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ANALYSIS OF A QUADRILATERAL CHEBYSHEV'S MECHANISM USING THE MINIMAX APPROXIMATIONS

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Abstract: *In this paper we consider a special quadrilateral mechanism of Chebyshev for which a certain point describes with a very good approximation of a straight line. First of all, one considers the general mathematical case in which a symmetrical function is approximated by a straight line using the minimax method of approximation. This general case is solved by numerical computations, but in certain particular cases it can be solved by analytical methods. For the mechanism considered in the paper is obtained the minimax straight line approximation by numerical methods and the corresponding diagram is plotted.*
Keywords: *quadrilateral mechanism; minimax approximation; numerical; straight line.*

1. INTRODUCTION

Let us consider the quadrangular mechanism in Fig. 1, where $AB = a$, $OA = b$, $BC = CM = OC = c$. It is asked to determine the distance Y_0 so that the straight line $Y - Y_0 = 0$ does approximate the trajectory of the point M on the interval $\varphi \in [-\pi/2, \pi/2]$ in the sense of the minimax method. Numerical application for $a = 0.1\,\mathrm{m}$, $b = 0.2\,\mathrm{m}$, $c = 0.25\,\mathrm{m}$.

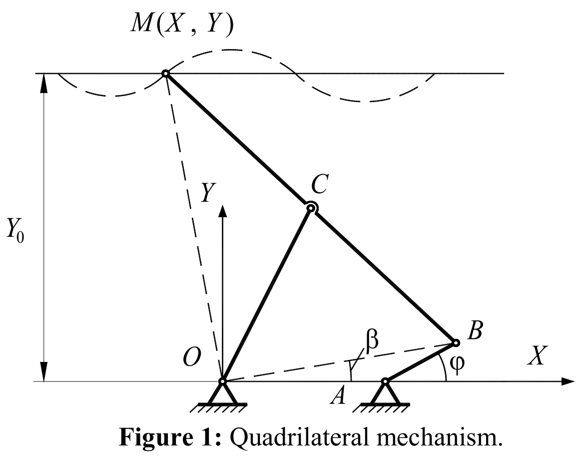

Figure 1: Quadrilateral mechanism.

2. GENERAL CASE

Let us consider the function $y = y(x)$, the graphic of which is symmetric on the interval $[-a, a]$ (Fig. 2). We wish to determine the straight line $y - y_0 = 0$ which does approximate this curve in the sense of the minimax method.

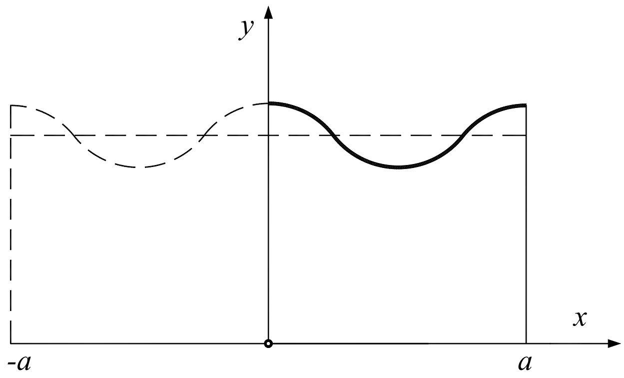

Figure 2: Theoretical aspects.

Let us choose, for example,

$$y_{0i} = y(0) + \Delta y_i \; ; \tag{1}$$

we calculate then

$$\max\left|y(x_i) - y_{0i}\right| = y^i_{\max} \, . \tag{2}$$

One constructs a table as the following one for each Δy_i .

Table 1: Example.

x	$y^i_{\max}$
0	0.2
0.01	0.3
0.02	0.5
⋮	⋮
a	0.01

The above table has been created for $\Delta y = 0.2$.
One obtains thus a sequence of data of the following form.

Table 2: Example.

Δy_i	$y^i_{\max}$
0	0.5
0.01	0.8
⋮	⋮
0.5	0.125
⋮	⋮

The minimum in this table is obtained (in the case given by us) for $\Delta y_i = 0.5$ and has the value

$$y^i_{\max} = 0.125 = \text{minimum} \, . \tag{3}$$

We deduce the searched straight line of equation

$$y_0 - y(0) = 0.125 \, . \tag{4}$$

Sometimes, the problem may be solved analytically too.
Let be (Fig. 3)

$$y = 2x^2 \, , \; x \in [-1, 1], \tag{5}$$

for which one considers

$$y_0 < f(1) = 2 \, . \tag{6}$$

It results, immediately,

$$g(x) = \left|y_0 - 2x^2\right| = \begin{cases} y_0 - 2x^2 \text{ for } |x| \le \dfrac{y_0}{\sqrt{2}}, \\ 2x^2 - y_0 \text{ for } |x| > \dfrac{y_0}{\sqrt{2}}. \end{cases} \tag{7}$$

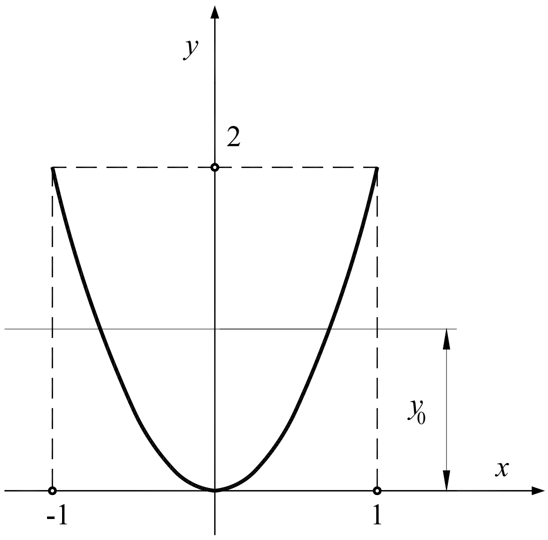

Figure 3: Function (5).

In the first case of the formula (7), one deduces $g_{\max} = y_0$, while in the second case one has $g_{\max} = 2 - y_0$. It results that the searched straight line is given by

$$y_0 = 1,\ y - 1 = 0. \tag{8}$$

3. QUADRILATERAL MECHANISM

Let us return to the problem in Fig. 1. The triangle OBM is rectangular at O, so that there result the relations

$$OM = \sqrt{BM^2 - OB^2} = \sqrt{4c^2 - \left(a^2 + c^2 + 2ac\cos\varphi\right)}. \tag{9}$$

Thus also result

$$\cos\beta = \frac{b + a\cos\varphi}{\sqrt{a^2 + b^2 + 2ab\cos\varphi}},\ \sin\beta = \frac{a\sin\varphi}{\sqrt{a^2 + b^2 + 2ab\cos\varphi}}; \tag{10}$$

hence

$$X_M = OM\cos\left(\frac{\pi}{2} + \beta\right) = -\frac{a\sin\varphi\sqrt{4c^2 - \left(a^2 + b^2 + 2ab\cos\varphi\right)}}{\sqrt{a^2 + b^2 + 2ab\cos\varphi}}, \tag{11}$$

$$Y_M = OM\sin\left(\frac{\pi}{2} + \beta\right) = \frac{\left(b + a\cos\varphi\right)\sqrt{4c^2 - \left(a^2 + b^2 + 2ab\cos\varphi\right)}}{\sqrt{a^2 + b^2 + 2ab\cos\varphi}}. \tag{12}$$

Because $X_M(-\varphi) = -X_M(\varphi)$, $Y_M(-\varphi) = Y_M(\varphi)$, it results that the trajectory of the point M is symmetric with respect to the OY-axis.

4. NUMERICAL CALCULATION

The expressions (11) and (12) become

$$X_M = -\frac{0.1\sin\varphi\sqrt{0.2 - 0.04\cos\varphi}}{\sqrt{0.05 + 0.04\cos\varphi}}, \tag{13}$$

$$Y_M = \frac{\left(0.2 + 0.1\cos\varphi\right)\sqrt{0.2 - 0.04\cos\varphi}}{\sqrt{0.05 + 0.04\cos\varphi}}. \tag{14}$$

Denoting now

$$\varphi = \frac{\pi}{2}\varphi^*,\ \varphi^* \in [-1, 1], \tag{15}$$

one obtains the following table of values.

Table 3: Numerical results.

φ^*	X_M	Y_M
–1	0.200000	0.400000
–0.9	0.183292	0.400183
–0.8	0.164973	0.400529
–0.7	0.145533	0.400825
–0.6	0.125354	0.400968
–0.5	0.104726	0.400934
–0.4	0.083858	0.400758
–0.3	0.062893	0.400505
–0.2	0.041912	0.400251
–0.1	0.020947	0.400067
0	0.000000	0.400000
0.1	–0.020947	0.400067
0.2	–0.041912	0.400251
0.3	–0.062893	0.400505
0.4	–0.083858	0.400758
0.5	–0.104726	0.400934
0.6	–0.125354	0.400968
0.7	–0.145533	0.400825
0.8	–0.164973	0.400529
0.9	–0.183292	0.400183
1	–0.200000	0.400000

We consider now the step

$$\Delta Y = 10^{-6}\ \mathrm{m} \tag{16}$$

and the interval $0.4\ \mathrm{m} \le Y \le 0.401\ \mathrm{m}$.

For each Y one has constructed a table of the following form (in this case the table has been created for $Y = 0.4\ \mathrm{m}$)

Table 4: Numerical results.

X_M^i	Y_M^i	$\left\|Y_M^i - Y\right\|$
–0.200000	0.400000	0.000000
0.183292	0.400183	0.000183
0.164973	0.400529	0.000529
0.145533	0.400825	0.000825
0.125354	0.400968	0.000968
0.104726	0.400934	0.000934
0.083858	0.400758	0.000758
0.062893	0.400505	0.000505
0.041912	0.400251	0.000251
0.020947	0.400067	0.000067
0.000000	0.400000	0.000000
–0.020947	0.400067	0.000067
–0.041912	0.400251	0.000251
–0.062893	0.400505	0.000505
–0.083858	0.400758	0.000758
–0.104726	0.400934	0.000934
–0.125354	0.400968	0.000968
–0.145533	0.400825	0.000825
–0.164973	0.400529	0.000529
–0.183292	0.400183	0.000183
–0.200000	0.400000	0.000000

From the above table it results

$$\max\left|Y_M^i - Y\right| = 0.000968\,. \tag{17}$$

Analyzing each table, one deuces the value

$$\min \max \left| Y_M^i - Y \right| = 0.000484 \tag{18}$$

obtained for

$$Y_0 = 0.400484 \text{ m} ; \tag{19}$$

hence the equation of the searched straight line is

$$Y - 0.400484 = 0 . \tag{20}$$

In Fig. 4 the trajectory of the point M has been drawn (with a continuous line), as well as the straight line (20) (with a broken line).

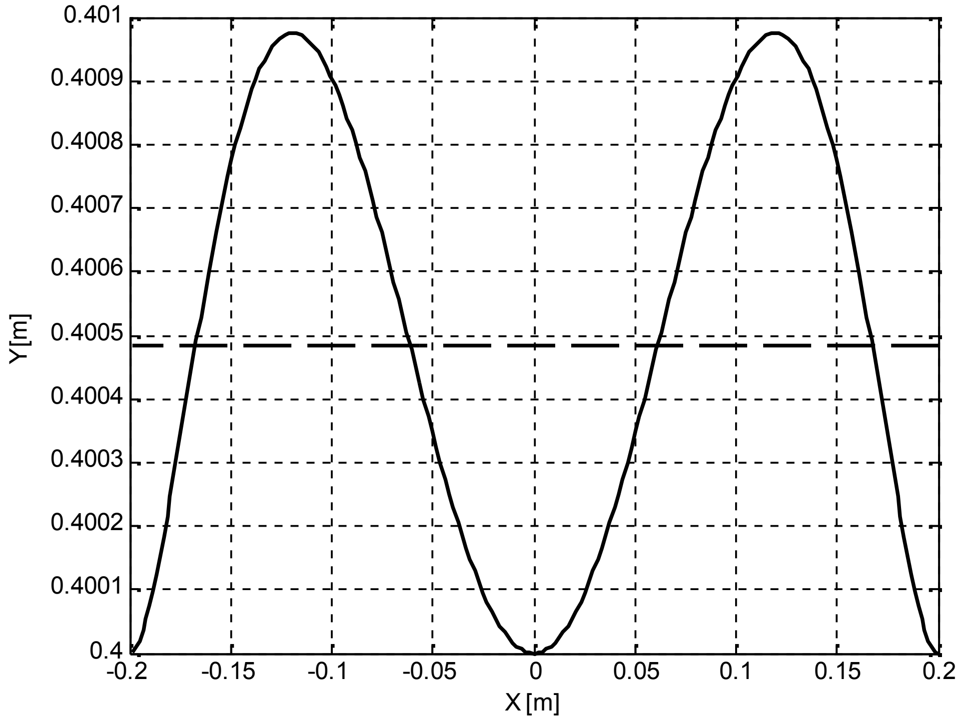

Figure 4: Trajectory of the point M (continuous line) and its approximation by the straight line (20) (broken line).

5. CONCLUSION

In our paper we determined the best linear approximation in the sense of minimax principle for a Chebyshev mechanism. We presented the theory in the most general case and we applied it at a quadrilateral Chebyshev mechanism.

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