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THE INFLUENCE OF FORCED STEERING VIBRATIONS ON A WHEEL AND DYNAMIC EFFECT OF A WHEEL WITH ABS BRAKING ON UNDULATED ROAD I

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Abstract: *A steering/suspension system of an automobile exhibits a rather complex configuration and possesses many degrees of freedom. A simplification is necessary to conduct a sensible analysis to gain insight into its general dynamic behavior and into the influence of important parameters of the system. Investigation of the steering mode of vibration requires at least the steering degree of freedom of the front wheel, possibly extended with the rotation degree of freedom of the steering wheel. For the sake of simplicity, one degree of freedom may be suppressed by holding the steering spring clamped at the node of the natural mode of vibration.*
Keyword: *vibration, degrees of freedom, motion, wheel, brake*

1. INTRODUCTION

We will consider the influence of two more degrees of freedom: the vertical axle motion and the longitudinal deflection of the suspension with respect to the steadily moving vehicle mass. The picture of Fig.1 shows the lay-out of system. Due to the assumed orthogonally of the system (wheel axis, king-pin, road plane) the dynamically coupled horizontal motions ($x\psi$) are not coupled with the vertical axle motion when small displacements are considered and tyre contact forces are disregarded. First, we will examine the dynamics of the free system not touching the road. After that, the tyre is loaded and tyre transient models for the in-plane and out-of-plane behavior are introduced and the system response to wheel unbalance will be assessed and discussed.

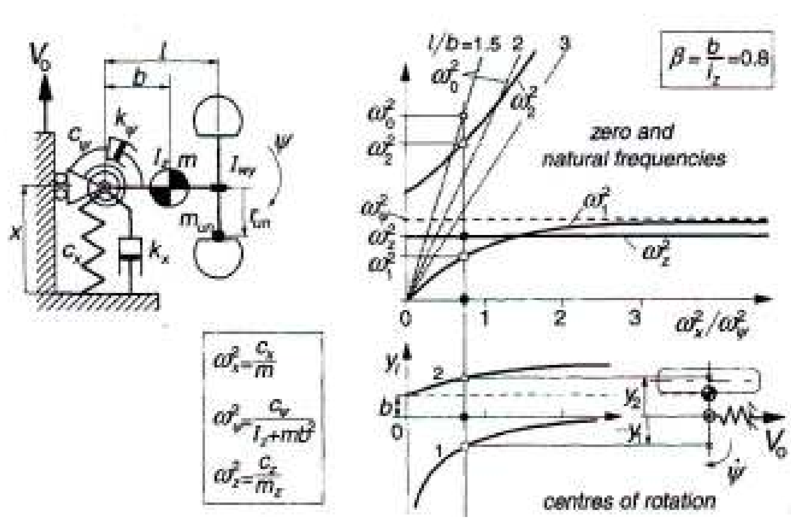

Figure 1: Configuration of a simple steering/suspension system and the resulting natural frequencies and vibration modes.

2. DYNAMICS OF THE UNLOADED SYSTEM EXCITED BY WHEEL UNBALANCE

The simple system depicted in Fig.1 possesses two horizontal degrees of freedom: the rotation about the vertical steering axis, ψ , and the fore and aft suspension deflection, *x*. The figure provides details about the geometry, stiffnesses, damping and inertia. The mass *m* represents the total mass of the horizontally moving parts. The length i_z denotes the radius of inertia: $i^2{}_z = i_z / m$.

The wheel rim that revolves with a speed Ω is provided with an unbalance mass m_{un} (in the wheel centre plane at a radius r_{un}). The centrifugal force has a component in forward direction:

$$F_{un,x} = -m_{un} r_{un} \Omega^2 \sin \Omega t \tag{1}$$

The equations of motion of this fourth-order system read:

$$m\ddot{x} - mb\ddot{\psi} + k_x + \dot{x} + c_x x = F_{un,x} \tag{2}$$

$$m(i_z^2 + b^2)\ddot{\psi} - mb\ddot{x} + k_\psi \dot{\psi} + c_\psi \psi = -lF_{un,x} \tag{3}$$

With damping disregarded, the magnitude of the frequency response function becomes:

$$\frac{\dot{\psi}}{m_{un} r_{un}} = \frac{1}{c_\psi} \frac{\Omega^2 \{1 - (\frac{\Omega}{\omega_0})^2\}}{\{1 - (\frac{\Omega}{\omega_1})^2\}\{1 - (\frac{\Omega}{\omega_2})^2\}} \tag{4}$$

in which we have the zero frequency:

$$\omega_0^2 = \frac{l}{l-b} \omega_x^2 \tag{5}$$

and the two natural frequencies:

$$\omega_{1,2}^2 = \frac{1}{2}(1+\beta)\{(\omega_x^2 + \omega_\psi^2) \pm (\omega_x^2 - \omega_\psi^2)\sqrt{1 + \frac{4\beta}{1+\beta} \frac{\omega_x^2 \omega_\psi^2}{(\omega_x^2 - \omega_\psi^2)^2}}\} \tag{6}$$

where we have introduced the 'uncoupled' natural frequencies:

$$\omega_x^2 = \frac{c_x}{m} \quad \text{and} \quad \omega_\psi^2 = \frac{c_\psi}{m(i^2 + b^2)} \tag{7}$$

and the coupling factor:

$$\beta = \frac{b^2}{i_z^2} \tag{8}$$

For the analysis, we are interested in the influence of the fore and aft compliance of the suspension. In the right-hand diagram of Fig.1 the two natural frequencies have been plotted as a function of the longitudinal natural frequency ratio (squared), which is proportional to the longitudinal stiffness $\boldsymbol{c_x}$, together with the constant vertical natural frequency and the zero frequency for three different values of *f/b*. In addition, the location of the two centers of rotation according to the two modes of the undraped vibration have been indicated.

The two steer natural frequencies ω_1 and ω_2 increase with increasing longitudinal suspension stiffness. The lower natural frequency with a centre of rotation located at the inside of the king-pin, approaches the uncoupled natural frequency ω_ψ .

From (5) it is seen that a zero does not occur if the unbalance arm length *l* lies in the range $0<l<b$ which does not represent a usual configuration. For the normal situation with $b>0$ the zero frequency line may cross the second natural frequency curve if *l* is not too large. If the two frequencies coincide, the second resonance peak of the steer response to unbalance will be suppressed. In that case, the unbalance force line of action passes through the centre of rotation of the higher vibration mode.

It may be noted that the situation with contact between wheel and road can be simply modeled if the wheel is assumed to be rigid. The same equations apply with inertia parameters adapted according to the altered system with a point mass attached to the axle in the wheel plane. The point mass has the value l_w / r^2 where l_w denotes the wheel polar moment of inertia and *r* the wheel radius.

3. DYNAMICS OF THE LOADED SYSTEM WITH TIRE PROPERTIES INCLUDED

In a more realistic model the in-plane and out-of-plane slip, compliance and inertia parameters should be taken into account. A possible important aspect is the interaction between vertical tyre deflection and longitudinal slip which may cause the appearance of a third resonance peak near the vertical natural frequency of the wheel system. The longitudinal carcass compliance gives rise to an additional natural frequency around 40 Hz of the wheel rotating against the foot print Due to damping, originating from tangential slip of the tyre, a supercritical condition will arise beyond a certain forward velocity. This causes the additional natural frequency to disappear.

For the extended system with road contact, the following complete set of linear equations apply:

$$m\,(\ddot{x} + 2\zeta_x \omega_x \dot{x} + \omega_x^2 x) - mb\,\ddot{\psi} - F_x = F_{un,x} \tag{9}$$

$$m_z(\ddot{z} + 2\zeta_x \omega_z \dot{z} + \omega_z^2 z) = F_{un,x} \tag{10}$$

$$l_\psi(\ddot{\psi} + 2\zeta_\psi \omega_\psi \dot{\psi} + \omega_\psi^2 \psi) - mb\ddot{x} + lF_x - M_z = -lF_{un,x} \tag{11}$$

$$l_w(\dot{\Omega} + r_0 + F_x = 0 \tag{12}$$

$$\sigma_k \dot{F}_x + V_0 F_x = -C_{Fk} V_{sx} \tag{13}$$

$$\sigma_a \dot{F}_y + V_0 F_y = -C_{Fa} V_{sy} \tag{14}$$

$$V_{sx} = \dot{x} - l\dot{\psi} - r_0(\Omega - \Omega_0) + \Omega_0 \eta z \tag{15}$$

$$V_{sy} = -V_0 \psi \tag{16}$$

$$M_z = -t_a F_y - \frac{1}{V_0} k * \dot{\psi} - C_{gyr} V_0 \dot{F}_y \tag{17}$$

$$F_{un,x} = -m_{un} r_{un} \Omega_0^2 \sin \Omega_0 t \tag{18}$$

$$F_{un,z} = m_{un} r_{un} \Omega_0^2 \cos \Omega_0 t \tag{19}$$

$$V_0 = r_0 \Omega_0 \tag{20}$$

Table 1. Parameter values of wheel suspension system and type considered

m	30 kg	ω_x	70 rad/s	σ_κ	0.15 m	F_{z0}	3500N
m_z	40 kg	ω_ψ	85 rad/s	σ_α	0.3 m	C_{Fz}	160 kN/m
I_ψ	1.2 kgm^2	ζ_x	0.01	t_α	0.03 m	$C_{F\kappa}$	60 kN
I_w	0.8 kgm^2	ζ_z	0.06	κ^*	80 Nm^2	$C_{F\alpha}$	40 kN/rad
m_{un}	0.1 kg	ζ_ψ	0.08	C_{gyr}	2x10^{-5} s^2	η	0.4

Note that mechanical caster has not been considered so that the lateral slip speed is simply expressed by (16). The rolling resistance moment has been neglected in (12) and the average effective rolling radius has been taken equal to the average axle height or loaded radius r0 (in reality re is usually slightly larger than r0).

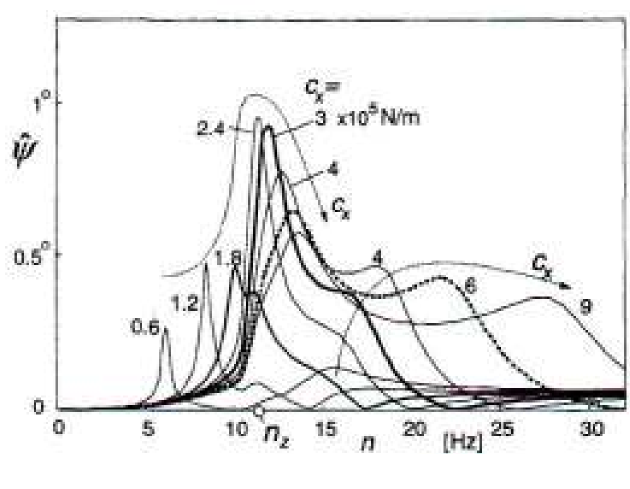

Figure 2: Steer vibration amplitude due to wheel unbalance as a function of wheel frequency of revolution $n = \Omega_0 / 2\pi$ for various values of the longitudinal suspension stiffness.

In Table 1 the set of parameter values used in the computations have been listed. The moment of inertia about the steering axis is denoted with i_ψ and equals $m(b^2 + i_z^2)$. The amplitude of the steer angle that occurs as a response to a wheel unbalance mass of 0.1kg has been plotted as a function of the wheel speed of revolution in Fig.2. To examine the influence of the longitudinal suspension compliance a series of values of the longitudinal stiffness c_x has been considered.

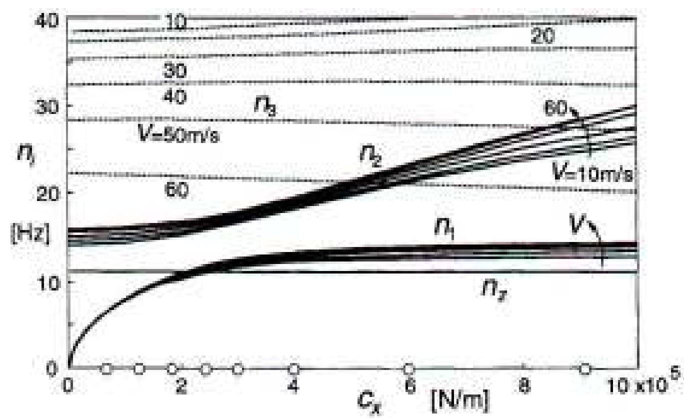

Figure 3: Variation of the three natural frequencies with longitudinal suspension stiffness at different speeds together with the constant vertical natural frequency. The circles on the horizontal axis mark the stiffness cases of Figure 2.

In figure 3 these values have been indicated by marks on the stiffness axis. Clearly, in agreement with the variation of the natural frequencies assessed in this figure, the two resonance peaks move to higher frequencies when the stiffness is raised. A third resonance peak may show up belonging to the vertical natural frequency. This peak remains at the same frequency. It is of interest to observe that when the lowest steer natural frequency n_1 coincides with n_z the interaction between vertical and horizontal motions causes the peak to reach relatively high levels. The zero frequency closely follows the formula (5) of the free system. At the lowest stiffness the zero frequency n_0 almost coincides with the second natural frequency n_2 and suppresses the second peak.

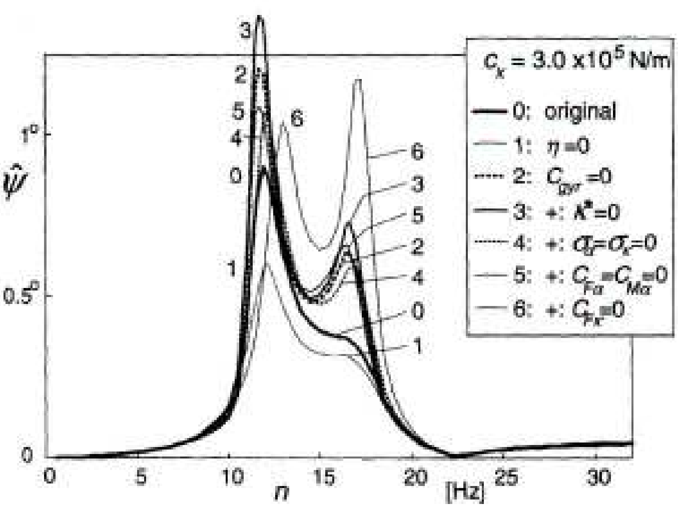

Figure 4: Influence the varies type parameters on the steer angle amplitude response curve

4 CONCLUSION

In Fig.4 the result of making these parameters equal to zero has been depicted for the stiffness case $c_x = 3X10^5\,N/m$. Neglecting the factor η (case 1) meaning that the effective rolling radius would not change with load, appears to have a considerable effect indicating that, with η, the vertical motion does amplify the steering oscillation. Omitting the gyroscopic tyre moment (2), while reinstating η, appears, as expected, to effectively decrease the steer damping. This is strengthened by additionally deleting the moment due to tread width (3). Omitting the relaxation lengths (4) lowers the peaks, thus removing the negative damping due to tyre compliance. Deleting, in addition, the side force and the aligning torque (5) raises the peaks again indicating that some energy is lost through the side slip. Disregarding the horizontal tyre forces altogether (6) brings us back to the (horizontally) free system. As predicted by the analysis, two sharp resonance peaks arise as well as the dip at the zero frequency.

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