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PLOTTING EFFORT DIAGRAMS IN POLAR COORDINATES FOR CURVED CIRCULAR BEAMS LOADED WITH COPLANAR UNIFORMLY DISTRIBUTED LOADS USING MATHCAD

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Abstract. *Plotting axial, shear and bending efforts for the plane curved circular beam in polar coordinates using computer software means a significant improvement of the student's or engineer's activity to show the variation of those functions. This can be done using the Step FunctionΦ in Mathcad, which allows computation at the left and right of a force(or couple of forces) section. This function can limit a function's representation for a given interval. This actual work presents some numerical results for this type of computations, considering different uniformly distributed loads coplanar with the beam's circular axis.*

Key Words: *effort diagrams, polar coordinates , step-function*

1. MATHEMATIC EXPRESSIONS OF THE AXIAL FORCE N, SHEAR FORCE T AND BENDING MOMENT M, FOR UNIFORMLY DISTRIBUTED RADIAL LOADS

One considers the curved beam with the geometric circular axis of radius R and $3\pi/2$ angle. The beam has a free end at right and a fixed one at left. The loads p are radial and uniformly distributed, coplanar with the beams circular axis, as it can be seen in figure 1.a.

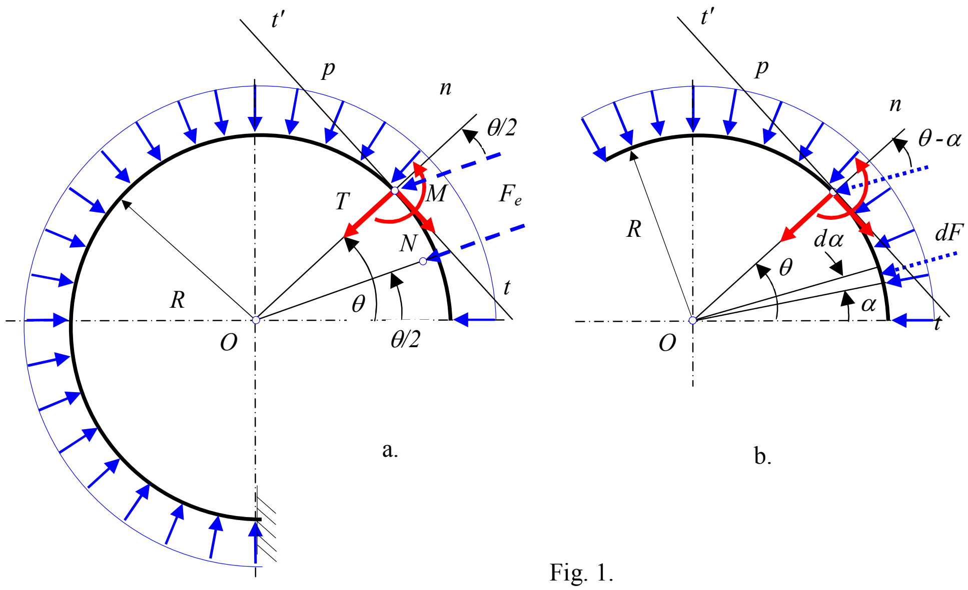

Fig. 1.

The axial force N and shear force T expressions are obtained by projecting the elementary force $dF=p\cdot ds$ on the normal direction *On,* and the tangential one *tt'* and integrating along the entire length of the circle sector having the central angle θ, as shown in fig. 1.b.
The expression of the bending moment *M* can be determined computing the elementary force $dF=p\cdot ds$ moment, with respect to the normal axis *On* and integrating along the length of the circle sector as seen in fig. 1.b.
Taking into account the sectional efforts sign rule for straight bars [4], one obtains:

$$\begin{cases} N(\theta)=-\int_0^\theta (pRd\alpha)\sin(\theta-\alpha)=-pR(1-\cos\theta); \\ T(\theta)=\int_0^\theta (pRd\alpha)\cos(\theta-\alpha)=pR\sin\theta; \\ M(\theta)=-\int_0^\theta (pRd\alpha)R\sin(\theta-\alpha)=-pR^2(1-\cos\theta). \end{cases} \quad (1)$$

One can obtain the same expressions of the efforts by computing the equivalent force F_e corresponding to the beam sector with central angle θ, applied on the direction of the θ angle bisector and with the application point in the gravity center *C* of the circular sector [4], as it can be seen in fig.1.a:

$$F_e=\int_0^\theta p\cos\left(\frac{\theta}{2}-\alpha\right)ds=2pR\sin\left(\frac{\theta}{2}\right) \quad (2)$$

The shear force *T* can be obtained projecting the equivalent force F_e on the normal direction *On* and the tangential one *tt'*, in the point corresponding to the current section *A* on the beam. The bending moment will be obtained as the moment of the equivalent force F_e with respect to a normal axis passing through A, as seen in fig. 1.a:

$$\begin{cases} N=-F_e\sin\left(\frac{\theta}{2}\right)=-pR(1-\cos\theta); \\ T=F_e\cos\left(\frac{\theta}{2}\right)=pR\sin\theta; \\ M=-F_eR\sin\left(\frac{\theta}{2}\right)=-pR^2(1-\cos\theta) \end{cases} \quad . \quad (3)$$

Between the sectional efforts *N, T* and *M* and the exterior loads *p* there are differential relations:

$$\frac{dN}{d\theta}=-T;\quad \frac{dT}{d\theta}=N+p\cdot R;\quad \frac{dM}{d\theta}=-T\cdot R \quad (4)$$

Using the differential relations (2) one can check the expressions of the sectional efforts (1):

$$\begin{cases} N=-pR\cdot(1-\cos\theta); \\ T=pR\cdot\sin\theta; \\ M=-pR^2(1-\cos\theta) \end{cases} \Rightarrow \begin{cases} \frac{dN}{d\theta}=-pR\cdot\sin\theta \\ \frac{dT}{d\theta}=pR\cdot\cos\theta \\ \frac{dM}{d\theta}=-pR^2\sin\theta \end{cases} \quad (5)$$

The efforts diagrams in *polar coordinates* will be plotted on both sides of the beam's geometrical axis, by the same sign rules as in the case of straight beams: positive *N* and *T* are on the exterior side and positive *M* is on the interior side.
This can be done using the step function Φ from Mathcad 14, which allows:

- Limitation of a function representation for a given angular interval
- Representation of the diagram jumps corresponding to concentrated forces (or force couples) by computing efforts at left and right limit of the specific section.

In figures 2, 3 and 4 one plotted the shear force T(θ), axial force N(θ) and bending moment M(θ) diagrams in polar coordinates ρ - θ, using Mathcad 14.
The function representation limitation for the angular interval (0, 3π/2) was done using: L(θ)=Φ(θ) - Φ(θ-3π/2)
The term *4pR* added to the shear and axial forces and $4pR^2$ for the bending moments (fig. 2, 3, 4), given by the relations (3), allowed the displacement of the diagrams origin in the point (*4pR,0*) and their representation on both sides of the beam's geometrical axis [4].

$R := 1$

$p := 1$

$$\mathrm{Axis}(\theta) := 4 \cdot R\left(\Phi(\theta) - \Phi\left(\theta - 3\frac{\pi}{2}\right)\right)$$

$$T(\theta) := (4 \cdot p \cdot R + p \cdot R \cdot \sin(\theta)) \cdot \left(\Phi(\theta) - \Phi\left(\theta - 3 \cdot \frac{\pi}{2}\right)\right)$$

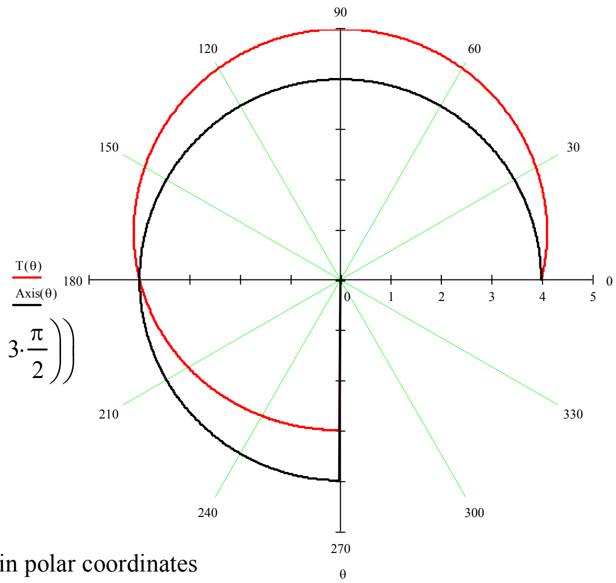

Fig. 2. Shear force diagram in polar coordinates

$$N(\theta) := [4 \cdot p \cdot R - p \cdot R \cdot (1 - \cos(\theta))] \cdot \left(\Phi(\theta) - \Phi\left(\theta - 3 \cdot \frac{\pi}{2}\right)\right)$$

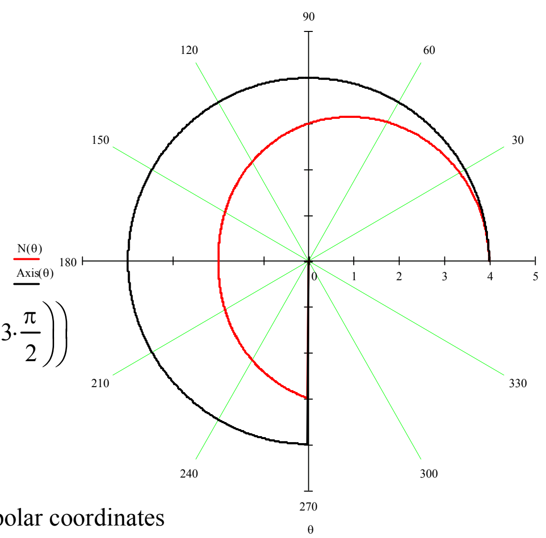

Fig. 3. Axial force diagram in polar coordinates

$$M(\theta) := \left[4 \cdot p \cdot R^2 - p \cdot R^2 \cdot (1 - \cos(\theta))\right] \cdot \left(\Phi(\theta) - \Phi\left(\theta - 3 \cdot \frac{\pi}{2}\right)\right)$$

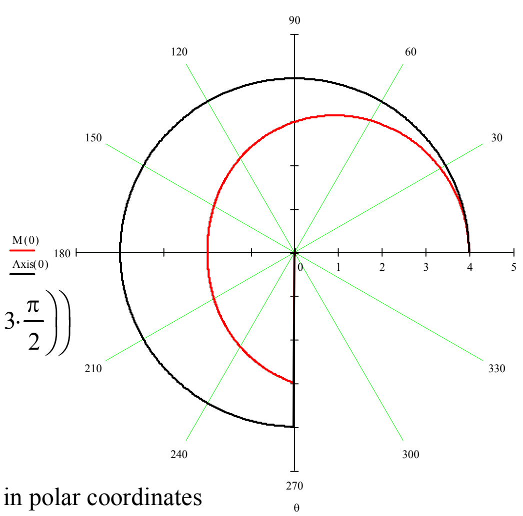

Fig. 4. Bending moment diagram in polar coordinates

2. MATHEMATIC EXPRESSIONS OF THE AXIAL FORCE N, SHEAR FORCE T AND BENDING MOMENT M, FOR UNIFORMLY DISTRIBUTED VERTICAL LOADS

One considers the curved beam with the geometric circular axis of radius R and $3\pi/2$ angle. The beam has a free end at right and a fixed one at left. The loads p are vertical and uniformly distributed, coplanar with the beams circular axis, as it can be seen in figure 5.a.

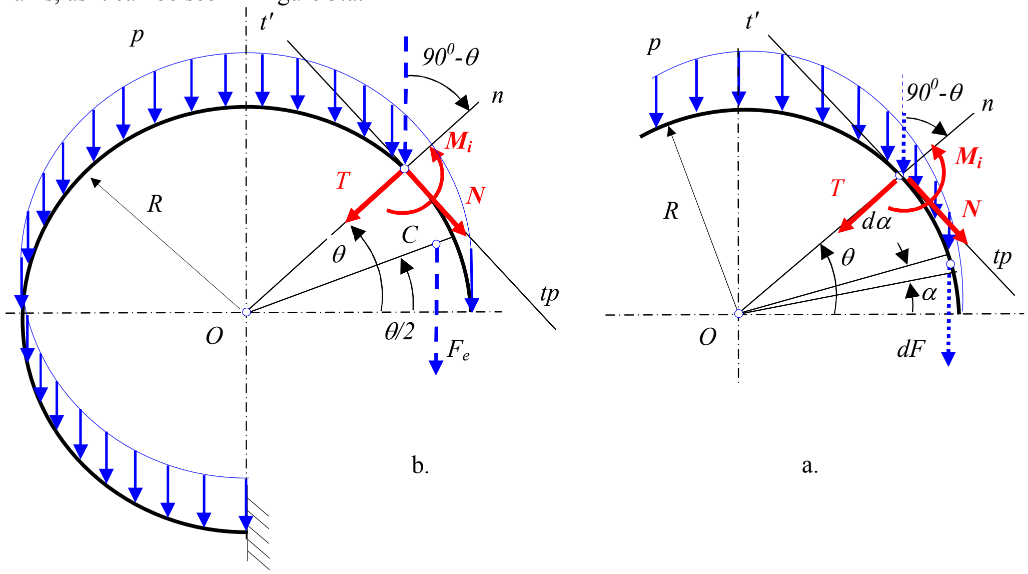

Fig. 5

The axial force N and shear force T expressions are obtained by projecting the elementary force $dF=p\cdot ds$ on the normal direction *On,* and the tangential one *tt'* and integrating along the entire length of the circle sector having the central angle θ, as shown in fig. 5.b [4].

The expression of the bending moment M_i can be determined computing the elementary force $dF=p\cdot ds$ moment, with respect to the normal axis *On* and integrating along the length of the circle sector as seen in fig. 5.b.

Taking into account the sectional efforts sign rule for straight bars [4], one obtains:

$$\begin{cases} N(\theta)=\int_0^\theta (pRd\alpha)\cos\theta = pR\cdot\theta\cdot\cos\theta; \\ T(\theta)=\int_0^\theta (pRd\alpha)\sin\theta = pR\cdot\theta\cdot\sin\theta; \\ M(\theta)=-\int_0^\theta (pRd\alpha)\cdot(R\cos\alpha - R\cos\theta) = pR^2(\theta\cdot\cos\theta-\sin\theta). \end{cases} \tag{6}$$

One can obtain the same expressions of the efforts by computing the equivalent force F_e corresponding to the beam sector with central angle θ, applied on the direction of the θ angle bisector and with the application point in the gravity center C of the circular sector, as it can be seen in fig.5.a: $F_e=\int_0^\theta pds = pR\cdot\theta$ (2)

The axial and shear force N and T can be obtained projecting the equivalent force F_e on the normal direction *On* and the tangential one *tt'*, in the point corresponding to the current section *A* on the beam. The bending moment will be obtained as the moment of the equivalent force F_e with respect to a normal axis passing through A, as seen in fig. 5.a:

$$\begin{cases} N = F_e\sin(90-\theta) = pR\cdot\theta\cdot\cos\theta; \\ T = F_e\cos(90-\theta) = pR\cdot\theta\cdot\sin\theta; \\ M \;= -F_e\left[-R\cos\theta+\frac{2R}{\theta}\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right] = pR^2(\theta\cdot\cos\theta-\sin\theta); \end{cases} \tag{8}$$

In figures 6, 7 and 8 one plotted the shear force $T(\theta)$, axial force $N(\theta)$ and bending moment $M(\theta)$ diagrams in polar coordinates ρ - θ, using Mathcad 14 [4].

$$p := 1$$

$$R := 1$$

$$Axis(\theta) := 4R\left(\Phi(\theta) - \Phi\left(\theta - 3\frac{\pi}{2}\right)\right)$$

$$T(\theta) := (6 \cdot p \cdot R + p \cdot R \cdot \theta \sin(\theta)) \cdot \left(\Phi(\theta) - \Phi\left(\theta - 3 \cdot \frac{\pi}{2}\right)\right)$$

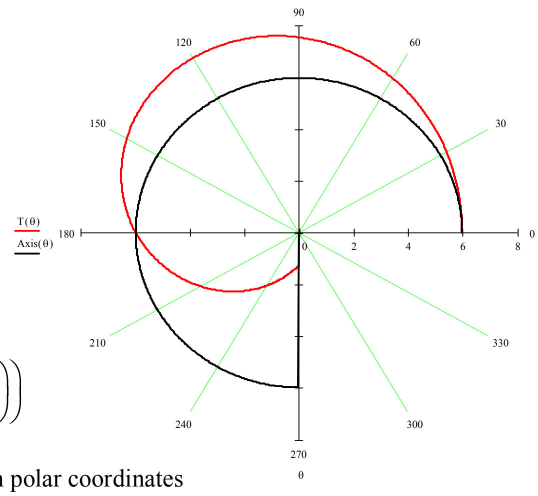

Fig. 6. Shear force diagram in polar coordinates

$$N(\theta) := (4 \cdot p \cdot R + p \cdot R \cdot \theta \cdot \cos(\theta)) \cdot \left(\Phi(\theta) - \Phi\left(\theta - 3 \cdot \frac{\pi}{2}\right)\right)$$

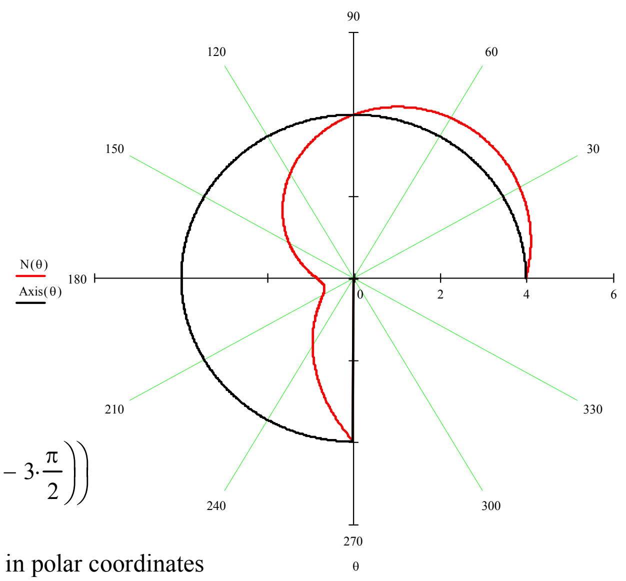

Fig. 7. Axial force diagram in polar coordinates

$$M(\theta) := \left[4 \cdot p \cdot R^2 + p \cdot R^2 \cdot (\theta \cdot \cos(\theta) - \sin(\theta))\right] \cdot \left(\Phi(\theta) - \Phi\left(\theta - 3 \cdot \frac{\pi}{2}\right)\right)$$

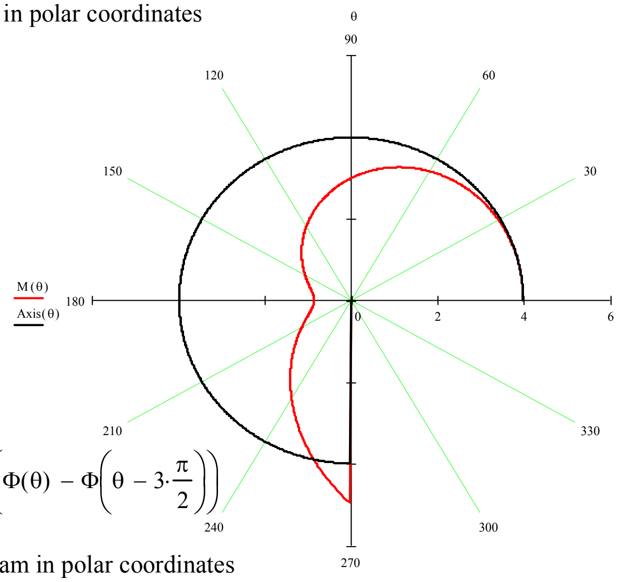

Fig. 8. Bending moment diagram in polar coordinates

3. CONCLUSIONS

- If one compared the diagrams obtained in the first case of radial loads, for the two efforts *N* and *M*, one can notice they are identical, because the corresponding analytical expressions (3) are the same, excepting the factor *R*
- If one compared the expressions N, T and M for the two loading cases one can notice that in the first case they are actually trigonometric constant functions sums. In the second case products between trigonometric and linear functions of θ appear
- In order to plot the efforts diagram one used the analytical expressions determined for each case, multiplied with the factor $L(\theta)=\Phi(\theta)-\Phi(\theta-3\pi/2)$, which actually limits the function representation in the interval $[0, 3\pi/2]$;
- In order to plot the circular axis, one used the constant function Axis(θ), multiplied with the same factoring function $L(\theta)=\Phi(\theta)-\Phi(\theta-3\pi/2)$, which actually limits the function representation in the interval $[0, 3\pi/2]$;
- The above presented method is really easy to approach and allows the plotting of the efforts diagrams for uniformly distributed loads and for concentrated ones as well.

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