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CONECTIVE DIFFUSIVE TRANSPORT WITH CHEMICAL REACTION IN NATURAL CONVECTION IN A POROUS MEDIUM

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Abstract: *The present work is concerned with the natural convection flow in the presence of a vertical plate in a fluid-saturated porous medium, driven by a chemical reaction and diffusion. The reaction-rate term is modelled in this study by a power-law model. Performing a suitable change of variables, the problem is reduced to a set of nonlinear ordinary differential equations, which are solved using a regular perturbation technique. The parameters of the reduced problem are the Lewis number and the order of the chemical reaction. Detailed discussions of the obtained results are presented, in terms of: a) stream function and concentration variations within the boundary layer and b) wall shear-stress, when the two parameters are varied. Comparisons with the analogue problem in clear fluids are also provided.*

Key words: *porous media, convection, diffusion, chemical reaction.*

1. Introduction

Fluid flow in porous media has been an area of intensive investigation for the last decades. The growing emphasis on effective granular and fibrous insulation systems for the successful containment of the transport of radio-nuclide from deposits of nuclear waste materials has stimulated various studies in fluid saturated porous media and many results were obtained for the forced and convective flow in the fundamental geometries of internal (cavities, annuli, etc.) and external flows. In comprehensive reviews of heat transfer mechanisms in geothermal systems, Recently, Nield and Bejan [6], Ingham and Pop [2-4], Vafai [8-9] identified many applications which highlight the directions where further theoretical and experimental developments and investigations are required.

The present paper aims to study the natural convection flow in the presence of diffusion and chemical reaction in a porous medium saturated with a Newtonian fluid. We mention to this end that the corresponding problem for clear (viscous) fluids was reported and solved by Rahman and Mulolani [7].

2. Analysis

On a vertical plate there is a chemical species maintained at a given concentration and immersed in a fluid-saturated porous medium. Far away from the plate the concentration is constant, at a value C_∞ The species on the plate is first transferred from the plate to the adjacent medium by diffusion. In the bulk of the medium there occurs a chemical reaction.

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In these conditions, adopting a Dacy model for the flow in porous medium, along with the Boussinesq approximation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u=\frac{gK}{\upsilon}\left(C-C_{\infty}\right) \quad (2)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}+\dot{C}''' \quad (3)$$

where x and y are the coordinates along and normal to the plate, respectively, the orientation of the x-axis being upwards. C is the concentration and D is the diffusion coefficient. Other notations are usual. The reaction-rate term is modeled in the present work by a power-law model

$$\dot{C}'''=-k\left(C-C_{\infty}\right)^n \quad (4)$$

where k is the reaction-rate constant and n is the order of the reaction, see for instance Aris [8].

The boundary conditions are

$$u=v=0,\ C=C_0(x),\ \text{at } y=0 \quad (5a)$$

$$v\to 0,\ C\to C_{\infty},\ \text{as } y\to\infty \quad (5a)$$

We look for the solution in the form

$$\psi=\alpha Ra_{x,C}^{1/2}f(\eta),\ \eta=\frac{y}{x}Ra_{x,C}^{1/2},$$

$$\phi=\frac{C-C_{\infty}}{C_0-C_{\infty}} \quad (6)$$

where ψ is the stream function which satisfies identically the continuity equation (1) and

$$Ra_{x,C}=\frac{gK\beta\left(C-C_{\infty}\right)x}{\upsilon D}$$

is the concentration Rayleigh number. Inserting (5) we obtain:

$$f'=\phi \quad (7)$$

$$\frac{1}{Le}\phi''+f\phi'-\frac{k}{D}\cdot\frac{x^2}{Ra_{x,C}}\left(C-C_{\infty}\right)^n\phi^n=0 \quad (8)$$

The last term in (7) can be rewritten as

$$\varepsilon(x)=\frac{k}{D}\cdot\frac{x^2}{Ra_{x,C}}\left(C-C_{\infty}\right)^n\phi^n$$

$$=\frac{k\upsilon\left(C-C_{\infty}\right)^{n-1}x}{gK_C\beta} \quad (9)$$

and, on physical basis, this quantity is small.

Since we are considering a regular perturbation problem, there is no need for matching of layers or for multiple scales. The unknown functions of the problem are expanded as follows

$$f(\eta)=f_0(\eta)+\varepsilon f_1(\eta)+\ldots,$$

$$\phi(\eta)=\phi_0(\eta)+\varepsilon\phi_1(\eta)+\ldots \quad (10)$$

and retaining the terms up to the first order, we obtain the following problems

$$f_0'=\phi_0 \quad (11a)$$

$$\frac{1}{Le}\phi_0''+f_0\phi_0'=0 \quad (11b)$$

$$f_0(0)=0,\ \phi_0(0)=1,\ \phi_0(\infty)=0 \quad (11c)$$

$$f_1'=\phi_1 \quad (12a)$$

$$\frac{1}{Le}\phi_1''+f_0\phi_1'+\phi_0'f_0-\phi_0^n=0 \quad (12b)$$

$$f_1(0)=0,\ \phi_1(0)=0,\ \phi_1(\infty)=0 \quad (12c)$$

As is readily seen, two parameters, Le and n, are involved in the present problem.

3. Numerical Approach and Results

Equations (11a)-(12a) have been solved using a shooting method, while for (11b)-(12b) the superposition method was used, see for instance [5]. The maximum value of η (t infinity) sufficient to achieve an accuracy of 10^{-6} was 10, except for Le = 0.1, when it was set to 15.

In Figs.1-2 there is shown the variation of the dimensionless stream function across the boundary layer, when *n* = 1 and 3, for Le = 0.1, 1, 10 and 100.

Figs. 3 to 5 depict the velocity profiles across the boundary layer for *n* = 1 and 3. It is worth to remark that the dimensionless velocity is equal to the dimensionless concentration according to Eq. (7).

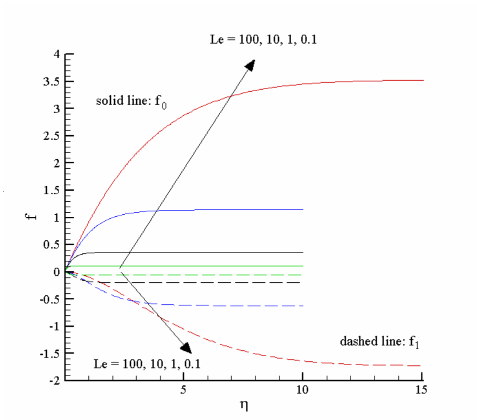

Fig.1. *Dimensionless stream function when n = 1.*

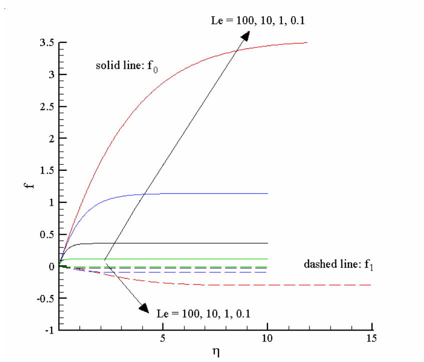

Fig.2. *Dimensionless stream function, when n = 3.*

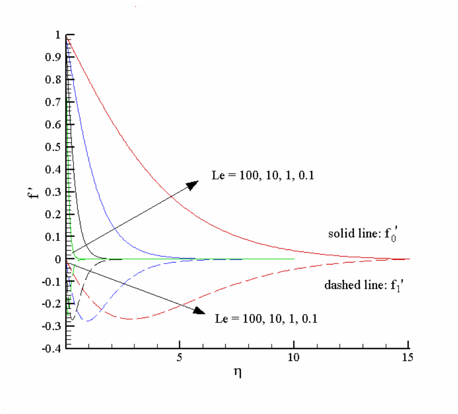

Fig.3. *Velocity profiles, n = 1, zeroth and first order solutions.*

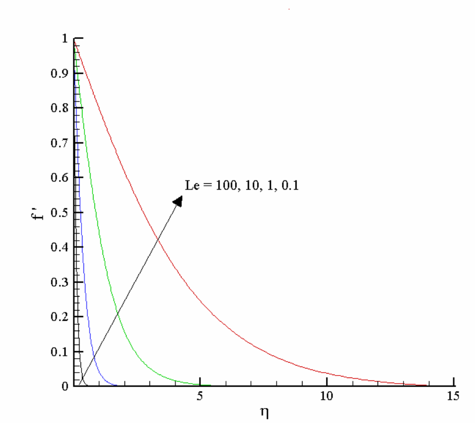

Fig.4. *Velocity profiles, n = 1*

A final comment is on the small parameter ε, quantity that dictates on the final solution. In [7] it was chosen to be 0.01. From our Table 1 and from the figures it is seen that the order of magnitude of the first order solution is roughly the same as that of the zeroth order solution. So, a choice of a maximum 0.01 should be appropriate, in order to get appropriate results. On the other hand, the definition of ε given in (8) does not give us much chances to calculate it on a physical basis. So, it was decided to show graphically the stream function and velocity across the boundary layer for the zeroth and first order solutions.

Dimensionless wall shear-stress. Table 1

	Le = 0.1	Le = 1	Le = 10	Le = 100
$f_0''(0)$	-0.19948	-0.62756	-1.98450	-6.27554
n	$f_1''(0)$	$f_1''(0)$	$f_1''(0)$	$f_1''(0)$
1	-0.218055	-0.69979	-2.17852	-6.53320
2	-0.13900	-0.43593	-1.32077	-3.62634
3	-0.10427	-0.32418	-0.95866	-2.42976

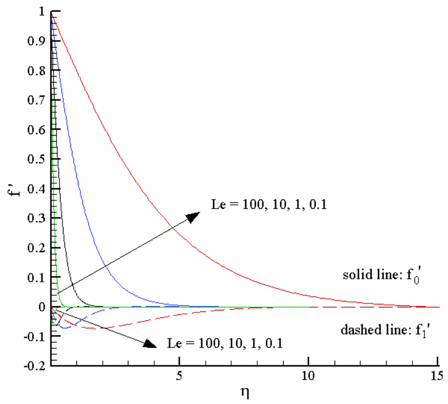

Fig.5. *Velocity profiles, n = 3, zeroth and first order solutions*

3. Conclusion

Steady-state free convection flow over a semi-infinite vertical flat plate located in a fluid saturated porous medium was studied in this paper. The plate is maintained at a given concentration and the processes involved in the physics are diffusion and a chemical reaction in the porous medium. In its final formulation, the problem is rendered in dimensionless quantities (stream function and concentration) and it depends on two parameters (Lewis number and order of the chemical reaction). The effects of these parameters are highlighted by numerical simulations.

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