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AN OPTIMAL CONTROL FOR NONLINEAR ACTIVE SUSPENSION SYSTEM

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Abstract: *This paper studies an optimal control strategy for nonlinear active suspension. The nonlinear model of car suspension includes electro-hydraulic actuators and the nonlinear characteristics of damping and springs.*
Keywords: *nonlinear system, optimal control, active suspension*

1. INTRODUCTION

The suspension system is classified as a passive, semi-active, and active suspension, according to its ability to add or extract energy. An automotive active suspension control has been one of the favorite subject in the automotive research area. The advantages of an automotive active suspension system have been promised for many years. The objectives of control scheme are to improve the ride quality and handling performance within a given suspension stroke limitation. The ride quality is measured by the vertical acceleration of the vehicle body called sprung mass. The handling performance is determined by the tire deflection, which is the difference of position between the wheel and the road surface input.

This paper will analyze the aspects of nonlinear active suspensions. The linear feedback control strategies for nonlinear systems are selected to control an actuator in active suspension. The result shows that the ride quality can be improved using an active suspension.

In any vehicle suspension system, there are variety of performance parameters to be considered. The most important parameters are [4], [5], and [7]:

- Ride comfort; is directly related to the acceleration sensed by passengers when traveling on a rough road.
- Body motion; which are known as bounce, pitch and roll of the sprung mass are created primarily by cornering and braking maneuvers.
- Road handling; is associated with the contact forces of the tires and the road surface.
- Suspension travel; refers to the relative displacement between the sprung and the unsprung masses.

The advantage of controlled suspension is that a better set of design trade-offs are possible compared with passive suspension.

2. NONLINEAR SUSPENSION MODEL

Since many of the proposed electronic suspension being considered today are independent, i.e. using local sensor information and control law, the quarter car model show in Fig. 1 has been considered in this paper.

We used the following notation: m_{us} is the equivalent unsprung mass consisting of the wheel and its moving parts; m_s is the sprung mass, i.e., the part of the whole body mass and the load mass pertaining to only one wheel; k_t is the elastic constant of the tire, whose damping characteristics have been neglected.

2.1 Nonlinear force characteristics

The real physical systems always include nonlinear components which must be taken into consideration. The suspension force generated by the hydraulic actuator is inherently nonlinear; thedynamic characteristics of suspension components, i.e., dampings and springs, have nonlinear properties. Suspension consists of a nonlinear spring, a damper and a hydraulic actuator to generate a pushing force between the body and axle. The nonlinear suspension stiffness is denoted by k_s and nonlinear suspension damping is denoted by c_s .

Firstly, the nonlinear characteristics of the damping and the spring are examined. The nonlinear damping characteristics are not symmetric with respect to positive and negative relative velocity.
When moving upwards the wheel generates a smaller damping force than when moving downwards. The nonlinearity allows an upward bump from the road profile to have a small impact on the car body, while vertical wheel oscillations are still effectively damped during the downward movement of the wheel [4].
The suspension damping forces:

$$F_{c_s} = c_s^l(\dot{x}_u - \dot{x}_s) - c_s^{sym}|\dot{x}_u - \dot{x}_s| + c_s^{nl}|\dot{x}_u - \dot{x}_s|^{1/2}\operatorname{sgn}(\dot{x}_u - \dot{x}_s) \quad \textbf{(1)}$$

Parts of the nonlinear suspension damping c_s are c_s^l, c_s^{nl} and c_s^{sym}. The c_s^l coefficient effects the damping force linearly while c_s^{nl} has nonlinear impact on the damping characteristics. The c_s^{sym} describes the asymmetric behavior of the characteristic.
The suspension spring forces are:

$$F_{k_s} = k_s^l(x_u - x_s) + k_s^{nl}(x_u - x_s)^3 \quad \textbf{(2)}$$

The tire stiffness can be considered as a linear spring so the tire force is:

$$F_{k_t} = k_t(x_u - x_0) \quad \textbf{(3)}$$

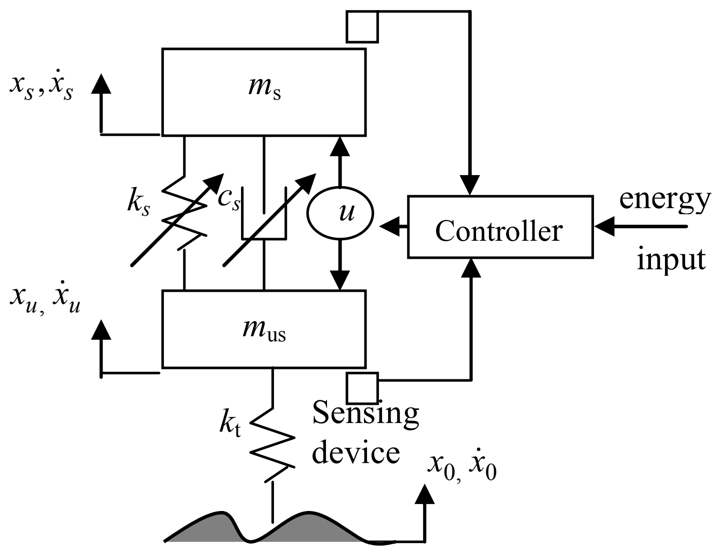

Figure 1: Nonlinear quarter car model and control configuration

Using the nonlinear characteristics of springs and dampers, the force balance equations acting on suspension are as follows:

$$\begin{aligned} m_s\ddot{x}_s &= F_{k_s} + F_{c_s} - u \\ m_u\ddot{x}_u &= -F_{k_s} - F_{c_s} - F_{k_t} + u \end{aligned} \quad \textbf{(4)}$$

If we neglect the last term, $c_s^{nl}|\dot{x}_u - \dot{x}_s|^{1/2}\operatorname{sgn}(\dot{x}_u - \dot{x}_s)$ of (3) we obtain the equation of motion of the vehicle active suspension

$$\begin{aligned} m_s\ddot{x}_s &= k_s^l(x_u - x_s) + k_s^{nl}(x_u - x_s)^3 + c_s^l(\dot{x}_u - \dot{x}_s) - c_s^{sym}|\dot{x}_u - \dot{x}_s| - u \\ m_u\ddot{x}_u &= -k_s^l(x_u - x_s) + k_s^{nl}(x_u - x_s)^3 - c_s^l(\dot{x}_u - \dot{x}_s) - c_s^{sym}|\dot{x}_u - \dot{x}_s| - k_t(x_u - x_0) + u \end{aligned} \quad \textbf{(5)}$$

3. LINEAR DESIGN FOR NONLINEAR SYSTEMS

We consider the nonlinear controlled system

$$\dot{x} = A(t)x + H(x)x + Bu, \; x(0) = x_0 \quad \textbf{(6)}$$

where $x \in R^n$ is a state vector, $A(t) \in R^{n\times n}$ is a bounded matrix, which elements are time dependent, $B \in R^{n\times m}$ is a constant matrix, $u \in R^m$ is a control vector, and $H(x) \in R^{n\times n}$ is a bounded matrix, which elements depend nonlinear on *x*.
Next, we present an important result, concerning a control law that guarantees stability for a nonlinear system and minimizes a nonquadratic performance functional [6].
Theorem 1. If there exist matrices *Q(t)* and *R(t)*, positive definite, being *Q* symmetric, such that the matrix:

$$\widetilde{Q}(x,t) = Q(t) - G^T(x)P(t) - P(t)G(x) \qquad \textbf{(7)}$$

is positive definite for the bounded matrix *G,* then the linear feedback control:

$$u = R^{-1}B^T P(t)\,x \qquad \textbf{(8)}$$

is optimal, in order to transfer the non-linear system (6) from an initial to final state: $x(t_f) = 0$ minimizing the functional:

$$J = \int_0^{t_f} (x^T\widetilde{Q}\,x + u^T R\,u)\,dt \qquad \textbf{(9)}$$

where the symmetric matrix *P*(*t*) is evaluated through the solution of
the matrix Riccati differential equation:

$$\dot{P} + PA + A^T P - PBR^{-1}B^T P + Q = 0 \qquad \textbf{(10)}$$

satisfying the final condition: $P(t_f) = 0$.
In addition, with the feedback control (8), there exists a neighborhood $U_0 \subset U, U \subset R^n$, of the origin such that if $x_0 \in U_0$, the solution $x(t) = 0$, $t > 0$, of the controlled system (6) is locally asymptotically stable, and $J_{\min} = x_0^T P(0)\,x_0$.
Finally, if $U = R^n$ then the solution $x(t) = 0$, $t > 0$, of the controlled system (6) is globally asymptotically stable.
We remark that according to the optimal control theory of linear systems with quadratic functional the solution of the nonlinear differential Riccati equation (10) is positive definite and symmetric matrix $P > 0$ for all $R > 0$ and $Q \geq 0$ given, one can conclude the Theorem proof.
If the time interval is infinite and *A, B, Q* and *R* are matrix with constant elements, the positive defined matrix *P* is the solution of the nonlinear, matrix algebraic Riccati equation [7]

$$PA + AT\,P - PBR^{-1}BT\,P + Q = 0 \qquad \textbf{(11)}$$

4. NUMERICAL SIMULATION

The equation of motion of the vehicle active suspension can be written in form (6). Defining the state space components: $x_1 = x_s - x_u$ is suspension deflection (rattle space); $x_2 = \dot{x}_s$ is absolute velocity of sprung mass; $x_3 = x_u - x_0$ is tire deflection; $x_4 = \dot{x}_u$ is absolute velocity of unsprung mass we will obtain the following equations in form of

$$\dot{x} = A(t)\,x + h(x) + B\,u,\; x(0) = x_0 \qquad \textbf{(12)}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{c_s}{m_u} & -\frac{k_t}{m_u} & -\frac{c_s}{m_u} \end{bmatrix}; \quad B = \begin{bmatrix} 0 & \frac{1}{m_s} & 0 & -\frac{1}{m_u} \end{bmatrix}^T; \quad h(x) = \begin{bmatrix} 0 \\ -\frac{k_s^{nl}}{m_s}x_1^3 + \frac{c_s^{sym}}{m_s}|\dot{x}_1| \\ 0 \\ \frac{k_s^{nl}}{m_s}x_1^3 - \frac{c_s^{sym}}{m_s}|\dot{x}_1| \end{bmatrix} \qquad \textbf{(13)}$$

We can present the equation (12) in the form (6) by considering $h(x) = H(x)x$, where

$$H(x) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\frac{k_s^{nl}}{m_s}x_1^2 & \frac{c_s^{sym}}{m_s}\delta(x_2,x_4) & \frac{k_s^{nl}}{m_s}x_1^2 & -\frac{c_s^{sym}}{m_s}\delta(x_2,x_4) \\ 0 & 0 & 0 & 0 \\ \frac{k_s^{nl}}{m_s}x_1^2 & -\frac{c_s^{sym}}{m_s}\delta(x_2,x_4) & -\frac{k_s^{nl}}{m_s}x_1^2 & \frac{c_s^{sym}}{m_s}\delta(x_2,x_4) \end{bmatrix} \quad \textbf{(14)}$$

$$\delta(x_2,x_4) = \begin{cases} 0 & \text{if } x_4 - x_2 = 0 \\ sign(x_4 - x_2) & \text{if } x_4 - x_2 \neq 0 \end{cases}$$

The linear feedback procedure proposed in paragraph 3 has been applied to the quarter car active suspension with the following parameter value [5], [6]:

$$m_s = 240\,Kg, m_u = 36\,Kg, c_s^l = 1000\,N\cdot \sec/m,\ c_s^{sym} = 400N\cdot\sec/m,$$
$$k_s^l = 16000\,N/m, k_t = 160000\,N/m, k_s^l = 160\cdot 10^4\,N/m$$

Figure 2 and Figure 3 show the sprung mass and unsprung mass displacement for numerical simulations.

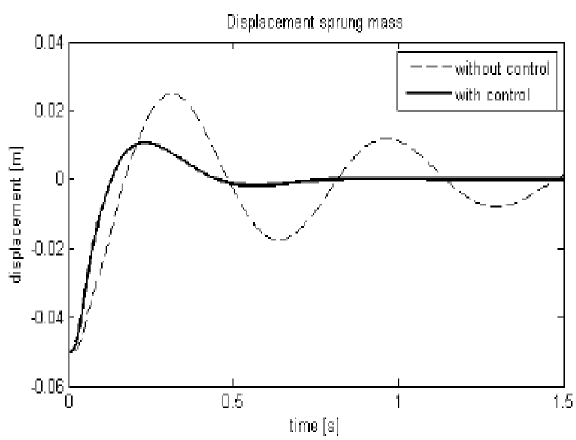

Figure 2: sprung mass displacement

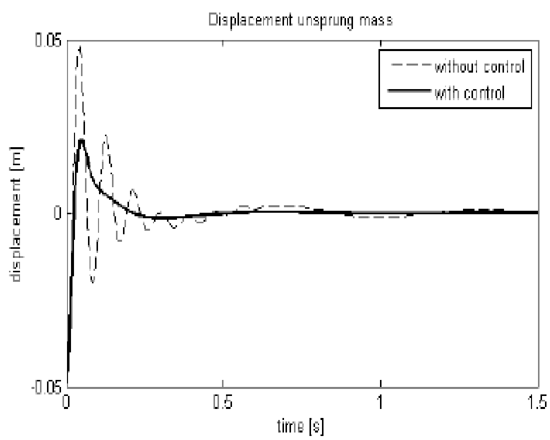

Figure 3: unsprung mass displacement

5. CONCLUSION

In this paper we have presented a framework for designing an optimal linear control for nonlinear active suspension. The optimal linear feedback strategies for nonlinear systems where used to achieve the desired nonlinear response of the vehicle suspension. The numerical simulations are provided the effectiveness of the method.

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