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# SOME APPLICATIONS IN LASER FIELD 

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#### Abstract

Movement of an electron around the atomic nucleus has today a great importance in many engineering fields. In this paper one determines the pumping frequency required to achieve the transition of the electrons between two energetically levels or sub levels. The LASER pumping frequencies presented in the table 1 are all outside the visible area. It can make Ultraviolet Frequency-X ray LASER. The bold value can be used to make a Rubin (Crystal) LASER. In any cases it can be made a LASER located in the visible area (see the table 2). The paper realizes a new quantum theory. It determines as well the frequency of pumping for the transition between two energetically levels, with possible applications in LASER, MASER industry. Keywords: LASER, electron, atom, quantum field, MASER


## 1. INTRODUCTION

This paper briefly describes how to determine the relationships by which it calculates the ray of an electron moving on an orbit around an atom. Then it determines the velocity of an electron when the electron is moving on an orbit of an atom. Practical we work with the squared electron velocity. When is known the electron velocity it can determine the mass of the electron which is in moving. It may now determine and total energy of the electron.

Relationships are genuine and they differ from the classic relations, known. In addition, we now have two instead of one relationship.

Finally, we can write the frequency of pumping between the two energetic sub levels, adjacent.
We can write more relationships to the laser frequencies of pumping.

## 2. THE RELATIONSHIPS

### 2.1. Determining the ray of an electron moving on an orbit around an atom

The main relationships 1 and 2 are written [2]:
$\left.\left.\begin{array}{r}\text { Kinetic energy } E_{c}=\frac{1}{2} \cdot m \cdot v^{2} \\ \text { Coulomb form } E_{C}=\frac{1}{8} \cdot \frac{Z \cdot e^{2}}{\pi \cdot \varepsilon_{0} \cdot r}\end{array}\right\} \Rightarrow m=\frac{Z \cdot e^{2}}{4 \cdot \pi \cdot \varepsilon_{0} \cdot r \cdot v^{2}}\right\} \Rightarrow$

$$
\Rightarrow \frac{m_{0} \cdot c}{\sqrt{c^{2}-v^{2}}}=\frac{Z \cdot e^{2}}{4 \cdot \pi \cdot \varepsilon_{0} \cdot r \cdot v^{2}} \Rightarrow\left\{\begin{array}{l}
l \cdot r \cdot c \cdot v^{2}=\sqrt{c^{2}-v^{2}} \\
\text { with } \quad l=\frac{4 \cdot \pi \cdot m_{0} \cdot \varepsilon_{0}}{Z \cdot e^{2}}
\end{array}\right.
$$

$\left.\begin{array}{rl}\text { Niels Bohr relation } m & =\frac{\varepsilon_{0} \cdot h^{2} \cdot n^{2}}{\pi \cdot e^{2} \cdot Z \cdot r} \\ \text { Lorentz relation } m & =\frac{m_{0} \cdot c}{\sqrt{c^{2}-v^{2}}}\end{array}\right\} \Rightarrow \frac{m_{0} \cdot c}{\sqrt{c^{2}-v^{2}}}=\frac{\varepsilon_{0} \cdot h^{2} \cdot n^{2}}{\pi \cdot e^{2} \cdot Z \cdot r} \Rightarrow$

$$
\Rightarrow \frac{\pi \cdot m_{0} \cdot e^{2} \cdot Z}{\varepsilon_{0} \cdot h^{2} \cdot n^{2}} \cdot r \cdot c=\sqrt{c^{2}-v^{2}} \Rightarrow\left\{\begin{array}{l}
k \cdot r \cdot c=\sqrt{c^{2}-v^{2}}  \tag{2}\\
\text { with } k=\frac{\pi \cdot m_{0} \cdot e^{2} \cdot Z}{\varepsilon_{0} \cdot h^{2} \cdot n^{2}}
\end{array}\right.
$$

It put the relationship 2 at the square and we obtain the formula 3 .
$v^{2}=c^{2}-k^{2} \cdot r^{2} \cdot c^{2}$
3 is inserted in the relationship 1 and we obtain the relations 4 .

$$
\left\{\begin{array}{l}
l \cdot r \cdot c \cdot\left(c^{2}-k^{2} \cdot r^{2} \cdot c^{2}\right)=\sqrt{c^{2}-c^{2}+k^{2} \cdot r^{2} \cdot c^{2}} \Rightarrow l \cdot r \cdot c \cdot c^{2} \cdot\left(1-k^{2} \cdot r^{2}\right)=\sqrt{k^{2} \cdot r^{2} \cdot c^{2}} \\
l \cdot r \cdot c \cdot c^{2} \cdot\left(1-k^{2} \cdot r^{2}\right)= \pm r \cdot c \cdot k \Rightarrow l \cdot c^{2} \cdot\left(1-k^{2} \cdot r^{2}\right)= \pm k \Rightarrow 1-k^{2} \cdot r^{2}= \pm \frac{k}{l \cdot c^{2}} \Rightarrow \\
r^{2}=\frac{1}{k^{2}} \cdot\left(1 \mp \frac{k}{l \cdot c^{2}}\right) \Rightarrow r= \pm \frac{1}{k} \cdot \sqrt{1 \mp \frac{k}{l \cdot c^{2}}} \Rightarrow r=\frac{1}{k} \cdot \sqrt{1 \mp \frac{k}{l \cdot c^{2}}} \Rightarrow  \tag{4}\\
r=\frac{\varepsilon_{0} \cdot h^{2} \cdot n^{2}}{\pi \cdot m_{0} \cdot e^{2} \cdot Z} \cdot \sqrt{1 \mp \frac{\pi \cdot m_{0} \cdot e^{2} \cdot Z \cdot e^{2} \cdot Z}{n^{2} \cdot \varepsilon_{0} \cdot h^{2} \cdot 4 \cdot \pi \cdot m_{0} \cdot \varepsilon_{0} \cdot c^{2}}} \Rightarrow \\
r=\frac{\varepsilon_{0} \cdot h^{2} \cdot n^{2}}{\pi \cdot m_{0} \cdot e^{2} \cdot Z} \cdot \sqrt{1 \mp \frac{e^{4} \cdot Z^{2}}{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}}
\end{array}\right.
$$

The final form (in 4) determines the ray of an electron running on an orbit around an atom. We have two r values at a single principal quantum number, n. It obtains a new and doubled relationship [2].

### 2.2. Determining the velocities of an electron which is running around an atom

From relationship 1 it obtains the speed of an electron to the square. We determine relationships numbered with 5 .

$$
\begin{align*}
& \int v^{2}=\frac{2 \cdot c^{2}}{1+\sqrt{1+4 \cdot c^{4} \cdot r^{2} \cdot l^{2}}} \Rightarrow v^{2}=\frac{2 \cdot c^{2}}{1+R} \quad \text { with } \quad R=\sqrt{1+4 \cdot c^{4} \cdot r^{2} \cdot l^{2}} \\
& R=\sqrt{1+4 \cdot c^{4} \cdot r^{2} \cdot l^{2}}=\sqrt{1+\frac{4 \cdot c^{4} \cdot l^{2}}{k^{2}} \mp 2 \cdot \frac{2 \cdot c^{2} \cdot l}{k}}=\sqrt{\left(1 \mp 2 \cdot \frac{c^{2} \cdot l}{k}\right)^{2}}= \\
& =\left|1 \mp 2 \cdot \frac{c^{2} \cdot l}{k}\right|=\left\{\begin{array}{l}
\frac{2 \cdot c^{2} \cdot l}{k}-1=\frac{8 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}-1 \\
\frac{2 \cdot c^{2} \cdot l}{k}+1=\frac{8 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}+1
\end{array} \text { with } E=\frac{2 \cdot c^{2} \cdot l}{k}>1\right. \\
& v_{-}^{2}=\frac{2 \cdot c^{2}}{1+\frac{8 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}-1}=\frac{2 \cdot c^{2}}{\frac{8 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}}=\frac{c^{2}}{\frac{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}}=\frac{k}{l} \\
& v_{+}^{2}=\frac{2 \cdot c^{2}}{1+\frac{8 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}+1}=\frac{2 \cdot c^{2}}{\frac{8 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}+2}=\frac{c^{2}}{\frac{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}+1}=\frac{k c^{2}}{l c^{2}+k} \tag{5}
\end{align*}
$$

### 2.3. Determining the mass of the electron in movement

When the speeds are known is simple to find quickly the masses values (forms 6).

$$
\begin{equation*}
m_{-}=\frac{m_{0}}{\sqrt{1-\frac{1}{\frac{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}}}} \quad \quad m_{+}=\frac{m_{0}}{\sqrt{1-\frac{1}{\frac{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}+1}}} \tag{6}
\end{equation*}
$$

### 2.4. Determining the energy of the electron in movement

To determine the energy of an electron in movement, it multiplies the mass of an electron with the squared speed of light (using the Einstein relation), (forms 7).

$$
\begin{equation*}
W_{-}=\frac{m_{0} \cdot c^{2}}{\sqrt{1-\frac{1}{\frac{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}}}} \quad W_{+}=\frac{m_{0} \cdot c^{2}}{\sqrt{1-\frac{1}{\frac{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}+1}}} \tag{7}
\end{equation*}
$$

### 2.5. Determining the frequencies of pumping

Finally, we can write the frequency of pumping between the two energetic sub levels, adjacent (see the form 8).

$$
\begin{equation*}
v=\frac{W_{1}-W_{2}}{h}=\frac{m_{0} \cdot c^{2}}{h} \cdot\left(\frac{1}{\left.\sqrt{1-\frac{1}{\frac{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot c^{2} \cdot n^{2}}{e^{4} \cdot Z^{2}}}}-\frac{1}{\sqrt{1-\frac{1}{\frac{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot c^{2} \cdot n^{2}}{e^{4} \cdot Z^{2}}+1}}}\right)}\right. \tag{8}
\end{equation*}
$$

### 2.6. Notes utilized (used notations) (forms 9)

The permissive constant (the permittivity) : $\varepsilon_{0}=8.85418 \cdot 10^{-12}\left[\frac{C^{2}}{N \cdot m^{2}}\right]$
The Planck constant :

$$
h=6.626 \cdot 10^{-34} \quad[J \cdot s]
$$

The rest mass of electron

$$
\begin{equation*}
m_{0}=9.1091 \cdot 10^{-31} \quad[\mathrm{~kg}] \tag{9}
\end{equation*}
$$

The Pythagora's number :
$\pi=3.141592654$

The electrical elementary load: $\quad e=-1.6021 \cdot 10^{-19}[C]$

The light speed in vacuum :

$$
c=2.997925 \cdot 10^{8}\left[\frac{\mathrm{~m}}{\mathrm{~s}}\right]
$$

$\mathrm{n}=$ the principal quantum number (the Bohr quantum number)
$Z=$ the number of protons from the atomic nucleus (the atomic number)

## 3. THE LASER FREQUENCIES

In the table 1 it determines the frequency of pumping for the transition between the first two energetic adjacent sub levels, with possible applications in LASER, MASER industry (relation (8) with $\mathrm{n}=1$ ). All frequencies, calculated in the table one, are outside of the visible domain $\left(4.34 * 10^{14} \div 6.97 * 10^{14}[\mathrm{~Hz}]\right)$.

Table 1: The frequencies of pumping between the first sub levels adjacent, $n=1$

| The pumping frequencies, between the first two adjacent sub levels, $\mathbf{n}=1$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Z | y | Element | Z | y | Element | Z | y | Element |
| 1 | $1.75131 \mathrm{E}+11$ | H | 2 | $2.80231 \mathrm{E}+12$ | He | 3 | $1.41886 \mathrm{E}+13$ | Li |
| 4 | $4.48513 \mathrm{E}+13$ | Be | 5 | $1.09527 \mathrm{E}+14$ | B | 6 | $2.27181 \mathrm{E}+14 \mathrm{C}$ | C |
| 7 | $4.21028 \mathrm{E}+14$ | N | 8 | $7.18543 \mathrm{E}+14$ | O | 9 | $1.15149 \mathrm{E}+15 \mathrm{~F}$ | F |
| 10 | $1.75595 \mathrm{E}+15$ | Ne | 11 | $2.57234 \mathrm{E}+15$ | Na | 12 | $3.64547 \mathrm{E}+15$ | Mg |
| 13 | $5.02453 \mathrm{E}+15$ | Al | 14 | $6.7632 \mathrm{E}+15$ | Si | 15 | $8.91965 \mathrm{E}+15 \mathrm{P}$ | P |
| 16 | $1.15566 \mathrm{E}+16$ | S | 17 | $1.47414 \mathrm{E}+16$ | Cl | 18 | $1.8546 \mathrm{E}+16$ | Ar |
| 19 | $2.30473 \mathrm{E}+16$ | K | 20 | $2.83266 \mathrm{E}+16$ | Ca | 21 | $3.44704 \mathrm{E}+16$ | Sc |
| 22 | $4.15702 \mathrm{E}+16$ | Ti | 23 | $4.97224 \mathrm{E}+16$ | V | 24 | 5.90284E+16 | Cr |
| 25 | $6.95952 \mathrm{E}+16$ | Mn | 26 | $8.1535 \mathrm{E}+16$ | Fe | 27 | $9.49654 \mathrm{E}+16$ | Co |
| 28 | $1.1001 \mathrm{E}+17$ | Ni | 29 | $1.26797 \mathrm{E}+17$ | Cu | 30 | $1.45461 \mathrm{E}+17 \mathrm{Z}$ | Zn |
| 31 | $1.66145 \mathrm{E}+17$ | Ga | 32 | $1.88994 \mathrm{E}+17$ | Ge | 33 | $2.14162 \mathrm{E}+17$ | As |
| 34 | $2.41809 \mathrm{E}+17$ | Se | 35 | $2.72101 \mathrm{E}+17$ | Br | 36 | $3.05214 \mathrm{E}+17$ | Kr |
| 37 | $3.41326 \mathrm{E}+17$ | Rb | 38 | $3.80627 \mathrm{E}+17$ | Sr | 39 | $4.23312 \mathrm{E}+17 \mathrm{Y}$ | Y |
| 40 | $4.69585 \mathrm{E}+17$ | Zr | 41 | $5.19659 \mathrm{E}+17$ | Nb | 42 | $5.73753 \mathrm{E}+17$ | Mo |
| 43 | $6.32098 \mathrm{E}+17$ | Tc | 44 | $6.94931 \mathrm{E}+17$ | Ru | 45 | $7.62502 \mathrm{E}+17$ | Rh |
| 46 | $8.35067 \mathrm{E}+17$ | Pd | 47 | $9.12897 \mathrm{E}+17$ | Ag | 48 | $9.9627 \mathrm{E}+17 \mathrm{Cd}$ | Cd |
| 49 | $1.08548 \mathrm{E}+18$ | In | 50 | $1.18082 \mathrm{E}+18$ | Sn | 51 | $1.28262 \mathrm{E}+18$ | Sb |
| 52 | $1.39119 \mathrm{E}+18$ | Te | 53 | $1.50689 \mathrm{E}+18$ | I | 54 | $1.63006 \mathrm{E}+18$ | Xe |
| 55 | $1.76108 \mathrm{E}+18$ | Cs | 56 | $1.90034 \mathrm{E}+18$ | Ba | 57 | $2.04823 \mathrm{E}+18$ | La |
| 58 | $2.20518 \mathrm{E}+18$ | Ce | 59 | $2.37163 \mathrm{E}+18$ | Pr | 60 | $2.54804 \mathrm{E}+18$ | Nd |
| 61 | $2.73489 \mathrm{E}+18$ | Pm | 62 | $2.93267 \mathrm{E}+18$ | Sm | 63 | $3.14191 \mathrm{E}+18$ E | Eu |
| 64 | $3.36316 \mathrm{E}+18$ | Gd | 65 | $3.597 \mathrm{E}+18$ | Tb | 66 | $3.84402 \mathrm{E}+18$ | Dy |
| 67 | $4.10486 \mathrm{E}+18$ | Ho | 68 | $4.38017 \mathrm{E}+18$ | Er | 69 | $4.67067 \mathrm{E}+18$ T | Tm |
| 70 | $4.97706 \mathrm{E}+18$ | Yb | 71 | $5.30013 \mathrm{E}+18$ | Lu | 72 | $5.64068 \mathrm{E}+18$ | Hf |
| 73 | $5.99956 \mathrm{E}+18$ | Ta | 74 | $6.37768 \mathrm{E}+18$ | W | 75 | $6.77596 \mathrm{E}+18$ | Re |
| 76 | $7.19542 \mathrm{E}+18$ | Os | 77 | $7.63712 \mathrm{E}+18$ | Ir | 78 | $8.10216 \mathrm{E}+18$ | Pt |
| 79 | $8.59173 \mathrm{E}+18$ | Au | 80 | $9.10709 \mathrm{E}+18$ | Hg | 81 | $9.64956 \mathrm{E}+18$ | Tl |
| 82 | $1.02206 \mathrm{E}+19$ | Pb | 83 | $1.08216 \mathrm{E}+19$ | Bi | 84 | $1.14543 \mathrm{E}+19$ | Po |
| 85 | $1.21203 \mathrm{E}+19$ | At | 86 | $1.28215 \mathrm{E}+19$ | Rn | 87 | $1.35598 \mathrm{E}+19 \mathrm{Fr}$ | Fr |
| 88 | $1.43372 \mathrm{E}+19$ | Ra | 89 | $1.51562 \mathrm{E}+19$ | Ac | 90 | $1.60189 \mathrm{E}+19$ | Th |
| 91 | $1.6928 \mathrm{E}+19$ | Pa | 92 | $1.78864 \mathrm{E}+19$ | U | 93 | $1.88968 \mathrm{E}+19 \mathrm{~N}$ | Np |
| 94 | $1.99627 \mathrm{E}+19$ | Pu | 95 | $2.10875 \mathrm{E}+19$ | Am | 96 | $2.2275 \mathrm{E}+19$ C | Cm |
| 97 | $2.35293 \mathrm{E}+19$ | Bk | 98 | $2.4855 \mathrm{E}+19$ | Cf | 99 | $2.62568 \mathrm{E}+19$ | Es |
| 100 | $2.77402 \mathrm{E}+19$ | Fm | 101 | $2.93111 \mathrm{E}+19$ | Md | 102 | $3.09758 \mathrm{E}+19$ | No |
| 103 | $3.27416 \mathrm{E}+19$ | Lr | 104 | $3.46162 \mathrm{E}+19$ | Rf | 105 | $3.66084 \mathrm{E}+19$ | Db |

In the table 2 it determines the frequency of pumping for the transition between two energetic sub levels adjacent, with possible applications in LASER industry (relation (8) with $n=2,3,4,5$ ). All frequencies, calculated in the table two are inside of the visible domain $\left(4.34 * 10^{14} \div 6.97 * 10^{14}[\mathrm{~Hz}]\right)$.

Table 2: The LASER frequencies of pumping ( $\mathrm{n}=2-5$ )

| n | Z | $\nu[\mathrm{Hz}]$ | Element |
| :--- | :--- | :--- | :--- |
| 2 | 15 | $=5.54942 \mathrm{E} 14$ | P |
| 3 | 22 | $=5.072 \mathrm{E} 14$ | Ti |
|  | 23 | $=6.0598 \mathrm{E} 14$ | V |
|  | 29 | $=4.8452 \mathrm{E}+14$ | Cu |
|  | 30 | $=5.54942 \mathrm{E}+14$ | Zn |
|  | 31 | $=6.32782 \mathrm{E}+14$ | Ga |
| 5 | 37 | $=5.25911 \mathrm{E}+14$ | Rb |
|  | 38 | $=5.8516 \mathrm{E}+14$ | Sr |
|  | 39 | $=6.49284 \mathrm{E}+14$ | Y |

## 4. CONCLUSION

All frequencies, calculated in the table 1, are outside of the visible domain $\left(4.34 * 10^{14} \div 6.97 * 10^{14}[\mathrm{~Hz}]\right)$.
Only the atmospheric elements, N and O , are located near the visible frequencies when $\mathrm{n}=1$.
The bold value can be used to make a Rubin (Crystal) LASER.
For $\mathrm{n}=2-5$ there are nine values indicated to make a LASER in the visible domain (see the table 2).
The substance is structured in this mode, that, we can obtain more energy, if one can penetrate it deeply. In this mode, we can check and extract, small portions of energy, but the total obtained energy will be bigger.

The atomic electrons are coupled. The transition between the two coupled electrons can give us more energy, in small portions.

First, we can make a stronger "Electromagnetic Amplification by the Stimulated Emission of Radiation" LASER (MASER), by pumping the energy between two sub levels, adjacent.

This paper briefly describes how to determine the relationships by which it calculates the ray of an electron moving on an orbit around an atom.

Now, it's the time to correct the length of the $r$ radius (see the Cap. 5. (4) $\rightarrow$ (12)).

## 5. CORRECTING THE LENGTH OF THE R RADIUS

The main expression (2) can be written in the form (10).
$r=\frac{1}{k} \cdot \sqrt{1-\frac{v^{2}}{c^{2}}}$
The velocities have the forms (11), known.
$\left\{\begin{array}{l}v_{-}^{2}=\frac{k \cdot c^{2}}{l \cdot c^{2}}=\frac{k}{l} \\ v_{+}^{2}=\frac{k \cdot c^{2}}{l \cdot c^{2}+k}\end{array}\right.$
With the relations (11) the expression (10) takes the forms (12).

$$
\left\{\begin{array}{l}
r_{-}=\frac{1}{k} \cdot \sqrt{1-\frac{k}{l \cdot c^{2}}}=\frac{\varepsilon_{0} \cdot h^{2} \cdot n^{2}}{\pi \cdot m_{0} \cdot e^{2} \cdot Z} \cdot \sqrt{1-\frac{e^{4} \cdot Z^{2}}{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}}  \tag{12}\\
r_{+}=\frac{1}{k} \cdot \sqrt{1-\frac{k}{l \cdot c^{2}+k}}=\frac{\varepsilon_{0} \cdot h^{2} \cdot n^{2}}{\pi \cdot m_{0} \cdot e^{2} \cdot Z} \cdot \sqrt{1-\frac{e^{4} \cdot Z^{2}}{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}+e^{4} \cdot Z^{2}}}
\end{array}\right.
$$

The values imposed by relations 12 are probably the real physical values, because the main relations 1 and 2 are verified in the same time by the relationships 12 .

The velocities, masses, energies and frequency of pumping have not changed (see a recap in cap. 6, relations 13-16).

## 6. RECAP

$$
\begin{align*}
& \left\{\begin{array}{l}
r_{-}=\frac{1}{k} \cdot \sqrt{1-\frac{k}{l \cdot c^{2}}}=\frac{\varepsilon_{0} \cdot h^{2} \cdot n^{2}}{\pi \cdot m_{0} \cdot e^{2} \cdot Z} \cdot \sqrt{1-\frac{e^{4} \cdot Z^{2}}{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}} \\
r_{+}=\frac{1}{k} \cdot \sqrt{1-\frac{k}{l \cdot c^{2}+k}}=\frac{\varepsilon_{0} \cdot h^{2} \cdot n^{2}}{\pi \cdot m_{0} \cdot e^{2} \cdot Z} \cdot \sqrt{1-\frac{e^{4} \cdot Z^{2}}{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}+e^{4} \cdot Z^{2}}}
\end{array}\right.  \tag{13}\\
& \left\{\begin{array}{l}
v_{-}^{2}=\frac{2 \cdot c^{2}}{1+\frac{8 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}-1}=\frac{2 \cdot c^{2}}{\frac{8 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}}=\frac{c^{2}}{\frac{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}}=\frac{k}{l} \\
v_{+}^{2}=\frac{2 \cdot c^{2}}{1+\frac{8 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}+1}=\frac{2 \cdot c^{2}}{\frac{8 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}+2}=\frac{c^{2}}{\frac{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}+1}=\frac{k \cdot c^{2}}{l \cdot c^{2}+k}
\end{array}\right.  \tag{14}\\
& m_{-}=\frac{m_{0}}{\sqrt{1-\frac{1}{\frac{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}}}}  \tag{15}\\
& m_{+}=\frac{m_{0}}{\sqrt{1-\frac{1}{\frac{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}+1}}} \\
& W_{-}=\frac{m_{0} \cdot c^{2}}{\sqrt{1-\frac{1}{\frac{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}}}}  \tag{16}\\
& W_{+}=\frac{m_{0} \cdot c^{2}}{\sqrt{1-\frac{1}{\frac{4 \cdot \varepsilon_{0}^{2} \cdot h^{2} \cdot n^{2} \cdot c^{2}}{e^{4} \cdot Z^{2}}+1}}}
\end{align*}
$$

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