D}\}; \\ \{\ddot{y}\} = \{\ddot{D}\}. \end{gathered} \tag{6}$$

If the matricial equation (1) is being multiplied at left with the $[M]^{-1}$ matrix, it yields:

$$\{\ddot{D}\} = -[M]^{-1}[C]\{\dot{D}\} - [M]^{-1}[K]\{D\} + [M]^{-1}\{F\} \tag{7}$$

From the (6) and (7) equations, it yields the following system of equations:

$$\{\dot{x}\} = \begin{Bmatrix} \dot{y} \\ \ddot{y} \end{Bmatrix} = \begin{bmatrix} [0] & [I] \\ [M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix}\begin{Bmatrix} y \\ \dot{y} \end{Bmatrix} + \begin{Bmatrix} 0 \\ [M]^{-1}\{F\} \end{Bmatrix}. \tag{8}$$

In order to determine the structure's dynamic response, the mass, stiffness and damping matrices are calculated with the Matlab program and with the block scheme in fig.2, the displacement, velocity and acceleration response are being determined. The response in velocity and acceleration for the structure in fig.1, is being determined by integrating the motion equation of the structure represented in space state - with (7) equation. The dynamic response of the structure is being represented in fig.1. In fig. 3, 4 and 5 the displacement, velocity and acceleration are represented for each node and for each dynamic degree of freedom.

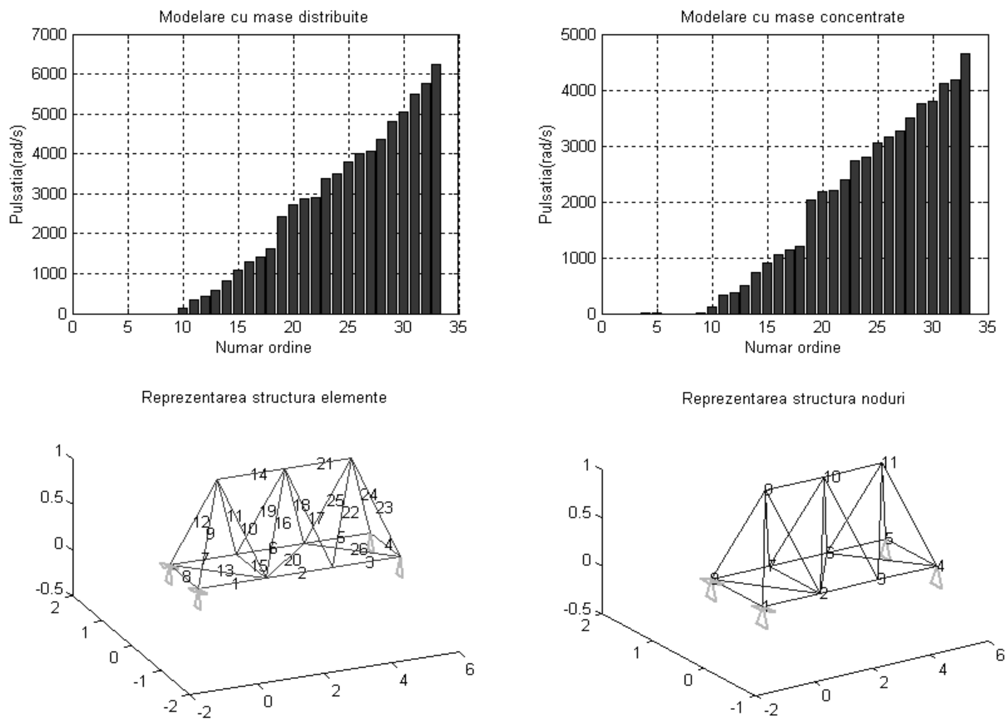

Fig.1 Representation of angular frequencies for modeling with concentrated mass and distributed mass

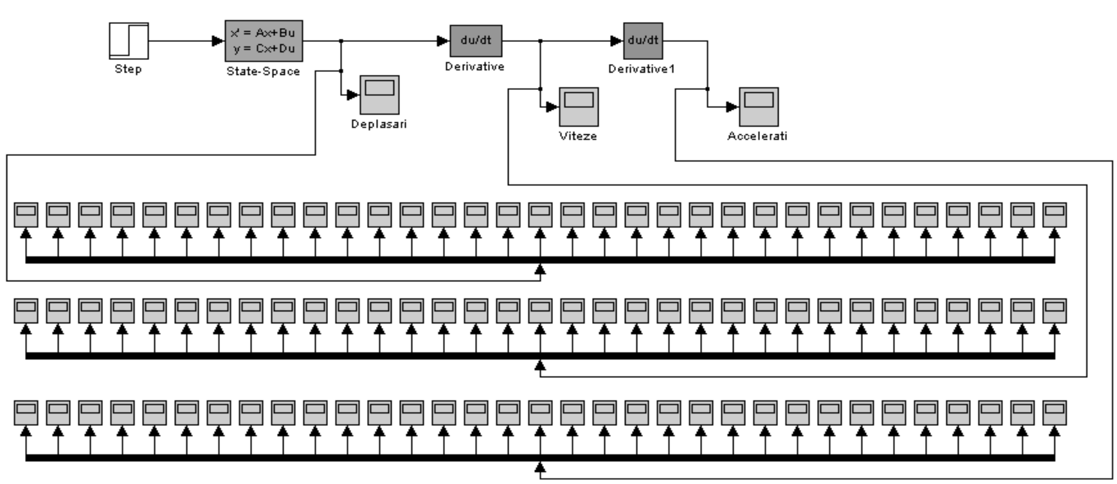

Fig.2 Block scheme for dynamic analysis – Simulink

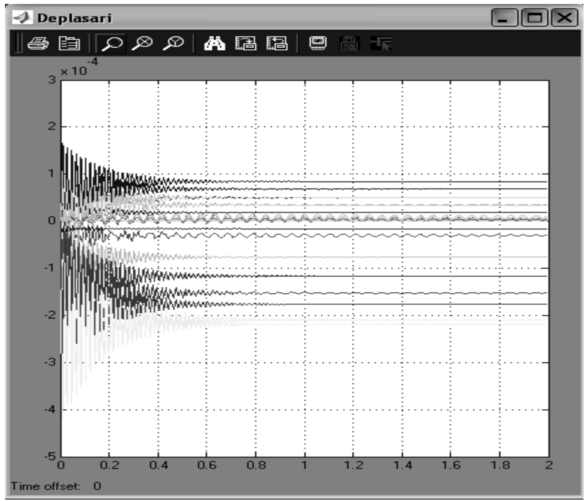

Fig.3 Nodal displacements

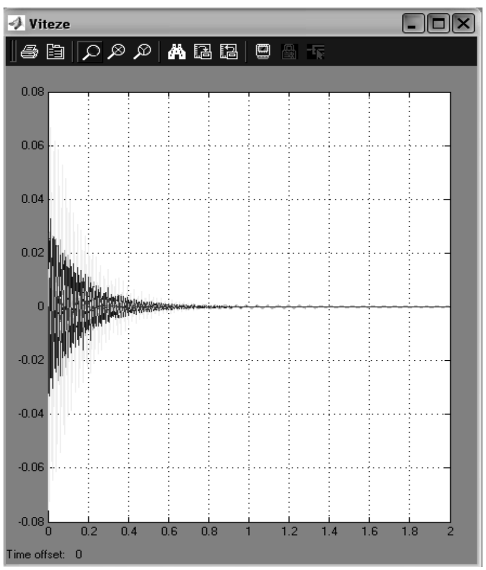

Fig.4 Nodal velocity

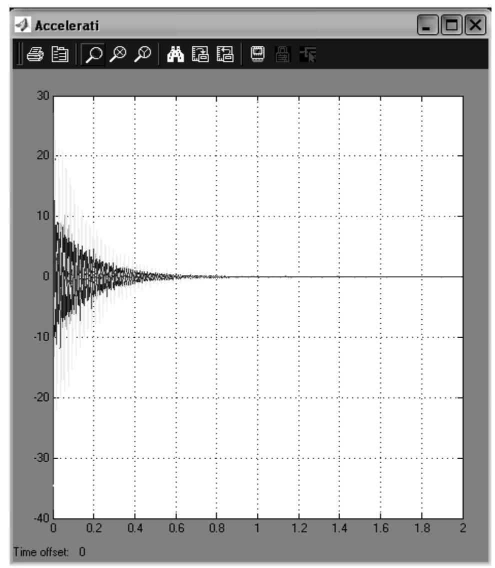

Fig.5 Nodal acceleration

2. CONCLUSIONS AND RESULTS

The presented method allows determining the dynamic characteristics for spatial jointed bar structures, as well as the dynamic response in displacement, velocity and acceleration in each node and on each degree of freedom, regardless of the number of bars;

The presented procedure is very useful in the case in which the mass, stiffness and damping matrices must be evaluated for new materials.

REFERENCES

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