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ON THE STATE OF STRESS AND STRAIN AT DISKS IN THERMAL REGIME, IN THE PRESENCE OF RESIDUAL STRESSES PART I – GENERAL THEORETICAL STUDY

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***Abstract**: The present paper presents the theoretical method for determination of the deformations and the loss of stability of the disk stressed by an axial – symmetric thermal field, variable according to disk radius and thickness, superposed with a field of membrane tensions given by the rotational motion. The experimental results confirm the theoretical hypothesis. This paper also presents the tension and deformation state of the disks being in a non – stationary field of temperature. The study is done until the plastic deformations occur.*
***Keywords**: disk, thermal field, stress, membrane tension, stability, non – stationary field*

The residual stresses exist in the bodies materials without the presence of any load. Their magnitude and distribution depend on the material, on the dimensions and on the working process. The residual stresses appear because of the elastic – plastic local strains and structural transformations. The physical properties of the material depend in a great manner on time and temperature, as parameters. The magnitude and the distribution of the residual stresses are determined not only by σ(ε) characteristic of the material, but also by the distribution of the thermal field on time. At high level temperatures, the yielding limit is very low, so that it will be easily transcend. In the strength of materials calculations, the residual stresses of first species. These are due to the nonhomogeneous loads. In order to determine the residual stresses , one may use the 12 equations from the elasticity theory (six equations for stresses and 6 equations for strains). For the determinations appropriate to the Hook's low, because the residual stresses are elastic stresses.

$$\begin{aligned} E\ \ \varepsilon_x &= \sigma_x - \upsilon(\sigma_y + \sigma_z) & G\ \ \gamma_{xy} &= \tau_{xy} \\ E\ \ \varepsilon_y &= \sigma_y - \upsilon(\sigma_z + \sigma_x) & G\ \ \gamma_{yz} &= \tau_{yz} \\ E\ \ \varepsilon_z &= \sigma_z - \upsilon(\sigma_x + \sigma_y) & G\ \ \gamma_{zx} &= \tau_{zx} \end{aligned} \qquad (1)$$

where the modulus of elasticity E and G and the transverse contraction coefficient ν are constant values for isotropic materials. From the six equations of equilibrium on the element of volume, one uses only the sums of forces projections. The abridgement of the number of equations is achieved by the subsisting of the tangential stresses duality.

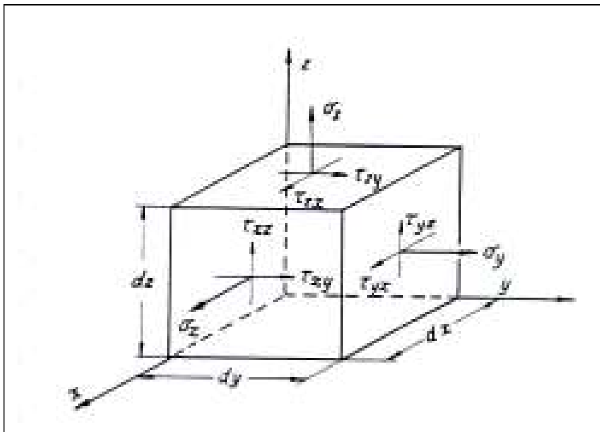

Figure 1

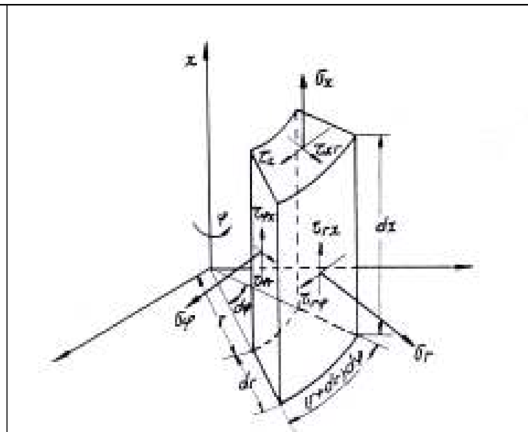

Figure 2

The sums of projections (fig. 1) and the compatibility conditions have the following expressions:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z}=0 \qquad \frac{\partial^2\varepsilon_x}{\partial y^2}+\frac{\partial^2\varepsilon_y}{\partial x^2}=\frac{\partial^2\gamma_{xy}}{\partial x\partial y}$$

$$\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\tau_{yz}}{\partial z}=0 \qquad \frac{\partial^2\varepsilon_y}{\partial z^2}+\frac{\partial^2\varepsilon_z}{\partial y^2}=\frac{\partial^2\gamma_{yz}}{\partial y\partial z} \tag{2}$$

$$\frac{\partial\sigma_z}{\partial z}+\frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y}=0 \qquad \frac{\partial^2\varepsilon_z}{\partial x^2}+\frac{\partial^2\varepsilon_x}{\partial z^2}=\frac{\partial^2\gamma_{xz}}{\partial x\partial z}$$

Except the equations of equilibrium for the treated element of volume, the residual stresses have to satisfy the equations of equilibrium for the entire material volume of the body which is unassigned by the action of a load (equations (3)).

$$\int\sigma_x dydz=0 \qquad \int\sigma_y z\,dz\,dx+\int\sigma_z y\,dx\,dy=0$$

$$\int\sigma_y dzdx=0 \qquad \int\sigma_z x\,dx\,dy+\int\sigma_x z\,dy\,dz=0 \tag{3}$$

$$\int\sigma_z dxdy=0 \qquad \int\sigma_x y\,dy\,dz+\int\sigma_y x\,dz\,dx=0$$

The last three equations are valid only if the x-, y- and z- axes of coordinates are principal axes of inertia of the treated body, too.

There are practical applications where the study is much easier, based on the cylindrical coordinates. For the general case, the equilibrium of the element of volume results from the following relations (fig. 2):

$$\frac{\partial\sigma_r}{\partial r}+\frac{\partial\tau_{r\varphi}}{r\partial\varphi}+\frac{\partial\tau_{rx}}{\partial x}+\frac{\sigma_r-\sigma_\varphi}{r}=0;$$

$$\frac{\partial\tau_{rx}}{\partial r}+\frac{\partial\tau_{x\varphi}}{r\partial\varphi}+\frac{\partial\sigma_x}{\partial x}+\frac{\tau_{rx}}{r}=0; \tag{4}$$

$$\frac{\partial\tau_{r\varphi}}{\partial r}+\frac{\partial\sigma_\varphi}{r\partial\varphi}+\frac{\partial\tau_{x\varphi}}{\partial x}+\frac{2\tau_{r\varphi}}{r}=0,$$

For the rotational disks in thermal regime, one gets axi-symmetrical distributions of the residual stresses. In this case $\tau_{r\varphi} = \tau_{x\varphi} = 0$, the above equations becoming:

$$\frac{\partial\sigma_r}{\partial r}+\frac{\partial\tau_{rx}}{\partial x}+\frac{\sigma_r-\sigma_\varphi}{r}=0;$$

$$\frac{\partial\tau_{rx}}{\partial r}+\frac{\partial\sigma_x}{\partial x}+\frac{\tau_{rx}}{r}=0. \tag{5}$$

The transition from the rectangular system of coordinates to cylindrical coordinates is done by conversion relations of stresses. If we take into account a direction n which makes with the axis the angles (nx), (ny) and (nz) and if n is principal axis too ($\tau_n = 0$), than one gets the equations:

$$(\sigma_z-\sigma)\cos(nx)+\tau_{xy}\cos(ny)+\tau_{zx}\cos(nz)=0;$$

$$\tau_{xy}\cos(nx)+(\sigma_y-\sigma)\cos(ny)+\tau_{yz}\cos(nz)=0 \tag{6}$$

$$\tau_{xz}\cos(nx)+\tau_{yz}\cos(ny)+(\sigma_z-\sigma)\cos(nz)=0$$

which give the principal direction of load; one adds the condition:

$$\cos^2(nx)+\cos^2(ny)+\cos^2(nz)=1 \tag{7}$$

The values of the principal normal stresses σ_1, σ_2, σ_3 yield by resolving of the equation:

$$\sigma^3-J_1\sigma^2+J_2\sigma-J_3=0 \tag{8}$$

One finally specifies the expression of the stress and strain upon a random direction n:

$$\sigma=\sigma_x\cos^2(nx)+\sigma_y\cos^2(ny)+\sigma_z\cos^2(nz)+2\tau_{xy}\cos(nx)\cos(ny)+ \\ +2\tau_{yz}\cos(ny)\cos(nz)+2\tau_{zx}\cos(nz)\cos(nx) \tag{9}$$

$$\varepsilon=\varepsilon_x\cos^2(nx)+\varepsilon_y\cos^2(ny)+\varepsilon_z\cos^2(nz)+\gamma_{xy}\cos(nx)\cos(ny)+ \\ +\gamma_{yz}\cos(ny)\cos(nz)+\gamma_{zx}\cos(nz)\cos(nx) \tag{10}$$

For the plane state of stress and strain, the calculation relations are simplified and take the form:

$$\sigma=\frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos 2(nx)+\tau_{xy}\sin 2(nx);$$

$$\tau=\frac{\sigma_x+\sigma_y}{2}-\frac{\sigma_x-\sigma_y}{2}\sin j2(nx)+\tau_{xy}\cos 2(nx). \tag{11}$$

$$\varepsilon_\alpha = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\alpha + \frac{\gamma_{xy}}{2}\sin 2\alpha;$$
$$\frac{\gamma_\alpha}{2} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\sin j2\varepsilon + \frac{\gamma_{xy}}{2}\cos 2\alpha. \tag{12}$$

The relations above can have representations by Mohr's circle, for stresses and strains, the angle (nx) being denoted by α. The graph is very useful for the strain gage measurement. In this case one defines the state of strain around a point, based on the known elongations along three random directions (axes) 1,2 and 3 (fig. 3).

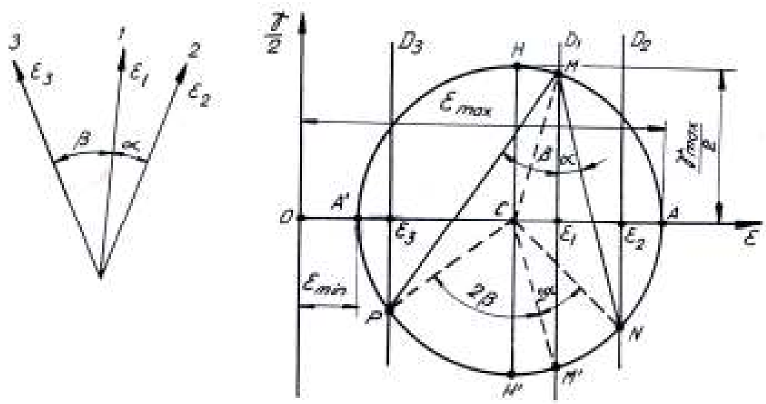

Figure 3

Based on the strains upon the three mentioned directions, one can find out analytically the principal stresses and principal directions. At the rectangular assembling of the transducers, between the three directions the angles have 60 °; the calculation relations have the expressions:

$$\sigma_{1,2} = \frac{E}{2}\left[\frac{\varepsilon_\alpha + \varepsilon_\gamma}{2} \pm \frac{1}{1+\upsilon}\sqrt{(\varepsilon_\alpha - \varepsilon_\gamma)^2 + (2\varepsilon_\beta - \varepsilon_\alpha - \varepsilon_\gamma)^2}\right];$$
$$\tau = \pm\frac{E}{1+\upsilon}\sqrt{(\varepsilon - \varepsilon_\gamma)^2 + (2\varepsilon_\beta - \varepsilon_\alpha - \varepsilon_\gamma)^2}; \tag{13}$$
$$tg2\varphi_1 = \frac{2\varepsilon_\beta - \varepsilon_\alpha - \varepsilon_\gamma}{\varepsilon_\alpha - \varepsilon_\gamma}$$

One gets similar relations for the delta assembling. One emphasizes the fact that the determination of the residual stresses by conversion of the strains is based on the elastic tempering of the material, after the cancellation of the causes which had lead to strains. If during the test some parts of the material are assigned by residual stresses which exceeds the limit flow value in those parts plastic strains will appear (with or without hardening). Further on one presents some theoretical considerations for a ring shaped disks, with the diameter 2a and 2b and thickness h. The determination of the state of of stress and strain during the heating or cooling of the disk is a thermo elasticity issue (load under the flow limit of the material). If the disk is subjected to a nonstationary thermal field, local plastic strains will appear, which lead to residual stresses. The nonstationary thermal field is:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{6}\frac{\partial T}{\partial r} - m^2 T_{st} = 0 \tag{14}$$

$$m^2 = \frac{2k}{\lambda h} \tag{15}$$

where k is the heat transmission coefficient, λ is the thermal conductivity coefficient.

The nonstationary thermal field T has a stationary component T_{st}, whose expression is similar to equation (14) and an additional component T_s; the last one goes null when the time t is big.

For the calculation of the state of stress one approaches the thermal fields by simple functions. One established that a simple expression which satisfies both the initial and the final state, has the form>

$$\frac{T}{T_0} = e^{-\left(\frac{r}{t}\right)\left(\frac{r}{a}-1\right)}\left[A + Be^{-\gamma\frac{r}{b}}\right] \tag{16}$$

where the stationary thermal T_{st}/T_0 is given by the square bracket is equation (16). Most of the above mentioned terms are determined by experimental tests. In the decond section of this paper one makes some concrete considerations on the disk in thermal regime.

The temperature's distribution upon the disk radius are:

- FEM software: $T = T_0\left(\frac{r}{R_e}\right)^{\gamma}$;

$$T = \Delta T\left[\frac{r - R_i}{r - R_e}(1 + C) + \left(\frac{r - R_i}{R_e - R_i}\right)^2 C\right]$$

- $\alpha = \frac{R_i}{R_e}$; D – nodal diameters; c – nodal circles;

From the experimental records, it has been found that C = -0.65…+0.6. The value of the dimensionless coefficient C is strongly affected by many factors, such as: temperature difference $\Delta T = T_2 - T_1$ and disk velocity.

Generally one can assume that the two (membrane and bending) loads are independent and can be studied separately. If the plate is thin, the membrane tensions exert a strong influence on flexures; this influence is also notable on natural frequencies and

buckling tensions. Theoretically the study has been carried out with von Karman's equations, which represent a dynamic version of the non-linear theory of plates.

Karman's equations describe the behavior of thin disks subjected to membrane tensions, flexion tensions as well as the reciprocal influence of the two types of tensions (second rank calculation).

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