RESEARCHES ON THE DIMENSIONAL SYNTHESIS OF THE THREE-POINT HITCH COUPLERS USED AT AGRICULTURAL TRACTORS

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ABSTRACT - The paper presents the analytical method applied to outline the dimensional synthesis of the three-point hitch couplers used at agricultural tractors. The dimensional synthesis is framed with the specific aim of determining the basic dimensions of all the elements of the three-point hitch couplers considering the kinematic properties (positions, speeds) imposed by both technological processes and agro-technical conditions.
The mechanism is divided in three kinematic groups. The study and the mathematical modeling is carried out for each distinct group taking into account the piston position in the lift cylinder of the three-point hitch couplers. The coordinates of each hitch point (to both rear axle and ground), the position and the instantaneous center of revolution are further determined. This mathematical modeling lies at the basis of the analysis into the kinematics and dynamics of the three-point hitch couplers used at agricultural tractors.
The paper shows the results of the researches conducted on wheeled tractors U 650 and U 650 DT.

INTRODUCTION

The questions related to the dimensional synthesis of three-point hitch couplers are raised when designing these mechanisms with the aim of appropriately choosing their geometrical and kinematic parameters. The geometrical parameters must be in compliance with the most advantageous ratios between the elements of the three-point hitch couplers that should meet at the same time the conditions imposed by national and international standards as well as agro-technical requirements.

For this very aim, starting from the kinematic diagram (fig.1), geometrical dimensions and piston motion in the lift cylinder, the following values are determined for a particular position of the three-point hitch couplers:

- coordinates of points of *B*, *D*, *K*, *F*, *M* of the mechanism elements;
- coordinates of the center of gravity (point S);
- coordinates of the instantaneous centre of revolution (CIR, point *I*).

The position of the three-point hitch couplers is determined for any piston position, i.e. *AB* dimension.
In order to deduce the formulas and to determine the parameters mentioned, the following were adopted:

- the system balance is analyzed in the longitudinal-vertical symmetry plan of the agricultural tractor (fig.1);
- the construction of the three-point hitch couplers is divided into several groups: group of elements *2–3*, *4–5* and group of elements *6–7;*
- the system of *xOy* coordinates axes is rigidly connected to the tractor: the *Oy* axis crosses the axis of the tractor's rear axle, and the *Ox* axis is positioned on the supporting surface of the agricultural tractor.

x_i, y_i denotes the coordinates of points *A*, *C*, *E*, *Q*; *CB*, *CD*, *DK*, *QM*, *MF*, *FE*, *QK*, *FS*, *MS* denotes the elements dimensions; β denotes the angle between *CB* and *CD* lever arms. These parameters stand for the initial calculus parameters.

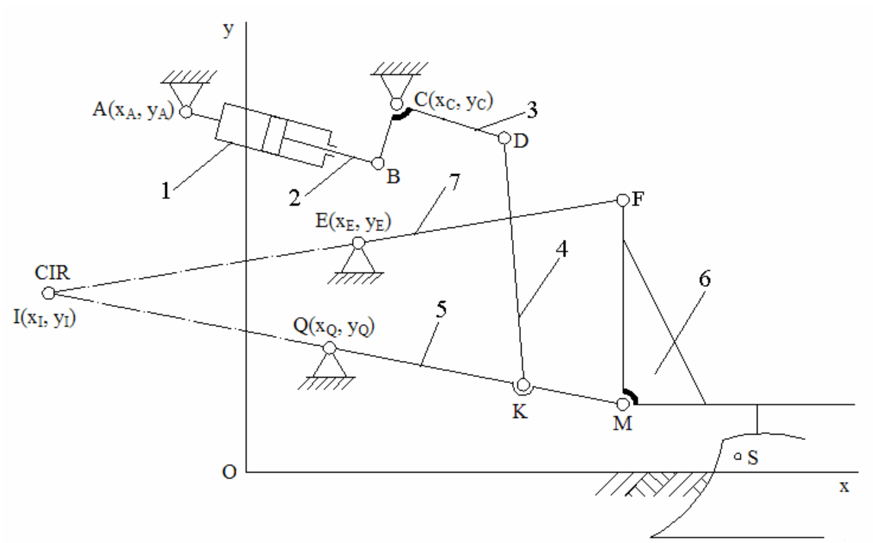

Fig. 1 Diagram for calculating the agricultural tractor's three-point hitch couplers

GEOMETRY OF THE THREE-POINT HITCH COUPLERS

The formulas for the kinematic analysis are deduced on separate groups: *2–3*, *4–5* and *6–7* (fig. 1). A single system of coordinates is used for each analyzed group, i.e. *xOy*, whose axis *Oy* crosses the axis of the tractor's rear axle, and the *Ox* axis is placed on the supporting surface of the agricultural tractor.

The analysis calls for the coordinate transformation method. In the general case of coordinate transformations (made up of a translation and a revolution of the coordinates axes), as shown in figure 2, we have the relations:

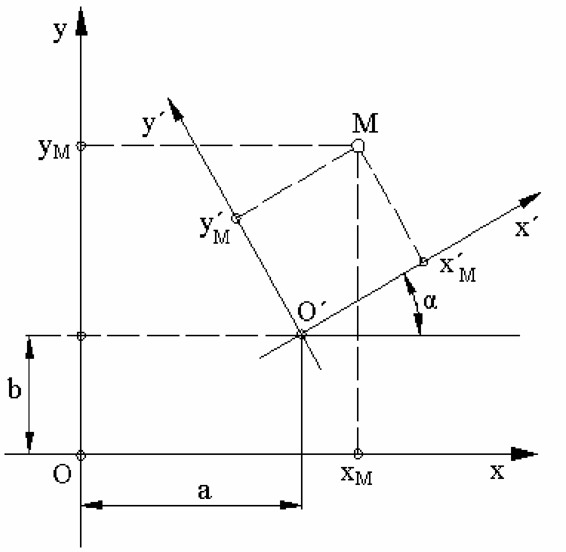

Fig. 2 General case of coordinate transformation

$$x_M = x'_M \cos\alpha - y'_M \sin\alpha + a; \quad y_M = x'_M \sin\alpha + y'_M \cos\alpha + b. \qquad (1)$$

Group 2-3. In case of the constructive model where the piston of the lift cylinder pushes the beat-up lever (fig.1), the piston moves a distance *s* from the position $A(x_{A0}, y_{A0})$ (of coordinates known in the *xOy* system) to the position $A(x_A, y_A)$ (fig. 3). In the new position, the coordinates of point *A* are determined with the relations:

$$x_A = x_{A0} + s\cos\alpha; \quad y_A = y_{A0} - s\sin\alpha. \qquad (2)$$

In order to determine the coordinates of point *B* a supplementary system of coordinates is chosen, i.e. *x′Cy′*, whose abscissa crosses points A and C, the last point having the coordinates known in the *xOy* system. Figure 3 is used to determine the x'_B and y'_B coordinates as well.

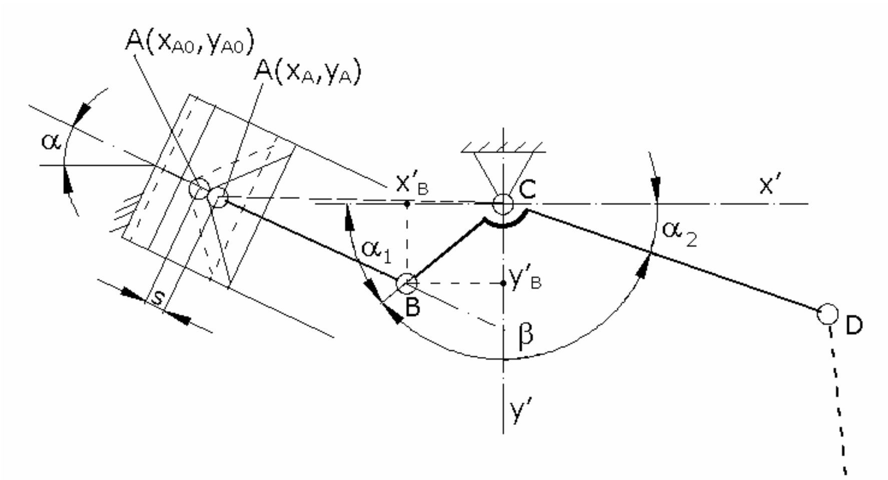

Fig. 3 Diagram for calculating the geometrical elements of group *2 – 3*

In triangle ABC the following relation is valid:

$$\cos C = \frac{AC^2 + BC^2 - AB^2}{2AC \cdot BC}.$$

On the other hand,

$$-x'_B = BC\cos C.$$

In the last relation, by replacing cos *C*, we obtain:

$$x'_B = \frac{AB^2 - AC^2 - BC^2}{2AC}. \qquad (3)$$

From the same figure there results:

$$y'_B = \sqrt{BC^2 - (x'_B)^2}. \qquad (4)$$

The distance value between points *A* and *C* is given by the relation:

$$AC = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2}. \qquad (5)$$

In the analyzed case, the coordinates of point *B* in the *xOy* system, after coordinate transformation, are determined with the relations:

$$x_B = x_C + \frac{1}{AC}\left(AC_x x'_B + AC_y y'_B\right); \quad y_B = y_C + \frac{1}{AC}(AC_y x'_B - AC_x y'_B). \qquad (6)$$

where: AC_x is the projection of line *AC* on axis *x*; AC_y – the projection of line *AC* on axis *y*.

In order to determine the coordinates of point *D,* in *xOy* system, we use the transformation relations of Cartesian coordinates into polar coordinates and the other way around:

$$-x'_B = BC\cos\alpha_1,$$

where

$$\alpha_1 = \arccos\left(\frac{-x'_B}{BC}\right) = \arccos\left(\frac{x'_B}{BC}\right).$$

From figure 3 there results:

$$\alpha_2 = 180 - (\alpha_1 + \beta).$$

The angle α_2 is turned into circular measure and the coordinates of point *D* are determined in *x´Oy´* system:

$$x'_D = CD\cos\alpha_2; \quad y'_D = CD\sin\alpha_2. \tag{7}$$

The coordinates of point *D* in *xOy* system will be determined with the relations:

$$x_D = x_C + x'_D; \quad y_D = y_C - y'_D. \tag{8}$$

Group 4 – 5. In order to determine the coordinates of point *K* a supplementary system of coordinates is chosen, i.e. $x'_k D y'_k$, whose abscissa crosses points *D* and *Q*, which has the coordinates known in the *xOy* system. Figure 4 is used to determine the x'_K and y'_K coordinates. The following relations may be written in triangle *DKQ*.

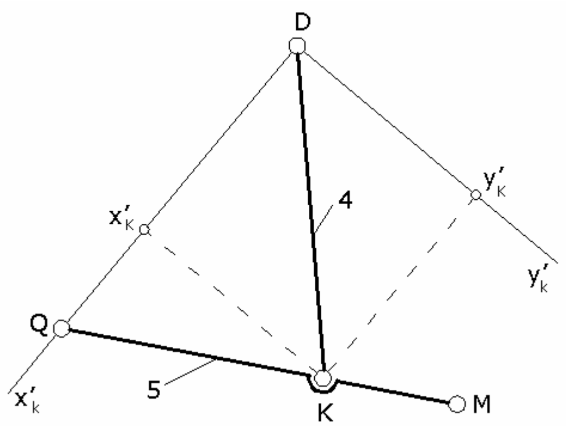

Fig. 4 Diagram for calculating the geometrical elements of group *4 – 5*

$$\cos D = \frac{DQ^2 + DK^2 - KQ^2}{2DQ \cdot DK} \text{ and } x'_K = DK\cos D.$$

Thus,

$$x'_K = \frac{DQ^2 + DK^2 - KQ^2}{2DQ \cdot}, \tag{9}$$

and

$$y'_K = \sqrt{DK^2 - (x'_K)^2}. \tag{10}$$

The coordinates of point *K* in *xOy* system, after coordinates transformation, is determined with the relations:

$$x_K = x_D + \frac{1}{DQ}\left(-DQ_x x'_K + DQ_y y'_K\right); \quad y_K = x_D + \frac{1}{DQ}\left(-DQ_y x'_K - DQ_x y'_K\right). \tag{11}$$

In order to identify the coordinates of point *M* (in case points *Q*, *K*, *M* are on the same line) the following ratios are used:

$$\frac{QM}{QK} = \frac{QM_x}{QK_x} = \frac{QM_y}{QK_y}.$$

The first equality may be written as follows:

$$\frac{QM}{QK} = \frac{x_M - x_Q}{x_K - x_Q} \quad \text{or} \quad x_M - x_Q = \frac{QM}{QK}\left(x_K - x_Q\right),$$

there results

$$x_M = x_Q + \frac{QM}{QK}\left(x_K - x_Q\right). \tag{12}$$

Similarly,

$$y_M = y_Q + \frac{QM}{QK}\left(y_K - y_Q\right). \tag{13}$$

Group 6 – 7. In order to determine the coordinates of point *F* a supplementary system of coordinates is chosen, i.e. $x'_f D y'_f$, whose abscissa crosses points *E,* with coordinates known in the *xOy* system, and *M*,. Figure 5 is used to determine the x'_F and y'_F coordinates. The following relations may be written in triangle *EFM.*

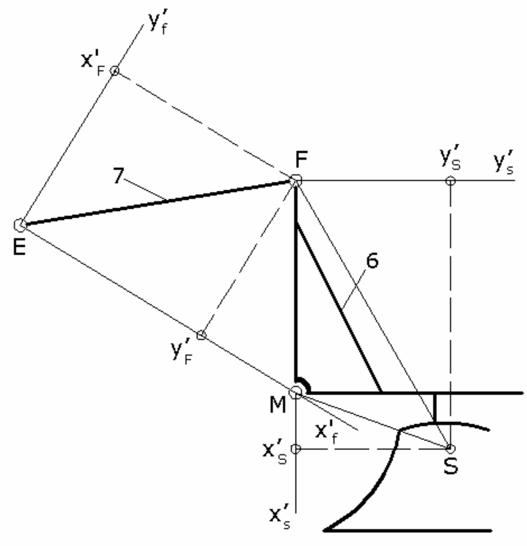

Fig. 5 Diagram for calculating the geometrical elements of group *6 – 7*

$$\cos E = \frac{EF^2 + EM^2 - FM^2}{2EF \cdot EM} \quad \text{and} \quad x'_F = EF \cos E,$$

and after replacing the cos *E* function there results:

$$x'_F = \frac{EF^2 + EM^2 - FM^2}{2EM}. \tag{14}$$

Using the same figure, we may write:

$$y'_F = \sqrt{EF^2 - (x'_F)^2}. \tag{15}$$

The coordinates of point *F* in *xOy* system, after coordinates transformation, is determined with the relations:

$$x_F = x_E + \frac{1}{EM}\left(EM_x x'_F + EM_y y'_F\right) \quad \text{and} \quad y_F = y_E + \frac{1}{EM}\left(-EM_y x'_F + EM_x y'_F\right). \tag{16}$$

In order to determine the coordinates x_S, y_S of the center of gravity S of the carried implement we use figure 5 as well. In a triangle FMS we have the relations:

$$\cos F = \frac{FM^2 + FS^2 - MS^2}{2FM \cdot FS} \text{ and } x'_S = FS\cos F.$$

After replacing the function cos F, we obtain:

$$x'_S = \frac{FM^2 + FS^2 - MS^2}{2FM}. \tag{17}$$

Figure 5 shows that:

$$y'_S = \sqrt{FS^2 - (x'_S)^2}. \tag{18}$$

The coordinates of point S in the xOy coordinates system, after coordinates transformation, is determined with the relations:

$$x_S = x_F + \frac{1}{FM}\left(FM_x x'_S - FM_y y'_S\right) \text{ and } y_S = y_F + \frac{1}{FM}\left(FM_y x'_S + FM_x y'_S\right). \tag{19}$$

In order to determine the coordinates of the instantaneous centre of revolution (point I in figure 1) we write the equation of the line crossing two given points (E and F, Q and M):

$$\begin{cases} \dfrac{x_I - x_E}{x_F - x_E} = \dfrac{y_I - y_E}{y_F - y_E}; \\ \dfrac{x_I - x_Q}{x_M - x_Q} = \dfrac{y_I - y_Q}{y_M - y_Q}. \end{cases} \tag{20}$$

By solving this system, in relation with x_I and y_I, we obtain the coordinates of point I. From the first equation we obtain:

$$y_I = y_E + (y_F - y_E)\frac{x_I - x_E}{x_F - x_E},$$

and from the second equation we obtain:

$$y_I = y_Q + (y_M - y_Q)\frac{x_I - x_Q}{x_M - x_Q}.$$

By equalizing the last two equations, we obtain the relation:

$$y_E + (y_F - y_E)\frac{x_I - x_E}{x_F - x_E} = y_Q + (y_M - y_Q)\frac{x_I - x_Q}{x_M - x_Q},$$

where

$$x_I = \frac{(x_F - x_E)[x_M y_Q - x_Q y_M - y_E(x_M - x_Q)] + x_E(x_M - x_Q)(y_F - y_E)}{(x_M - x_Q)(y_F - y_E) - (y_M - y_Q)(x_F - x_E)} \tag{21}$$

For the coordinate y_I we obtain the relation:

$$y_I = \frac{x_I(y_M - y_Q) + x_M y_Q - x_Q y_M}{x_M - x_Q}. \tag{22}$$

APPLICATIONS OF THEORETICAL RESEARCHES

In this part of the paper the author presents part of the results arisen from the theoretical researches carried out on the agricultural tractor U 650 DT, which is representative for the Romanian tractor fleet. The following input data, corresponding to the second category (SR ISO 730-1) were used:

- coordinates of fixed points (in mm): *A*(-1, 1156), *C*(230, 1143), *E*(400, 891), *Q*(230, 495);
- dimensions of elements (in mm): *AB* = 160, *CB* = 94,8, *CD* = 260, *DK* = 760, *QM* = 900, *MF* = 850, *FE* = 637, *QK* = 459, *MS* = 820;
- values of angles: α = 25° (inclination of the lift cylinder to the horizontal) β = 124° (angle between *CB* and *CD* lever arms).

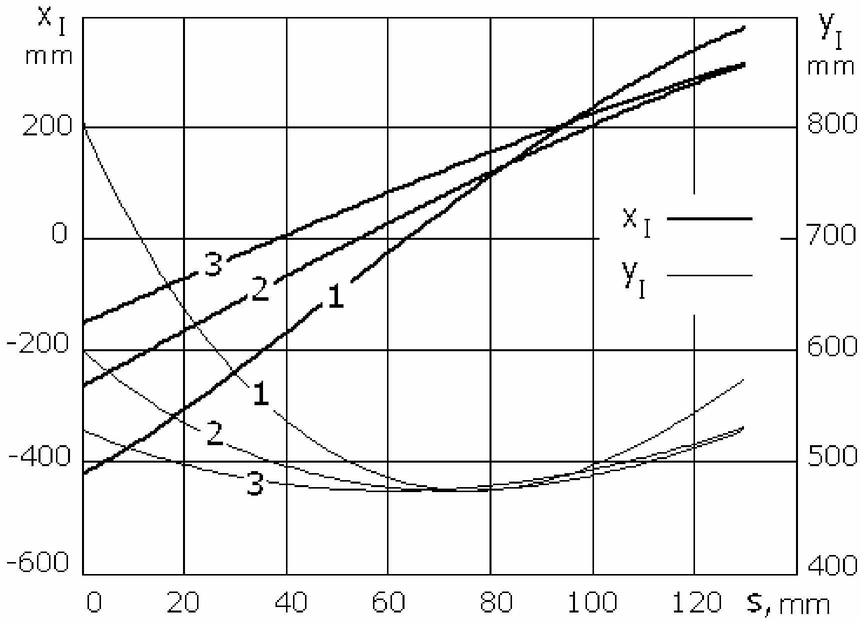

Fig. 6 Variation of coordinates of point *I* depending on piston position for 3 values *QK*: 1 – *QK* = 299 mm; 2 – *QK* = 459 mm; 3 – *QK* = 559 mm

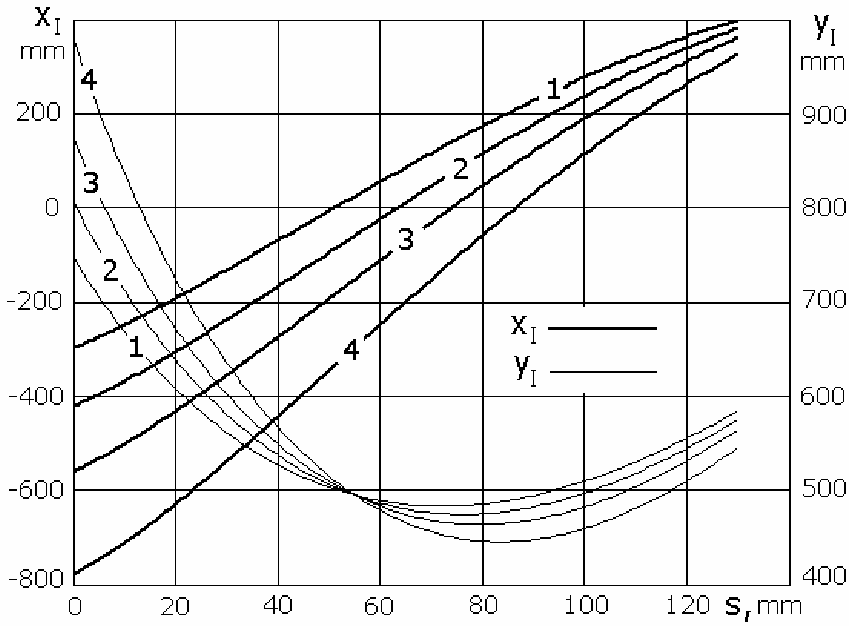

Fig. 7 Variation of coordinates of point *I* depending on piston position for 4 positions of point *E*: 1 – *E*(400, 857); 2 – *E*(400, 891); 3 – *E*(400, 923); 4 – *E*(400, 964)

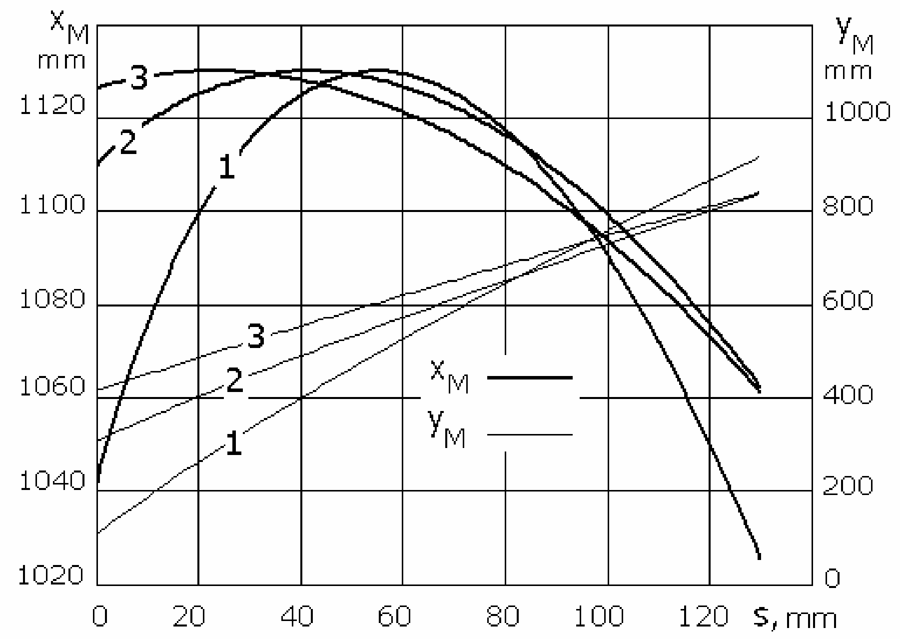

Fig. 8 Variation of coordinates of point *M* depending on piston position for 3 values *QK*: 1 – *QK* = 299 mm; 2 – *QK* = 459 mm; 3 – *QK* = 559 mm

CONCLUSIONS

- Using widely known geometrical relations and the coordinate transformation method, we obtained general mathematical relations in order to determine the coordinates of each point of the three-point hitch couplers. The method and the relations obtained are valid for any three-point hitch couplers with a kinematic diagram similar to the one shown in figure 1.
- The relations obtained in order to determine the coordinates of the instantaneous centre of revolution are of great utility as they greatly influence the traction qualities and tractor's dynamics.
- Choosing as generalized coordinate the piston motion in the lift cylinder, we may create a direct link between the mechanical process of the three-point hitch couplers and its hydrostatic drive.
- The application of mathematical relations obtained is carried out on one of the most representative Romanian agricultural tractor, i.e U 650 DT.

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