# LAGGING ANGLE BETWEEN THE DRIVING FLANGE AND THE CYLINDER BLOCK IN CASE OF AXIAL PISTON PUMP WITH ROTARY MOTION TRANSMITTING BY CONNECTING RODS 

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#### Abstract

In this paper the variation of the lagging angle between the cylinder block and driving shaft in case of axial piston pumps with transmitting of the rotary motion by connecting rods (APPTRMCR) is studied, accordingly to the peculiarity of these types of hydraulic machines, and namely, the transmitting of the rotary motion from the driving flange to the cylinder block by means of connecting rods. It is studied the case of a mechanism having a single ensemble connecting rod-piston, therefore, in the case where the connecting rod is always in contact with the inside wall of the piston cup. The performed analysis is exemplified for the pumps F132 and F232 realized by S.C. Plopeni S.A. The paper results are useful to study the accurate kinematics of APPTRMCR, by establishing the variation laws of the lagging angle in case of occurring az connecting rods/pistons, from which a single connecting rod is in the state of driving connecting rod, at a given moment. Keywords: axial piston pump, lagging angle


## 1. INTRODUCTION

In paper [1] the functional geometry of the mechanism in case of axial piston pump with rotary motion transmitting by connecting rods (APPTRMCR) is presented, taking into account the peculiarity of these hydraulic machine types, and namely the rotary motion transmitting from the driving flange to the cylinder block by means of a connecting rod, called driving connecting rod. In this way, on basis of some geometrical and functional, structural-kinematical, respectively, arguments and of some calculations, the geometrical, linear and angular sizes, which from functional/kinematical point of view characterize the analyzed mechanism, and a series of relationships among the different sizes are determined. Among these relationships are presented as follows:

- the length of the connecting rod on the plane ( $\Pi$ ) (the cylinder block plane where the joint, noted with $Q_{3}$, of the connecting rod with the piston is found), being the segment $\left|Q_{3} Q_{2}^{(\Pi)}\right|$ (see fig. 1),

$$
\begin{equation*}
p \equiv\left|Q_{3} Q_{2}^{(\Pi)}\right|=\sqrt{\left[R_{c} \cdot \cos \left(\varphi_{1}-\Delta \varphi\right)-R_{f} \cdot \cos \gamma \cdot \cos \varphi_{1}\right]^{2}+\left[R_{c} \cdot \sin \left(\varphi_{1}-\Delta \varphi\right)-R_{f} \cdot \sin \varphi_{1}\right]^{2}} \tag{1}
\end{equation*}
$$

- the characteristic angle of the connecting rod projection on the plane ( $\Pi$ ) $\left(\Delta \varphi_{p}\right)$ :

$$
\begin{equation*}
\cos \Delta \varphi_{p}=\frac{\cos \gamma \cdot \cos \varphi_{1} \cdot \cos \left(\varphi_{1}-\Delta \varphi\right)+\sin \varphi_{1} \cdot \sin \left(\varphi_{1}-\Delta \varphi\right)}{\sqrt{\cos ^{2} \gamma \cdot \cos ^{2} \varphi_{1}+\sin ^{2} \varphi_{1}}} \tag{2}
\end{equation*}
$$

- the relationship between the rotary angles of the driving shaft/flange $\left(\varphi_{1}\right)$ and of the cylinder block $\left(\varphi_{Q 3}\right)$ :

$$
\begin{equation*}
\varphi_{1}-\Delta \varphi=\varphi_{Q 3}-\left(\Delta \varphi_{I}^{\prime}-\varphi_{1 I}\right) \tag{3}
\end{equation*}
$$

- the connecting rod inclination angle against the piston/cylinder axis ( $\delta$ ),

$$
\begin{equation*}
\cos \delta=\frac{1}{l} \cdot \sqrt{l^{2}-p^{2}} \tag{4}
\end{equation*}
$$

which becomes maximum $\left(\delta_{M}\right)$, according to the expression

$$
\begin{equation*}
\cos \delta_{M}=\frac{1}{l} \cdot \sqrt{l^{2}-p_{M}^{2}} \tag{5}
\end{equation*}
$$

in case when the connecting rod comes in contact with the piston cup.
In the above mentioned relationships the following notations have been used: $R_{f}$ - the radius of the circle where the connecting rod joints with the driving flange are found; $R_{c}$ - the radius of the circle of layout of the cylinder
axis (of the connecting rod joint with the pistons); $\gamma-$ the tipple angle of the cylinder block (the angle between the cylinder block axis and the driving shaft/flange axis); $l$ - the connecting rod length.
In the same paper [1] it has been shown that there is a lagging between the rotary motions of the cylinder block and of the driving flange/shaft $(\Delta \varphi)$, the cylinder block staying behind the shaft, contrary to the case usually considered, by using the elementary kinematics theory. In this way, after the pump driving shaft has turned round (with the angle $\varphi_{1 . l}$, the piston arrives at the lower dead point (PMI) ( $Q_{3 . I}$ ), the initial lagging being $\Delta \varphi_{I}^{\prime}$ (see fig. 1). It is results that the piston dead points (PMI and PMS) are not found in the positions occupied by the pistons at $\varphi_{1} \in\left\{0^{\circ}, 180^{\circ}\right\}$. These positions are done by the angle $\varphi_{1 . Q 3 . I}$, for PMI, and by the angle $\varphi_{1 . Q 3 . S}$, for PMS, accordingly to expressions (see fig. 1):
$\varphi_{1 Q 3 I}=\varphi_{1 . I}-\Delta \varphi_{I}^{\prime} ; \varphi_{1, Q 3 S}=\varphi_{1 . S}-\Delta \varphi_{S}^{\prime}$,
and the angles $\varphi_{Q 3 . I}$, for PMI, and $\varphi_{Q 3 . S}$, for PMS, respectively, accordingly to the formula (see fig. 1):

$$
\begin{equation*}
\varphi_{Q 3}=\varphi_{1}-\Delta \varphi-\left(\varphi_{1 I}-\Delta \varphi_{I}^{\prime}\right) \tag{6}
\end{equation*}
$$

from which will result

$$
\begin{equation*}
\varphi_{Q 3 I}=0^{\circ} ; \varphi_{Q 3 S}=\varphi_{1 Q 3 S}-\varphi_{1 Q 3 I} . \tag{8}
\end{equation*}
$$



Figure 1: Representation of the connecting rod joint with the driving flange $\left(Q_{2}^{(\Phi)}\right.$ and $\left.Q_{2}^{(\Pi)}\right)$ and with the piston $\left(Q_{3}\right)$ in the plan (ח), of the connecting rod projection on the same plan $\left(\left|Q_{3} Q_{2}^{(\Pi)}\right|\right)$, of the driving flange rotation angle $\left(\varphi_{1}\right)$, of the cylinder block rotation angle $\left(\varphi_{4} \equiv \varphi_{Q_{3}}\right)$, of the lag angle between the cylinder block and the driving flange $(\Delta \varphi)$, of the angles ( $\varphi_{1 . Q 3 . I}$ and $\varphi_{1 . Q 3 . S}$ ) locating the lower neutral axis (ANI) and the upper ones
(AMS), a.s.o.
It means that there are (accordingly to [1] and [2]) two neutral axes of the distribution plate (see fig. 1): the lower neutral axis (ANI) and the upper neutral axis (ANS). ANI represents the axis from the distributing plate plane which passes through the points of projection on this plan of the cylinder block axis $\left(Q_{4}^{(\Pi)}\right)$ and of the cylinder axis ( $Q_{3 I}^{\prime}$ ), in the moments when the pistons are found, successively, at PMI. ANI coincides to the axis $O^{,(\mathrm{I})} X^{(\mathrm{II})}$ (ANI $\left.\equiv O^{,(\mathrm{I})} X^{,(\mathrm{I})}\right)$ and is determined by the angle $\varphi_{1 . Q 3 . I}$ against $O^{(\mathrm{T})} X^{(\mathrm{I})}$. ANS is the axis from the same distributing plate plane which passes through the points of projection on this plane of the cylinder block axis $\left(Q_{4}^{(\Pi)}\right)$ and of the cylinder axis $\left(Q_{3 . S}^{\prime}\right)$, in the moments when the pistons are found, successively, at PMS.

ANS is, at its turn, determined by the angle $\varphi_{1 . Q 3 . S}$ against the same axis, $O^{(\Pi)} X^{(\Pi)}$. The positions of the axes ANI and ANS are dependent on the tipple angle of the cylinder block $(\gamma)$.
The present paper object is the determination of the variation law of the lagging angle between the cylinder block and the driving flange, depending on the rotary angle of the driving flange, that is $\Delta \varphi=f\left(\varphi_{1}\right)$.

## 2. LAGGING ANGLE VARIATION IN CASE OF A SINGLE CONNECTING ROD-PISTON ENSEMBLE

In the case where there is a single connecting rod-piston ensemble between the driving flange and the cylinder block, may be ascertained that the connecting rod is always in contact with the piston cup (see [1]), meaning that the connecting rod is inclined with the maximal angle ( $\delta_{M}$ ) against the cylinder axis respectively, and as a following, the connecting rod projection on the plane ( $\Pi$ ) is maximal $\left(p_{M}\right)$, indifferent of the rotary angle $\varphi_{1}$. It results that the following equation is valid:

$$
\begin{equation*}
p_{M}=\sqrt{\left[R_{c} \cdot \cos \left(\varphi_{1}-\Delta \varphi\right)-R_{f} \cdot \cos \gamma \cdot \cos \varphi_{1}\right]^{2}+\left[R_{c} \cdot \sin \left(\varphi_{1}-\Delta \varphi\right)-R_{f} \cdot \sin \varphi_{1}\right]^{2}} \tag{9}
\end{equation*}
$$

from which the law $\Delta \varphi=f\left(\varphi_{1}\right)$ is determined.
In this way, processing this equation and solving it, the following solution is obtained:

$$
\begin{equation*}
\operatorname{tg} \frac{\Delta \varphi}{2}=\frac{D\left(\varphi_{1}\right)-B\left(\varphi_{1}\right)}{A\left(\varphi_{1}\right)} \tag{10}
\end{equation*}
$$

where $A\left(\varphi_{1}\right), B\left(\varphi_{1}\right)$ and $D\left(\varphi_{1}\right)$ have the expressions

$$
\begin{align*}
& A\left(\varphi_{1}\right)=\left(R_{c}+R_{f} \cdot \cos \gamma\right)^{2} \cdot \cos ^{2} \varphi_{1}+\left(R_{c}+R_{f}\right)^{2} \cdot \sin ^{2} \varphi_{1}-p_{M}^{2}  \tag{11}\\
& B\left(\varphi_{1}\right)=R_{c} \cdot R_{f} \cdot(1-\cos \gamma) \cdot \sin 2 \varphi_{1}  \tag{12}\\
& D\left(\varphi_{1}\right)=\sqrt{B^{2}\left(\varphi_{1}\right)+A\left(\varphi_{1}\right) \cdot C\left(\varphi_{1}\right)} \tag{13}
\end{align*}
$$

where

$$
\begin{equation*}
C\left(\varphi_{1}\right)=p_{M}^{2}-\left(R_{c}-R_{f} \cdot \cos \gamma\right)^{2} \cdot \cos ^{2} \varphi_{1}-\left(R_{f}-R_{c}\right)^{2} \cdot \sin ^{2} \varphi_{1} . \tag{14}
\end{equation*}
$$

In case of $\varphi_{1}=0$, the lagging angle is noted by $\Delta \varphi_{I}$ (see fig. 1) and it is given by the formula:

$$
\begin{equation*}
\operatorname{tg} \frac{\Delta \varphi_{I}}{2}=\sqrt{\frac{p_{M}^{2}-\left(R_{c}-R_{f} \cdot \cos \gamma\right)^{2}}{\left(R_{c}+R_{f} \cdot \cos \gamma\right)^{2}-p_{M}^{2}}} . \tag{15}
\end{equation*}
$$

For $\varphi_{1}=0$, also $\Delta \varphi_{p . I}$ is obtained from the relationship (2) in the case of a single connecting rod-piston ensemble ( $p=p_{M}$ ). It is found that the respective expression has the shape (15), that is there is the equality (see fig. 1 ):

$$
\begin{equation*}
\Delta \varphi_{p . I}=\Delta \varphi_{I} . \tag{16}
\end{equation*}
$$

It is observed that $\Delta \varphi_{I}$ depends on the geometrical constructive sizes of the pump: $R_{f}, R_{c}, l, \delta_{M}$ (the two last of them determining $p_{M}$ ) and $\gamma$. Taking into account the dependence on $\gamma$, it means that $\Delta \varphi_{I}$ will be changed at the same time with the pump capacity variation, determined by $\gamma$ (as it may be also happen in the case of the pump F232, built by the firm S.C. Plopeni S.A.).
From the relationship (15) will result the constructive condition which must be performed by the mechanism in case of APPTRMCR, in the way that the initial lagging should become null (that is PMI should be found on axis $\left.O^{(\mathrm{II})} X^{(\mathrm{T})}\right)$,

$$
\begin{equation*}
\Delta \varphi_{I}=0, \tag{17}
\end{equation*}
$$

and namely:

$$
\begin{equation*}
p_{M}=R_{c}-R_{f} \cdot \cos \gamma_{M} . \tag{18}
\end{equation*}
$$

If $R_{c}, R_{f}$ and $p_{M}$ are imposed (that is $l$ and $\delta_{M}$, according to (5)), $\gamma_{M}$ will result from the following equality:

$$
\begin{equation*}
\gamma_{M}=\arccos \frac{R_{c}-p_{M}}{R_{f}} \tag{19}
\end{equation*}
$$

In case of pumps having variable capacity, obtained by changing the cylinder block inclination angle ( $\gamma$ ), between a minimal value ( $\gamma_{m}$, usually $0^{\circ}$ ) and a maximal one ( $\gamma_{M}$, usually $25^{\circ}$ ), there is an angle $\gamma_{0}$ for which, at $\varphi=0$, the initial lagging angle $\Delta \varphi_{I}$ is minimal $\left(\Delta \varphi_{I . m}\right)$ or maximal $\left(\Delta \varphi_{I . M}\right)$. This angle is determined from the following condition:

$$
\begin{equation*}
\left.\left.\frac{\mathrm{d} \Delta \varphi(0)}{\mathrm{d} \gamma}\right|_{\gamma=\gamma_{0}} \equiv \frac{\mathrm{~d} \Delta \varphi_{I}}{\mathrm{~d} \gamma}\right|_{\gamma=\gamma_{0}}=0 . \tag{20}
\end{equation*}
$$

By derivation of expression (15) and imposing the condition (20), the following equation is obtained:

$$
\begin{equation*}
\sin \gamma_{0} \cdot\left(p_{M}^{2}-R_{c}^{2}+R_{f}^{2} \cdot \cos ^{2} \gamma_{0}\right)=0 \tag{21}
\end{equation*}
$$

having the practical solutions
$\gamma_{01}=0^{\circ}$ şi $\gamma_{02}=\arccos \frac{\sqrt{R_{c}^{2}-p_{M}^{2}}}{R_{f}}$,
in the way that

$$
\begin{equation*}
\Delta \varphi_{I}\left(\gamma_{01}\right) \equiv \Delta \varphi_{I . m} \text { si } \Delta \varphi_{I}\left(\gamma_{02}\right) \equiv \Delta \varphi_{I M} . \tag{23}
\end{equation*}
$$

Substituting each of the solution $\gamma_{0 i}, i=1,2$, given by (22) in formula (15), results:

$$
\begin{equation*}
\Delta \varphi_{I, m}=2 \cdot \operatorname{arctg} \sqrt{\frac{p_{M}^{2}-\left(R_{f}-R_{c}\right)^{2}}{\left(R_{f}+R_{c}\right)^{2}-p_{M}^{2}}} \tag{24}
\end{equation*}
$$

and

$$
\begin{equation*}
\Delta \varphi_{I M}=2 \cdot \operatorname{arctg} \frac{p \cdot \sqrt{R_{c}^{2}-p_{M}^{2}}}{R_{c}^{2}-p_{M}^{2}+R_{c} \cdot \sqrt{R_{c}^{2}-p_{M}^{2}}} . \tag{25}
\end{equation*}
$$

In table 1 the measures of the sizes, which characterize the mechanism in case of APPTRMCR of types F132 and F232, determined on the basis of the execution drawings of the respective pieces and by measuring, are presented. By their means the angles $\gamma_{01}, \gamma_{02} \gamma_{M}$ and the corresponding lagging angles $\left(\Delta \varphi_{I}\right)$ have been calculated, by using the prior formulas determination. These results are presented concentrated in table 2.

Table 1: The measures of the sizes characterizing the mechanism of the pumps of types F132 and F232

| $R_{f}, \mathrm{~mm}$ | $R_{c}, \mathrm{~mm}$ | $l, \mathrm{~mm}$ | $\delta_{M}$ | $p_{M}, \mathrm{~mm}$ | $\gamma$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 50.640 | 50.165 | 94 | $1.913^{\circ}$ | 3.138 | $25^{\circ}(\mathrm{F} 132) ;\left[0^{\circ}, 25^{\circ}\right](\mathrm{F} 232)$ |

Table 2: The values of the initial lagging angle (at $\varphi_{1}=0^{\circ}$ ) for different peculiar values of $\gamma$

| $\gamma$ | $\gamma_{01}=0^{\circ}$ | $\gamma_{02}=17.989^{\circ}$ | $\gamma_{M}=26.7^{\circ}$ |
| :---: | :---: | :---: | :---: |
| $\Delta \varphi_{I}$ | $\Delta \varphi_{I . m}=2.151^{\circ}$ | $\Delta \varphi_{I . M}=3.586^{\circ}$ | $\Delta \varphi_{I}=0^{\circ}$ |



Figure 2: The variation curves of the lagging angle between the driving flange and the cylinder block $(\Delta \varphi)$, for different values of the tipple angle $(\gamma)$

May be seen that the null initial lagging angle $\left(\Delta \varphi_{I}=0^{\circ}\right)$ is obtained for a tipple angle of the cylinder block of $26.7^{\circ}$, being greater than the maximal value, of $25^{\circ}$, which can be realized at the studied pumps. Although it has no practical importance for the pumps F132 and F232, it is taken into consideration also this value $\gamma_{M}=26.7^{\circ}$ for studying the kinematics of these hydrostatic units.
In fig. 2 the variation laws $\Delta \varphi=f\left(\varphi_{1}\right)$ for different values of the cylinder block tipple angles $\gamma \in\left\{0^{\circ}, 5^{\circ}, 10^{\circ}\right.$, $\left.15^{\circ}, 17,989^{\circ}, 20^{\circ}, 25^{\circ}, 26,7^{\circ}\right\}$, determined by means of a calculation program written in MATLAB language, are presented. For $\varphi_{1}=0^{\circ}, \Delta \varphi_{I}$, and for $\varphi_{1}=180^{\circ}, \Delta \varphi_{S}$, are obtained.
According to fig. 2, the lagging angle between the driving flange and the cylinder block is changed during the piston stroke, at the beginning having a decrease from $\Delta \varphi_{I}$ to a minimal value $\left(\Delta \varphi_{m}\right)$, then an increase until a maximal value $\left(\Delta \varphi_{M}\right)$, and at last, at the final part of the piston stroke, a decrease until $\Delta \varphi_{S}$. May be ascertained that

$$
\begin{equation*}
\Delta \varphi_{S}=\Delta \varphi_{I}, \tag{26}
\end{equation*}
$$

But the values of the two lagging angles are different depending on the tipple angle (see table 3), and namely, there is an increase of $\Delta \varphi_{I}$ at $2.151^{\circ}$ (for $\gamma=0^{\circ}$ ) until the maximal value of $3.586^{\circ}$ (for $\gamma=17,989^{\circ}$ ), and then a decrease until $2.286^{\circ}$, for $\gamma_{M}=25^{\circ}$, and $0^{\circ}$, respectively, for $\gamma_{M}=26.7^{\circ}$. Also, both $\Delta \varphi_{I . m}$ and $\Delta \varphi_{I . M}$ increase in the extent how $\gamma$ increases. At $\varphi_{1} \in\left\{90^{\circ} ; 270^{\circ}\right\}, \Delta \varphi$ has the same value irrespective of $\gamma$, being determined by an expression having the form (24). In this way, for a pump having the type F232 we have:
$\Delta \varphi\left(90^{\circ}\right)=\Delta \varphi\left(270^{\circ}\right)=\Delta \varphi_{I, m}=2.151^{\circ}$.
Table 3: The values of the initial lagging angle $\Delta \varphi_{I}\left(\right.$ for $\left.\varphi_{1}=0^{\circ}\right)$ and of the final one $\Delta \varphi_{S}$
(for $\varphi_{1}=180^{\circ}$ ) for different values of the tipple angle $(\gamma)$.

| $\gamma$, grade | 0 | 5 | 10 | 15 | 17.989 | 20 | 25 | 26,7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta \varphi_{I}=\Delta \varphi_{S}$, grade | 2.151 | 2.415 | 2.982 | 3.476 | 3.586 | 3.519 | 2.286 | 0 |

Further, the derivative expression of the lagging angle is determined, depending on $\varphi_{1}$. In this way, by derivation of the relationship (10), it is obtained:

$$
\begin{equation*}
\frac{\mathrm{d} \Delta \varphi}{\mathrm{~d} \varphi_{1}}=\frac{2 \cdot\left[\frac{A\left(\varphi_{1}\right) \cdot J\left(\varphi_{1}\right)}{2 \cdot D\left(\varphi_{1}\right)}-2 \cdot R_{c} \cdot R_{f} \cdot(1-\cos \gamma) \cdot A\left(\varphi_{1}\right) \cdot \cos 2 \varphi_{1}-D\left(\varphi_{1}\right) \cdot K \cdot \sin 2 \varphi_{1}+B\left(\varphi_{1}\right) \cdot K \cdot \sin 2 \varphi_{1}\right]}{A^{2}\left(\varphi_{1}\right) \cdot\left[1+E^{2}\left(\varphi_{1}\right)\right]}, \tag{28}
\end{equation*}
$$

where function $J\left(\varphi_{1}\right)$ has the following expression
$J\left(\varphi_{1}\right)=2 \cdot R_{c}^{2} \cdot R_{f}^{2} \cdot(1-\cos \gamma)^{2} \cdot \sin 4 \varphi_{1}+C\left(\varphi_{1}\right) \cdot K \cdot \sin 2 \varphi_{1}+A\left(\varphi_{1}\right) \cdot L \cdot \sin 2 \varphi_{1}$,
and $K$ and $L$ are given by the formulas

$$
\begin{align*}
& K=\left(R_{f}+R_{c}\right)^{2}-\left(R_{c}+R_{f} \cdot \cos \gamma\right)^{2}  \tag{30}\\
& L=\left(R_{c}-R_{f} \cdot \cos \gamma\right)^{2}-\left(R_{f}-R_{c}\right)^{2} \tag{31}
\end{align*}
$$

From the equation

$$
\begin{equation*}
\left.\frac{\mathrm{d} \Delta \varphi}{\mathrm{~d} \varphi_{1}}\right|_{\varphi_{1}=\varphi_{1} 0 i}=0 \tag{32}
\end{equation*}
$$

the solutions
$\varphi_{1.01} ; \varphi_{1.02}=\varphi_{10.1}+180^{\circ} ; \varphi_{1.03} ; \varphi_{103}=\varphi_{1.03}+180^{\circ}$.
are obtained.
For $\varphi_{1.0 .1}$ and $\varphi_{1.0 .2}$ function $\Delta \varphi=f\left(\varphi_{1}\right)$ allows a minimum (for the outlet stroke and for the intake stroke, respectively), noted by $\Delta \varphi_{m}$, and for $\varphi_{1.0 .3}$ and $\varphi_{1.0 .4}$, the same function allows a maximum (for each of the piston strokes), noted by $\Delta \varphi_{M}$.

Table 4: Values of the driving flange rotary angle $\left(\varphi_{1}\right)$ for which the lagging angle $\Delta \varphi$ allows a minimum $\left(\Delta \varphi_{m}\right)$ and a maximum $\left(\Delta \varphi_{M}\right)$, respectively

| $i$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $\varphi_{1.0 . i}$ | $\varphi_{1.0 .1}=45.576^{\circ}$ | $\varphi_{1.0 .2}=225.576^{\circ}$ | $\varphi_{1.0 .3} \cong 137^{\circ}$ | $\varphi_{1.0 .4} \cong 317^{\circ}$ |
| $\Delta \varphi\left(\varphi_{1.0 . i}\right)$ | $\Delta \varphi_{m}=0.767^{\circ}$ | $\Delta \varphi_{m}=0.767^{\circ}$ | $\Delta \varphi_{M}=6.403^{\circ}$ | $\Delta \varphi_{M}=6.403^{\circ}$ |

For example, for the pumps F132 and F232 and for $\gamma=25^{\circ}$, these angles have the values presented in table 4. They have been determined by means of a program, written in the MATLAB language.

## 3. CONCLUSIONS

In this paper the law variation, depending on the rotary angle of the driving shaft $\varphi_{1}$, of the lagging angle between the cylinder block and the driving shaft/flange in case of APPTRMCR ( $\Delta \varphi$ ), is presented, taking into account the peculiarity of these types of pumps, and namely the transmitting of the rotary motion from the driving flange to the cylinder block by means of connecting rods.
It is studied only the case of a mechanism having a single connecting rod-piston ensemble, therefore in the case where the connecting rod is found always in contact with the inside wall of the piston cup. In case of pumps with z pistons, there is, at a given moment only a connecting rod, the other $(z-1)$ connecting rods being not in contact with the respective pistons. In this way, the mechanism desmodromy is assured (see [3]). The case of mechanism having $z$ connecting rod-piston ensembles will be the object of another paper.
From the realized analysis regarding the variation $\Delta \varphi=f\left(\varphi_{1}\right)$, exemplified for the pump F132 and F232, made by S.C. Plopeni S.A., a series of interesting conclusions may be drawn, which are farther presented.

- The lagging angle $\Delta \varphi$ depends on the geometrical, constructive sizes of the pumps, $R_{f}, R_{c}, l, \delta_{M}$ (the last two determining $p_{M}$ ) and $\gamma$, and is variable accordingly to the rotary angle of the driving shaft.
- The initial lagging angle ( $\Delta \varphi_{I}$ ) depends also on the same sizes.
- The initial lagging angle is null only for a certain tipple angle $\gamma_{M}$ performing the condition (19) and depending on $R_{f}, R_{c}, l$ and on the contact angle between the connecting rod and the piston cup $\left(\delta_{M}\right)$. In case of pumps F132 and F232, this angle is of $26.7^{\circ}$, having no importance, because the maximal tipple angle of these pumps is $25^{\circ}$.
- In case of pumps having a variable capacity, obtained by changing the cylinder block inclination angle ( $\gamma$ ) between a minimal value ( $\gamma_{m}$, usually $0^{\circ}$ ) and a maximal one ( $\gamma_{M}$, usually $25^{\circ}$ ), there is an angle $\gamma_{0}$, for which the initial lagging angle $\left(\Delta \varphi_{I}\right)$ is minimal $\left(\Delta \varphi_{I . m}\right)$ or maximal $\left(\Delta \varphi_{I . M}\right)$. For the pump F232, $\Delta \varphi_{I . m}$ is $2.151^{\circ}$ (for $\gamma_{01}=$ $0^{\circ}$ ) and $\Delta \varphi_{I . M}$ is $3.586^{\circ}\left(\right.$ for $\left.\gamma_{02}=17.989^{\circ}\right)$.
- The variation laws $\Delta \varphi=f\left(\varphi_{1}\right)$, represented for different values of tipple angle, show that the lagging angle between the cylinder block and the driving flange will change during the piston stroke, presenting at the beginning a decrease from $\Delta \varphi_{I}$ (for $\varphi_{1}=0^{\circ}$ ) to a minimal value $\left(\Delta \varphi_{m}\right)$, then, an increase until a maximal value $\left(\Delta \varphi_{M}\right)$, and again then, in the last part of the piston stroke, a decrease until $\Delta \varphi_{S}\left(\right.$ for $\left.\varphi_{1}=180^{\circ}\right)$.
- It may be seen that $\Delta \varphi_{S}=\Delta \varphi_{I}$, but the values of the two lagging angles are different in dependence on the tipple angle, and namely, there is an increase of $\Delta \varphi_{I}$ from $2.151^{\circ}$ (for $\gamma=0^{\circ}$ ) until the maximal value $3.586^{\circ}$ (for $\gamma=17.989^{\circ}$ ) and then a decrease until $2.286^{\circ}$ (for $\gamma_{M}=25^{\circ}$ ) and $0^{\circ}$, respectively, for $\gamma_{M}=26.7^{\circ}$. Also, both $\Delta \varphi_{\text {I.m }}$ and $\Delta \varphi_{I . M}$ will increase as far as $\gamma$ increases. At $\varphi_{1} \in\left\{90^{\circ} ; 270^{\circ}\right\}, \Delta \varphi$ has the same value, irrespective of $\gamma$, being $2.151^{\circ}$, for F232.
- The function $\Delta \varphi=f\left(\varphi_{1}\right)$ allows a minimum (for each of the piston strokes), noted with $\Delta \varphi_{m}$, and a maximum (also, for each of the piston strokes), noted with $\Delta \varphi_{M}$. In this way, for the pump F232, $\Delta \varphi_{m}=0.767^{\circ}$ (for $\varphi_{1.0 .1}=$ $45.576^{\circ}$ and $\varphi_{1.0 .2}=225.576^{\circ}$ ) and $\Delta \varphi_{M}=6.403^{\circ}\left(\right.$ for $\varphi_{1.0 .3} \cong 137^{\circ}$ and $\varphi_{1.0 .4} \cong 317^{\circ}$ ).
The study performed in the frame of this paper allows to determine the variation law $\Delta \varphi=f\left(\varphi_{1}\right)$ for the mechanism having $z$ pistons, on its basis can be carried out the accurate kinematics study of APPTRMCR, taking into account the peculiarity of the rotation motion transmitting from the driving shaft to the cylinder block by means of the driving connecting rod, performing this role at the given moment.


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