Abstract: The paper deals with problems related to planetary and differential reducers for driving the coaxial propellers of aircraft. There are introduced a classification and the main kinematic schemes of the planetary reducers for coaxial propellers and there are established, also, the formulae for calculating the transmission ratio. There are pointed out some functional particularities of these reducers, which exhibit when functioning on aircrafts.

Keywords: planetary reducer, differential reducer, transmission ratio

1. GENERAL INTRODUCTION OF THE PLANETARY AND DIFFERENTIAL REDUCERS FOR COAXIAL PROPELLERS

For the turbo driving mechanism (TDM) with powers over 4,000 – 6,000 HP, it is necessary to use the coaxial propellers, because, by doing so, it can be obtained important advantages. Firstly, the use of 2 coaxial propellers ascertains the decrease of the propellers diameter. Secondly, the reducer, which assures to drive the propellers in opposite directions, allows increasing of their efficiency, generally, and decreasing of the strains of some parts, as a result of elimination of the reagent and gyroscope torques.

In this way it is achieved an improvement of the maneuverability of the aircraft, as well as, its longitudinal and lateral stability. Also, because the coaxial propellers reduce the twist of the air stream, there are improved the flow terms at the input in the admission device of the engine and on the aircraft wing. On the other side, it has to take into consideration that the use of the coaxial propellers needs more complex reducers, as constructive solutions, and more complicated command systems for the propellers step, as we shall describe further on.

The reducers used for driving of the coaxial propellers have diverse constructive schemes. A few schemes of simple reducers have been presented in [4]. Their essential drawback is that the same time when the transmission ratio is increasing, it is increasing rapidly their diameter. In principle, they can assure transmission ratio between 2.5,..., 5. Also, they have big losses by friction and it is difficult to achieve an uniform charging of bearings. From these reasons, for driving of the coaxial propellers, there are preferred the combined reducers, made by elementary gears and planetary mechanisms. Through this solution, it is obtained a reduction of the overall size of reducers, but it is increasing their constructive complexity. Besides, by constructive means, it has to be taken into consideration the general explanations concerning the aviation reducers, specified in the works [4, 5].

2. THE CLASSIFICATION AND THE MAIN KINEMATIC SCHEMES OF THE PLANETARY AND DIFFERENTIAL REDUCERS FOR COAXIAL PROPELLERS

A first type of reducers used for driving of coaxial propellers is obtained by connecting of a planetary mechanism to a simple reducer. The kinematic schemes of two such reducers are presented in Fig. 1. By setting up, actually, a distinct tread inside the reducer, through its transmission ratio, the planetary mechanism allows decreasing of the dimensions of the simple reducer. It can be seen that the second scheme assures a more compact construction of the reducer.
For driving the coaxial propellers of TDM, there are more frequently used the reducers made by using of the differential mechanisms of serial or parallel type. The kinematic schemes of some constructive solutions of these reducers are presented in Fig. 2. These reducers are superior, as regards the transmission ratio, efficiency and overall size to the other constructive solutions, but their use has to take into consideration a certain structural – kinematic specific feature of the differential mechanisms, which is reflected propellers working. Thus, by applying to the mechanisms from Fig. 2, the structural formula of the plane mechanisms, it can be easily observed that their DOF is \( M = 2 \). The kinematic study is achieved by using the Willis method and in the hypothesis that through the reverse motion it is stopped the front propeller, for instance, for the mechanism from Fig. 2, we obtain the equation:

\[
\frac{n_1 - n_f}{n_9 - n_f} = \frac{Z_2Z_4}{Z_1Z_3} \tag{2.1}
\]

where \( n_1 \) is the speed of the input shaft into the reducer (equal to the turbine shaft speed), \( n_f \) and \( n_9 \) the speed of the front, respectively, the rear propeller.

Because in the left part of the equation (2.1) is known only \( n_1 \), it results that the respective differential mechanism is kinematical unascertained, that means that for one rotation given to the turbine shaft, the propellers speeds can be different. That means, further on, that the powers sent to both propellers are different and dependent on the speed and on the attack angle (step) of the propellers blades.

Thus, if the speed \( n_f \) remains stable (the power delivered is stable) and the propeller step from the rear is decreasing, then its speed is increasing and the speed of the front propeller is decreasing and reverse (in the specified terms this behavior is valid and, also, for the front propeller).

If the rear propeller is broken until its complete stopping, the differential transmission becomes a planetary one and if the front propeller it stops, it is obtained a simple reducer. Therefore, when using differential reducers, the propellers must have variable step and they have to be equipped with distinct speed centrifugal regulators, each of them driven by the adequate propeller, which has to maintain a pre-established speed of the propellers, by modifying the attack angle of the blades. At a differential reducer, the equality of the propellers speed can be maintained even with one single regulator, but, in this case, it has to be applied the operation of “closing” of the differential mechanism (this type of reducers will be studied in another paper).

Through elementary changes the equation (2.1) can be written as:
\[ n_s = \left( 1 + \frac{z_3}{z_4} \right) \cdot n_t - \frac{z_3}{z_2} \cdot n_1 \]  \hspace{1cm} (2.2),

and it will help to establish the limit speeds of each propeller, from the condition that the other one being fixed:

- for \( n_s = 0 \), \[ n_t = 1 + \frac{z_2}{z_3} \cdot n_1 \]  \hspace{1cm} (2.3)

- for \( n_t = 0 \), \[ n_s = -\frac{z_2}{z_3} \cdot n_1 \]  \hspace{1cm} (2.4)

If the speeds of both propellers are equal and of opposite directions \( (n_t = -n_s) \), from (2.2) results:

\[ n_t/n_s = 1 + 2\left( \frac{z_2}{z_4}/z_1/z_4 \right) \]  \hspace{1cm} (2.5)

For these reducers, the minimum transmission ratio is \( i = 10, \ldots, 12 \).

For the differential reducer, the satellites speed is depending on the turbine shaft speed \( n_t \) and on the front propeller speed \( n_1 \). In the general case, it can be established with:

\[ n_{sat} = -\frac{z_1}{z_2} \]  \hspace{1cm} (2.6)

To establish the satellites speed, in case of the equality of the propellers speeds, there are using the relations (2.6) and (2.5) and result:

\[ n_{sat} = -2n_t/\left( z_3/z_4 + 2z_2/z_1 \right) \.]  \hspace{1cm} (2.7)

Same result we can obtain, also, for the reducer from Fig. 2 b, but using a faster method. Because \( i_{1s} = n_t/n_s \), by taking into consideration the relation

\[ i_{1s} = \frac{n_1 - n_5}{n_3 - n_5} \]  \hspace{1cm} (2.8)

and imposing the term \( n_s = n_3 = \text{const.} \), from the respective relation we obtain:

\[ n_1 = i_{1s}n_3 + n_5\left( 1 - i_{1s}^s \right) = n_s\left( 1 - 2i_{1s}^s \right) \]  \hspace{1cm} (2.9)

Result:

\[ i = n_t/n_s = n_t/n_5 = 1 - 2i_{1s}^s = 1 + 2\cdot\left( z_3/z_1 \right) \]  \hspace{1cm} (2.10)

The transmission ratio at the rear propeller is equal and of opposite direction to \( i \), namely \( i = -i_{1s} \).

We have to notice that the transmission ratio from (2.10) can be obtained from (2.5) for \( z_2 = z_3 \) and by changing \( z_4 \) with \( z_3 \). The distribution of powers between both propellers is accomplished according to the relations:

\[ P_1 = \frac{1}{716.2} M_3 n_5; \quad P_{II} = \frac{1}{716.2} M_3 n_3. \]  \hspace{1cm} (2.11)

From the functional terms of the differential reducer, the torques and speeds signs are different. From the equilibrium terms of the satellite, we easily obtain the relations:

\[ M_3 = -i_{1s}^s \cdot n_3; \quad M_5 = -M_1 \left( 1 - i_{1s}^s \right) \]  \hspace{1cm} (2.12)

and then

\[ P_1 = M_1 \left( i_{1s}^s - 1 \right) \cdot n_5 = i_{1s}^s - 1 = \frac{z_3 + z_1}{z_3} \]  \hspace{1cm} (2.13)

By comparing (2.10) with the relation for the transmission ratio of the simple planetary reducer of same type \( i_{1s}^{(3)} = 1 + z_3/z_1 \) (see the relation (2.4) from [5]), it is observed that the best kinematic effect is obtained at the differential mechanism:

\[ i = 1 + 2\cdot\left( z_3/z_1 \right) \]  \hspace{1cm} (2.14)

It is, also, the lightest, because upon its carcasse is acting only the torque difference \( M_5 - M_3 \). The smallest transmission ratio belongs to the serial simple 2 steps reducer \( (i = -z_3/z_1) \), being stressed, also, by a reactive torque.
\[ M_{1S} = -M_{1} \cdot \left(1 - i_{13}^{S}\right) \]  
\[ M_{1S} = M_{1} \cdot \left(1 - i_{13}^{S}\right) \]  
It means that the simple reducer is, also, the heaviest. Therefore, the serial simple planetary reducer, as regards the transmission ratio, occupies an intermediary position between the serial simple 2 steps reducers and the serial simple differential reducer.

More advantageous, as regards the transmission ratio and the relieving of the bearings of the turbine gas engines from the action of the torsion torque, are the symmetrical differential – planetary reducers [12]. The kinematic schemes of two such reducers are presented in Fig. 3. Under structural aspect, they represent a development of the planetary mechanisms on axial direction that leads to kinematic schemes with 3 central gears [17]. For kinematic study, these mechanisms can be divided in two simpler mechanisms. For instance, the mechanism from Fig. 3 a will be divided in a differential mechanism \( z_1 - z_2 - z_3 - z_4 - S \) and a planetary mechanism \( z_1 - z_2 - z_5 - z_6 - S \) (\( z_6 \) is fixed gear), the port-satellite arm \( S \) being common to both mechanisms.

Based on Willis method, for the differential mechanism we obtain the equation:

\[ i_{41}^{S} = \frac{n_4 - n_5}{n_1 - n_5} = \frac{z_3 \cdot z_1}{z_4 \cdot z_2} \]  
\[ i_{41}^{S} = n_1 \cdot i_{41}^{S} + n_5 \cdot \left(1 - i_{41}^{S}\right) \]  

For the planetary mechanism we can write:

\[ i_{15}^{(n)} = \frac{n_1}{n_5} = 1 - i_{16}^{S} = 1 - \left(\frac{z_2}{z_1}\right) \cdot \left(\frac{z_6}{z_5}\right) \]  
\[ i_{15}^{(n)} = \frac{n_1}{n_5} = 1 - i_{16}^{S} = 1 - i_{16}^{S} \]  
\[ n_5 = \frac{n_1}{1 - i_{16}^{S}} \]  

By introducing the expression of \( n_5 \) in (2.18), results:

\[ n_4 = n_1 \left(\frac{i_{41}^{S}}{1 - i_{16}^{S}}\right), \text{ from where} \]  
\[ \frac{n_4}{n_1} = \frac{1 - i_{16}^{S}}{1 - i_{16}^{S}} \]  
relation that allows us to establish the front propeller speed, because \( n_1 = n_4 \).

If in the last relation it is achieved the condition: \( i_{41}^{S} i_{16}^{S} = 2 \), the propellers will have equal speeds, but of opposite directions. Similarly, we analyze the mechanism from Fig. 3 b, too. It is made from a serial differential mechanism \( z_1 - z_2 - z_3 - S \) and a serial planetary mechanism \( z_6 - z_5 - z_4 - S \), in which the gear \( z_6 \) is leading element.
and the port-satellite arm $S_2$ is output element. The equations that describe the kinematics of these mechanisms are:

\[ i_{13}^{S} = i_{13}^{f} = \frac{n_1 - n_f}{n_3 - n_f} = \left( -\frac{z_2}{z_1} \right) \cdot \left( \frac{z_3}{z_2} \right) = -\frac{z_3}{z_1} \]  

(2.22)

namely

\[ n_1 = n_3 \cdot i_{13}^{S} + n_1 \left( 1 - i_{13}^{S} \right) \]  

(2.23)

respectively

\[ i_{6}^{(4)} = i_{6}^{(4)} = \frac{n_3}{n_6} = 1 - i_{6}^{(3)} = 1 - \left( -\frac{Z_5}{Z_6} \right) \cdot \left( \frac{Z_4}{Z_5} \right) = 1 + \frac{Z_4}{Z_6} \]  

(2.24)

namely

\[ n_3 = \left( 1 - i_{6}^{(3)} \right) \cdot n_4 \text{, where } i_{6}^{(3)} = -\frac{Z_4}{Z_6} \]  

(2.25)

From (2.23) and (2.25) we obtain the relation between the speed of the shaft engine $n_1$ and the speeds of the two propellers:

\[ n_1 = n_3 \cdot \left( 1 - i_{6}^{(3)} \right) \cdot n_4 + \left( 1 - i_{13}^{S} \right) \cdot n_f \]  

(2.26)

If in (2.26) we admit $z_1 = z_4$, $z_2 = z_5$, $z_3 = z_6$ and $n_3 = -n_f$, it results:

\[ i_{1f} = i_{1s} = \frac{n_1}{n_f} = -\frac{n_1}{n_s} = -2 \cdot \left( 1 + \frac{z_3}{z_1} \right) \]  

(2.27),

where $i_{1f}$ and $i_{1s}$ are the transmission ratios from the turbine shaft to the front propeller, respectively, to the rear propeller. This relation demonstrates, also, the possibility to obtain some bigger transmission ratios.

It has to be mentioned that there are, also, other kinematic – structural alternatives of these mechanisms (Fig.4).

Typical for this alternative is that the port-satellite arm is rotating free in bearings and doesn’t transmit motion (the respective alternative can be used to drive a simple propeller). If the gear $z_4$ is fixed, the DOF of the mechanism is $M = 1$ and with the notations from the figure, the transmission ratio is:

\[ i_{14}^{(5)} = \frac{n_1}{n_4} = \frac{n_1}{n_5} \cdot \frac{n_5}{n_4} = i_{1s}^{(5)} \cdot i_{4s}^{(5)} = \left( 1 - i_{15}^{S} \right) \cdot \frac{1}{1 - i_{45}^{S}} = \left( 1 + \frac{z_5}{z_1} \right) \cdot \frac{1}{1 - z_5 z_5 / z_4 z_2} = \frac{z_2 z_4 \left( z_1 + z_4 \right)}{z_2 z_4 - z_3 z_3} \]  

(2.28)

By a right selection of the teeth of gears, it can be obtained transmission ratios $i \geq 100$, at acceptable values of efficiency. Reasonable constructions of such type of reducers are obtained for $i = 20...100$, although their efficiency is a little smaller than the efficiency of planetary reducers with two central gears. The reducers for coaxial propellers can be achieved, also, based on planetary-differential mechanism with conical gears (Fig. 5), but their use is much more reduced than of the similar reducers with cylindrical gears, because of the smaller transmission ratios that can be obtained and because of an efficiency quite reduced than of the reducers with cylindrical gears. The kinematic calculus of this mechanism is achieved similar to that one belonging to the mechanisms of Fig. 3. For this reason, we consider that the mechanism is made of the planetary mechanism $z_1 z_2 z_3 z_4 S$ ($z_4$ fixed) and the differential mechanism $z_1 z_2 z_5 z_6 S$. Thus, for the planetary mechanism, by starting from the general relation:
where \( n_4 = 0 \), we obtain:

\[
\frac{n_4}{n_8} = 1 - i_{14}^S = 1 + \frac{z_2 z_4}{z_1 z_3}
\]  

(2.30)

For the differential mechanism we first write the relation

\[
i_{51}^S = \frac{n_5 - n_8}{n_1 - n_8} = \left( -\frac{z_2}{z_5} \right) \cdot \frac{z_1}{z_2} = -1
\]  

(2.31)

In the first equality we divide by \( n_8 \) and consider the relation (2.30), we obtain:

\[-i_{51}^S \cdot i_{14}^S = n_5 / n_1 - 1\]  

(2.32)

Finally, if in the ratio from the right part we multiply and divide by \( n_1 \), using once again the relation (2.30), it results:

\[
\frac{n_5}{n_1} = 1 - i_{51}^S \cdot i_{14}^S
\]  

(2.33)

3. CONCLUSIONS

Driving of the coaxial propellers needs more complex reducers, as constructive solutions, by comparison with the reducers with fixed axes. For these propellers, a much more advantageous solution is represented by the combined reducers, made from a simple reducer and a planetary mechanism or by reducers made by using of some differential mechanisms. By using these constructive schemes, there can be obtained necessary transmission ratios for overall sizes and efficiency quite reasonable.

The differential reducers are superior, as regards the transmission ratio, efficiency and overall size against the other constructive solutions, but, when using them, it appears some constructive and functional features, that are reflected in the propellers work.

REFERENCES