The 4th International Conference
″Advanced Composite Materials Engineering ″
COMAT 2012
18- 20 October 2012, Brasov, Romania

MATHEMATICAL MODEL FOR THE TOTAL SOLAR RADIATION DETERMINATION AT THE SOIL LEVEL

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Abstract: *In one minute the sun produces enough energy to cover the annual energy needs worldwide, and one day produce as much energy as energy demand worldwide for a period of 27 years. Sun has a radius equal to 695,000 km and a volume of 1.42 · 1018 km3 and develop space, the reaction thermonuclear transformation of hydrogen into helium, a radiant energy flux of 8.8 · 1025 cal / sec. The mathematical model presented in this paper takes into account all these parameters, less than the quantity of vapor and dust in the atmosphere that has a random evolution. The mathematical model developed for the determination anytime and anywhere in the world for the next values and parameters: apparent solar time, solar declination, solar azimuth and incidence angle, angle of sun elevation, solar declination, solar constant, solar flux density, diffuse solar radiation, global radiation, soil albedo, total radiant flux density and relational links of these values.*
Keywords: *solar radiation, mathematical model, geomembranes*

1. INTRODUCTION

In one minute the Sun produces enough energy to cover the annual energy needs worldwide and in one day produce more energy than the global energy needs for a period of 27 years.Sun has a radius equal to 695,000 km and a volume of $1.42 \cdot 10^{18}$ km^3 and develops into space from the reaction of thermonuclear conversion of hydrogen into helium flow of radiant energy of $8.8 \cdot 10^{25}$ cal / sec. Since over 50% of the solar mass consists of hydrogen requires the same intensity of solar activity for another 5 billion years [4].
Approximately 90% of the energy generated by the Sun in its central part is transmitted to the surface and then radiated into space through a series of complex radioactive and convective emission processes, absorption and subsequent radiation of different wavelengths in the spectrum continuous or discontinuous [1], [2].

1. SUN POSITION IN RELATION TO A FLAT SURFACE

Within a year, the Earth perform a rotation around the Sun on an elliptical path whose axes are inclined at an angle of 23 º 45 'to the Earth-Sun orbital plane. Meanwhile, the Earth rotates around its axis in a polar within 24 hours. At the spring equinox (March 21 and the autumn equinox (September 21) the sun is above the equator, and the entire world, except for poles, day equals night. Realized the sun angle to the plane of the equator is called declination. Declination angle varies between 23 ° 45 '(21 June) and -23 ° 45' (21 December) [2], [5].
In [2] solar time is defined as the time when the sun crosses the meridian of the observer and depends on the apparent angular motion of the Sun across the sky.
Based on this definition can convert local time to solar time, as follows:
- Determine a constant correction for the difference between analyzed and meridian longitude location dependent local time;
- The 1 º longitude equals four minutes of time with Earth perform knowing that someday a complete rotation of 360 °;
- Apply time correction equation that takes into account changes in Earth's rotation.

Thus, apparent solar time equation, denoted AST defined in [2] is:

$$AST = LST + ET\ 4\ (LSM-LON) \tag{1}$$

where:
ET - equation of time (in minutes);

LST - standard local time corresponding Greenwich time (minutes);
LSM - standard local meridian (°);
LON - local longitude (°);
4 - minute time corresponding to 1 and the Earth's rotation.
A relationship for ET time equation [2] can be written as:

$$ET = 9.87 \sin [4\pi (n-81) / 364] -7.53 \cos [2\pi (n-81) / 364]-1.5\sin [2\pi (n-81) / 364] \quad (2)$$

where:
n - day of the year, n = 1 ... 365.
Sun position and geometric relationships that occur between a plane (flat plate) and a beam incident on the plan can be determined by the following values:
- L - latitude, location is the angle relative to the equator (the north is considered positive);
- δ - solar declination, is the angular position of the sun at solar noon corresponding equatorial plane (δ = -23 ° 45 '... 23 º 45');
- α - solar altitude, is the angle between the solar beam and horizontal incidence (α = 0 ... 90 °);
- z - zenith, solar incidence is the angle between the radius and vertical ground;
- φ - solar azimuth, is the angle between the incident beam and horizontal projection southern direction (positive is afternoon);
- γ - surface solar azimuth, is the angle between the horizontal projection of the incident solar beam and projection of the surface normal inclined plane;
- ψ - azimuth surface, is the angle between the projection on the plane of the surface normal and the direction south (east direction is considered negative);
- β - angle of the flat plate to the horizontal plane of the Earth's surface (β = 0 ... 180 °);
- θ - angle of incidence, is the angle of incidence of the solar beam and the surface normal (flat plate) slope.

2. APPARENT MOTIONS OF THE SUN IN THE SKY [3]

Sun performs a apparent rotation in the sky with angular velocity $\omega = \pi$ / 12 rad / h = 15 deg / h The sun in the sky is determined by the angle H zone and elevation angle h is measured clockwise angle around the axis between the poles of the site plan and circle meridian zone of the star. The simplifying assumption that solar time is equal to the meridian legal ground, the angle zone is calculated with:

$$H = (\pi / 12) (\tau - \tau 0) \quad (3)$$

In (3) the meanings sizes are: τ - Legal time, τ_0 - time of the meridian passage of the sun, shall be deemed τ_0 = 12, H-angle zone.
Sun elevation angle from the horizontal plane of the ground is

$$\sin h = \sin \lambda \sin \delta + \cos \delta \cos \lambda \cos H \quad (4)$$

In relation (4):
- Size λ is the latitude of the place, the calculations can be considered a value aproximateive λ = 45 ° (for Bucharest λ = 44.3954 º λ = 44.26969 and Craiova, Sibiu λ = 45.779887 º);
- Size δ is declination, ie the angle measured between the Sun and the equatorial plane:

$$\delta = 23.4 \sin\left[\frac{2\pi(284+N)}{365}\right] \quad (5)$$

N is the Julian day, that day of the year counted from January 1, for which one can use the following equation:

$$N = 30,416 (\chi-1) + n, \quad (6)$$

where χ n indicates the number indicate the month and day of month.
Introducing relations (3) and (5) in relation (4) we obtain:

$$\sinh = \sin\lambda \cdot \sin\left\{23.4\sin\left[\frac{2\pi(284+N)}{365}\right]\right\} + \cos\left\{23.4\sin\left[\frac{2\pi(284+N)}{365}\right]\right\} \cdot \cos\lambda \cdot \cos\left[\frac{\pi(\tau-\tau_0)}{12}\right] \quad (7)$$

From relation (7) can assess the value of sun elevation angle from the horizontal plane of the ground, as follows:

$$h = \arcsin\left\{\sin\lambda \cdot \sin\left\{23.4\sin\left[\frac{2\pi(284+N)}{365}\right]\right\} + \cos\left\{23.4\sin\left[\frac{2\pi(284+N)}{365}\right]\right\} \cdot \cos\lambda \cdot \cos\left[\frac{\pi(\tau-\tau_0)}{12}\right]\right\} \quad (8)$$

Where: λ - is the latitude of the place;
N - number of the current day of the year;
τ - Legal time;
τ_0 - time of the meridian passage of the sun, consider τ_0 = 12.
Applying relation (8) the Sun elevation angle h to the horizontal for September 1, 2012 are obtained the diagram represented in Figures 1.

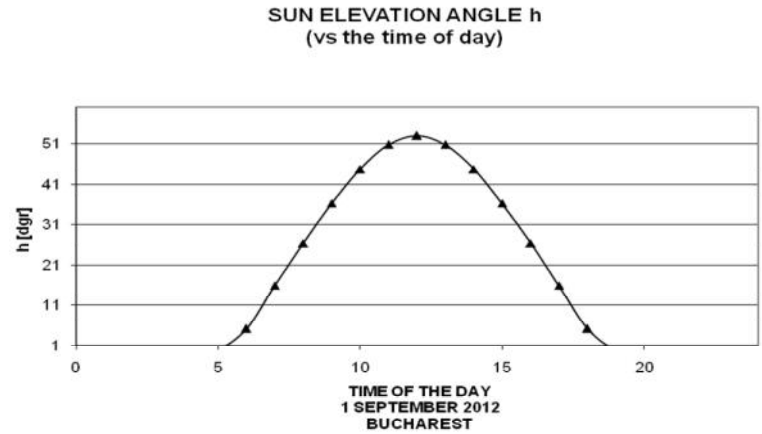

Figure 1: Sun elevation angle for 1st of September in Bucharest

3. SOLAR CONSTANT

The amount of solar energy to upper Earth's atmosphere, pass through a unit area normal to the direction of propagation of the unitary time I called global solar radiation intensity is:

$$I = \frac{4\pi r^2 \sigma T^4}{4\pi d^2} \tag{9}$$

where d is the distance Earth - Sun.
Earth-Sun distance variation causes changes in the size I from day to day. Measurements of N.A.S.A. indicates the intensity variation between the limits 1310 W/m2 (June) and 1400 W/m2 (January). Annual average solar radiation in the upper atmosphere is called the solar constant S_{SN} = 1367 W/m^2.
For N- day of the year, the solar constant correction is calculated as:

$$S = S_{sn}\ (\ 1 + 0{,}0034 \cos N\ ) \tag{10}$$

Using relation (10) for January 2012 was obtained the representation in Figures 2.

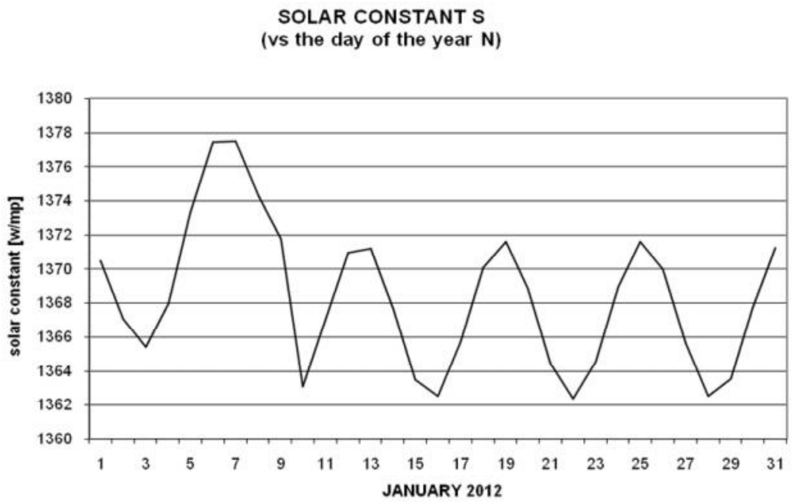

Figure 2: Solar constant S for January 2012

4. RADIANT FLUX DENSITY IN THE SOIL [3]

Crossing terrestrial atmosphere by solar radiation attenuation occurring phenomena leading radiant flux density. These phenomena are a fraction of radiation energy absorbed by the gas components of the atmosphere, molecular diffusion and Rayleigh scattering by aerosols. Ground radiation has two components: the direct component attenuated and diffuse component. Also included are called albedou ground reflected radiation.
Direct component of solar flux density B ground on a surface normal to the sun, under a clear sky, is:

$$B = S \cdot a \cdot e^{\left(-\frac{b}{\sinh}\right)} \tag{11}$$

Depends on place characteristics (concrete, lawn) and size b depends on attenuated influence factors. To a common place the values can be considered: a = 0.88 and b = 0.28.
Applying relation (11) to replace (8) and (10) we obtain direct component of solar flux density B to ground:

$$B = S_{sn}\,(1 + 0{,}0034 \cos N) \cdot a \cdot e^{\left(-\frac{b}{\sin\lambda \cdot \sin\left\{23.4 \sin\left[\frac{2\pi(284+N)}{365}\right]\right\} + \cos\left\{23.4 \sin\left[\frac{2\pi(284+N)}{365}\right]\right\} \cdot \cos\lambda \cdot \cos\left[\frac{\pi(\tau-\tau_0)}{12}\right]}\right)} \qquad (12)$$

Applying relation (12) to Bucharest variations were obtained direct component of solar flux on soil B shown in Figure 3 (1 September 2012).

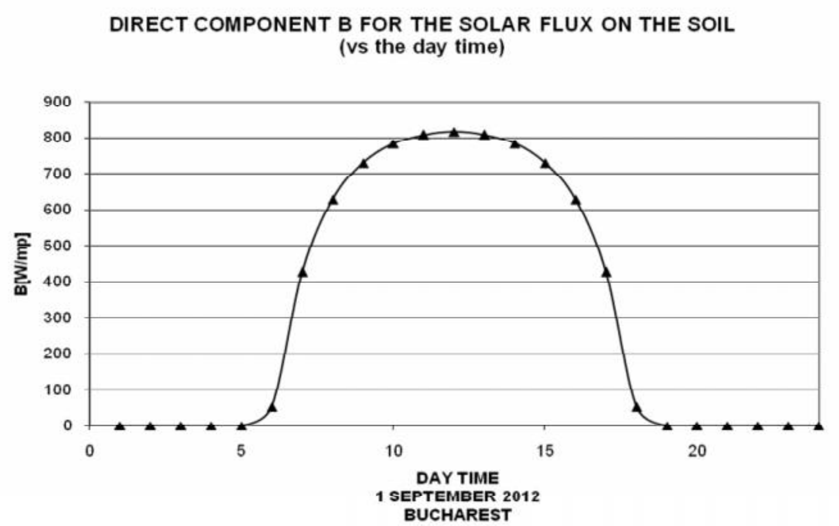

Figure 3: Direct component of solar flux density B to ground for 1st September 2012 in Bucharest

Diffuse component of solar radiation D is:

$$D = S \cdot \left[0.2710 - 0.2939 a \cdot e^{\frac{-b}{\sin h}}\right] \cdot \sin h \qquad (13)$$

Substituting relation (10) solar constant S in (13) we obtain:

$$D = S_{sn}(1 + 0.0034 \cdot \cos N) \cdot \left[0.2710 - 0.2939 a \cdot e^{\frac{-b}{\sin h}}\right] \cdot \sin h \qquad (14)$$

Also, the equation of the Sun elevation angle h to the horizontal (8) can be replaced in (14) we obtain the relationship diffuse solar radiation component D:

$$D = S_{sn}(1 + 0.0034 \cdot \cos N) \cdot \left[0.2710 - 0.2939 a \cdot e^{\frac{-b}{\sin\lambda \cdot \sin\left\{23.4 \sin\left[\frac{2\pi(284+N)}{365}\right]\right\} + \cos\left\{23.4 \sin\left[\frac{2\pi(284+N)}{365}\right]\right\} \cdot \cos\lambda \cdot \cos\left[\frac{\pi(\tau-\tau_0)}{12}\right]}}\right] \cdot$$

$$\cdot \sin\lambda \cdot \sin\left\{23.4 \sin\left[\frac{2\pi(284+N)}{365}\right]\right\} + \cos\left\{23.4 \sin\left[\frac{2\pi(284+N)}{365}\right]\right\} \cdot \cos\lambda \cdot \cos\left[\frac{\pi(\tau-\tau_0)}{12}\right] \qquad (15)$$

Applying relationship for diffuse component of solar radiation (15) are obtained for Bucharest the variation shown in Figure 4 (September 1, 2012).

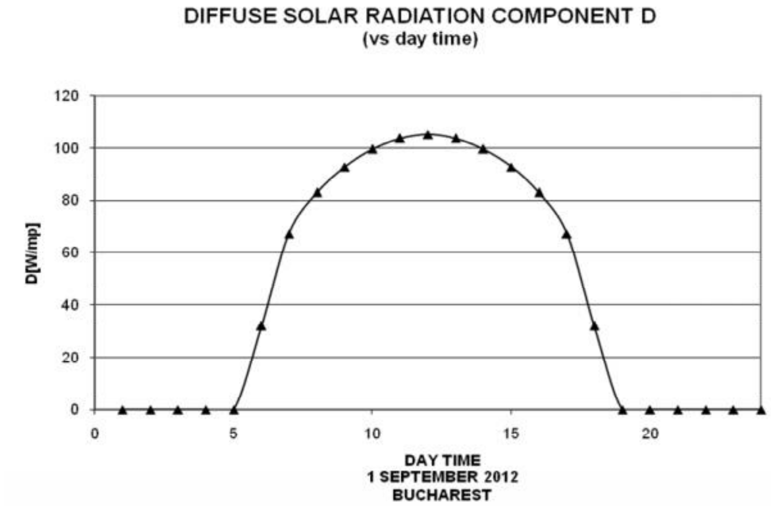

Figure 4: Diffuse solar radiation component D for 1st September 2012 in Bucharest

Total radiant flux density on a horizontal surface is called irradiance or global radiation G:

$$G = B\sin h + D \tag{16}$$

Replacing direct relationship component B (12) diffuse component D (15) and sinus elevation angle h of the Sun from the horizontal plane (7) is obtained relative global radiation G:

$$G = S_{sn}\ (1 + 0{,}0034\cos \mathrm{N}) \cdot a \cdot e^{\left(-\frac{b}{\sin\lambda \cdot \sin\left\{23.4\sin\left[\frac{2\pi(284+N)}{365}\right]\right\} + \cos\left\{23.4\sin\left[\frac{2\pi(284+N)}{365}\right]\right\}\cdot\cos\lambda\cdot\cos\left[\frac{\pi(\tau-\tau_0)}{12}\right]}\right)} \cdot$$

$$\cdot\left\{\sin\lambda\cdot\sin\left\{23.4\sin\left[\frac{2\pi(284+N)}{365}\right]\right\} + \cos\left\{23.4\sin\left[\frac{2\pi(284+N)}{365}\right]\right\}\cdot\cos\lambda\cdot\cos\left[\frac{\pi(\tau-\tau_0)}{12}\right]\right\} +$$

$$+\left\{S_{sn}(1+0.0034\cdot\cos N)\cdot\left[0.2710 - 0.2939 a\cdot e^{\frac{-b}{\sin\lambda\cdot\sin\left\{23.4\sin\left[\frac{2\pi(284+N)}{365}\right]\right\}+\cos\left\{23.4\sin\left[\frac{2\pi(284+N)}{365}\right]\right\}\cdot\cos\lambda\cdot\cos\left[\frac{\pi(\tau-\tau_0)}{12}\right]}}\right]\cdot\right.$$

$$\left.\cdot\sin\lambda\cdot\sin\left\{23.4\sin\left[\frac{2\pi(284+N)}{365}\right]\right\}+\cos\left\{23.4\sin\left[\frac{2\pi(284+N)}{365}\right]\right\}\cdot\cos\lambda\cdot\cos\left[\frac{\pi(\tau-\tau_0)}{12}\right]\right\} \tag{17}$$

Applying relation global radiation G (17) to Bucharest was obtained the variation shown in Figure 5 (September 1, 2012).

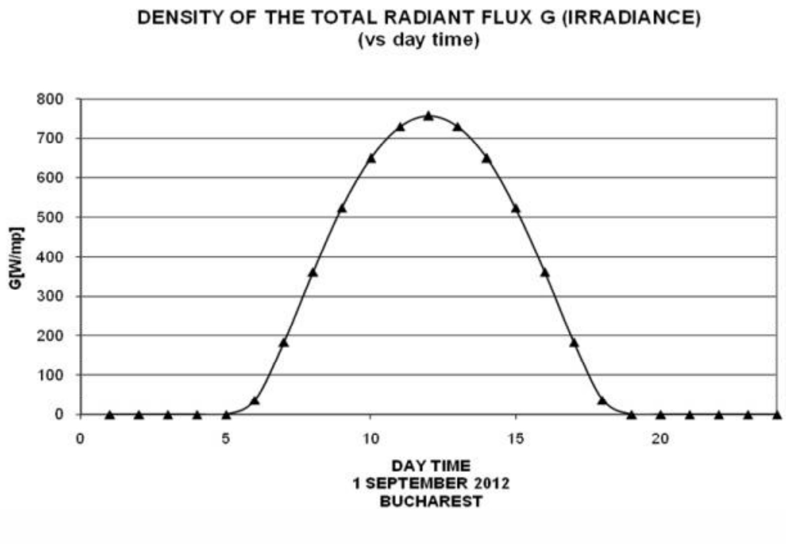

Figure 5: Density of the total radiant flux G for 1st September 2012 in Bucharest

Soil albedo is:

$$(Ab) = \frac{r \cdot B \cdot \sin h}{2} \quad (18)$$

where, for local conditions, r = 0.55.
Total radiant flux density is:

$$G_t = G + (Ab) \quad (19)$$

Applying total radiant flux density relationship G_t (19) are obtained Bucharest variations shown in Figure 6 (September 1, 2012).

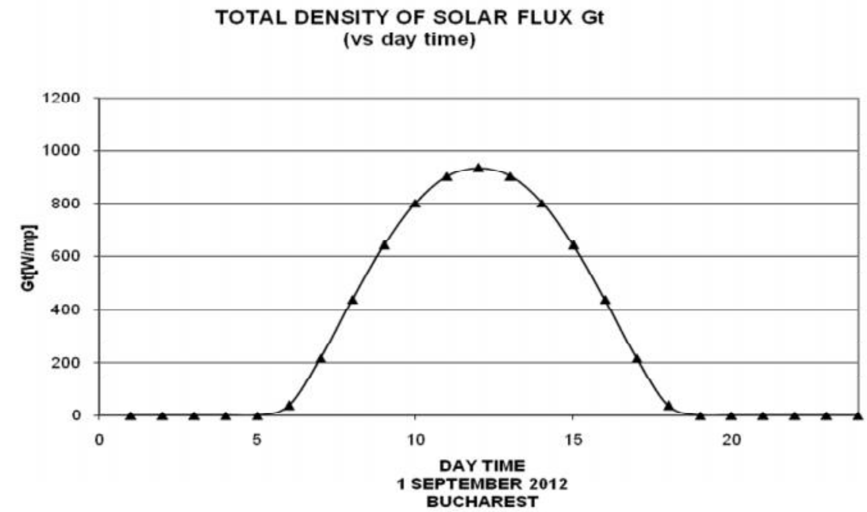

Figure 6: Total density of the solar flux G_t for 1st September 2012 in Bucharest

4. CONCLUSIONS

The mathematical model presented in this paper was adapted to Microsoft Excel and it can give the evolution of total radiant flux density at any time of the day and at any position on the Earth.
As we know, the geomembranes, used for ecological landfill during the assembly process, are exposed to the sun radiation. These elements due to thermal expansion may be folded and after exploitation of the ecological landfill, may have undesirable effects on ground water protection.
In that case, the mathematical model presented in this paper can be a good tool for the evaluation of the total solar radiation flux during the geomembranes assembling process. Also, the Excel file can be installed on a mobile computer and the initial data can be obtained from a GPS device.

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